Computer Algebra
Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.1-Hyperbolic-sine/161-6.1.3-e-x-^m-a+b-sinh-c+d-x^n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [102]. This is test number [161].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (102)	0.00 (0)
Mathematica	100.00 (102)	0.00 (0)
Maxima	82.35 (84)	17.65 (18)
Maple	78.43 (80)	21.57 (22)
Fricas	76.47 (78)	23.53 (24)
Giac	52.94 (54)	47.06 (48)
Sympy	31.37 (32)	68.63 (70)
Mupad	28.43 (29)	71.57 (73)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

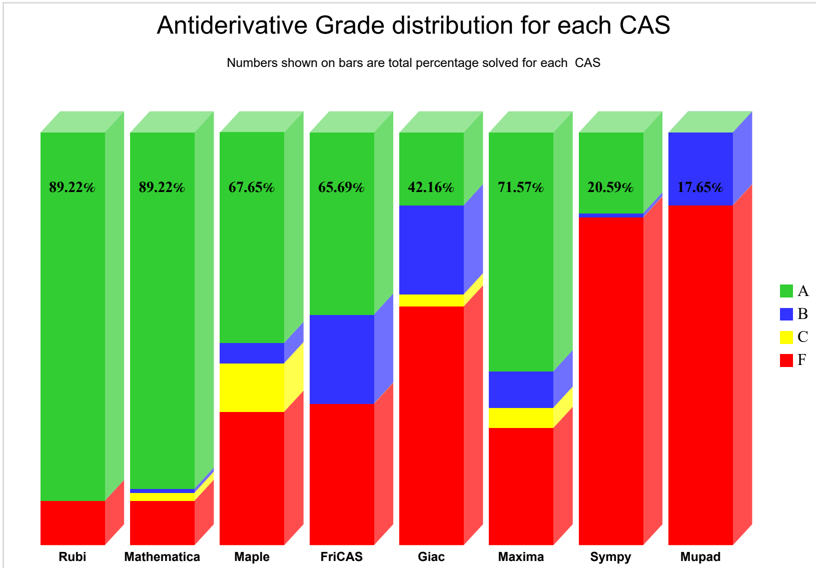
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

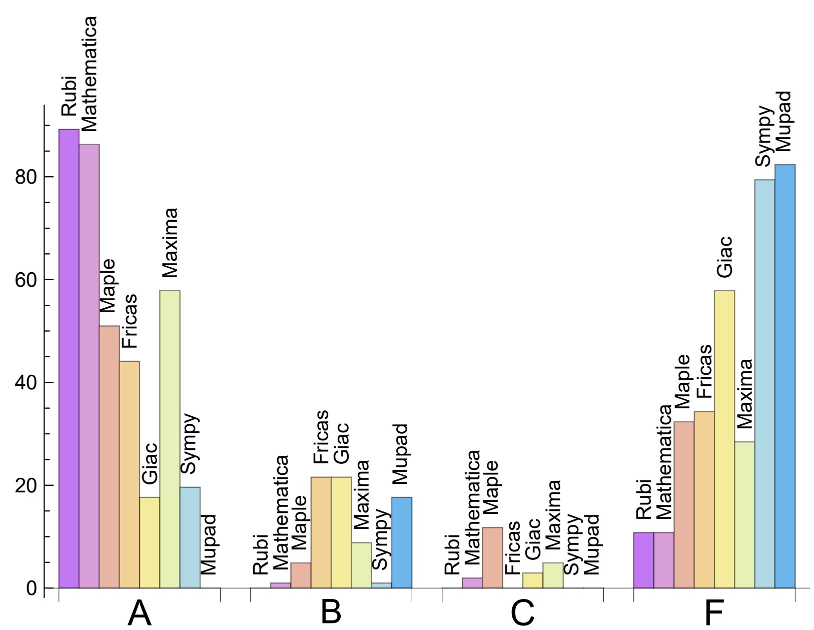
System	% A grade	% B grade	% C grade	% F grade
Rubi	89.216	0.000	0.000	10.784
Mathematica	86.275	0.980	1.961	10.784
Maxima	57.843	8.824	4.902	28.431
Maple	50.980	4.902	11.765	32.353
Fricas	44.118	21.569	0.000	34.314
Sympy	19.608	0.980	0.000	79.412
Giac	17.647	21.569	2.941	57.843
Mupad	0.000	17.647	0.000	82.353

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maxima	18	100.00	0.00	0.00
Maple	22	100.00	0.00	0.00
Fricas	24	100.00	0.00	0.00
Giac	48	100.00	0.00	0.00
Sympy	70	100.00	0.00	0.00
Mupad	73	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.08
Fricas	0.24
Maxima	0.27
Giac	0.61
Mathematica	0.77
Maple	0.88
Mupad	0.99
Sympy	3.32

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	32.45	1.06	22.00	1.08
Sympy	46.75	1.14	22.00	1.03
Mathematica	76.62	0.94	63.50	0.93
Rubi	87.50	1.00	68.00	1.00
Maxima	98.83	1.47	60.00	0.97
Maple	104.04	1.19	66.00	1.09
Fricas	140.92	1.77	70.00	1.41
Giac	166.72	2.10	61.00	1.56

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

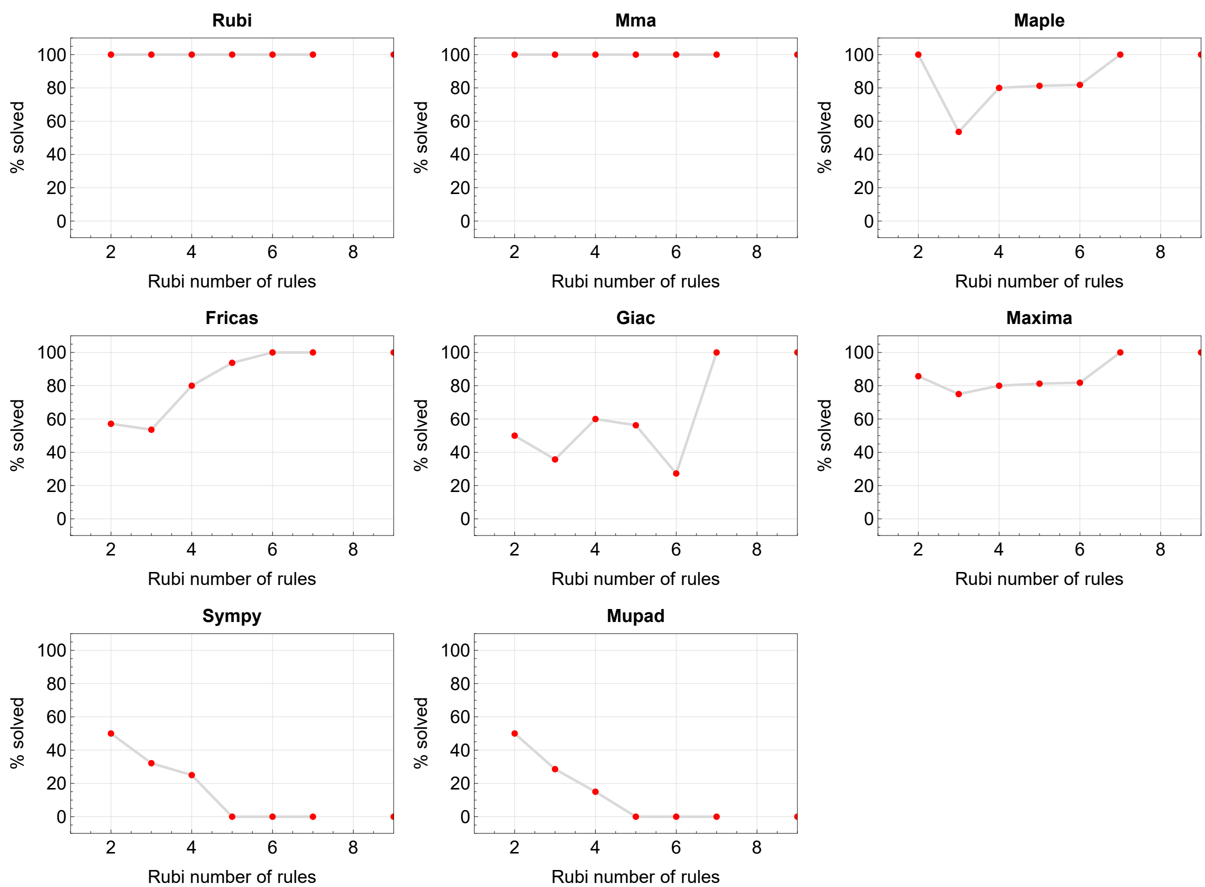


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

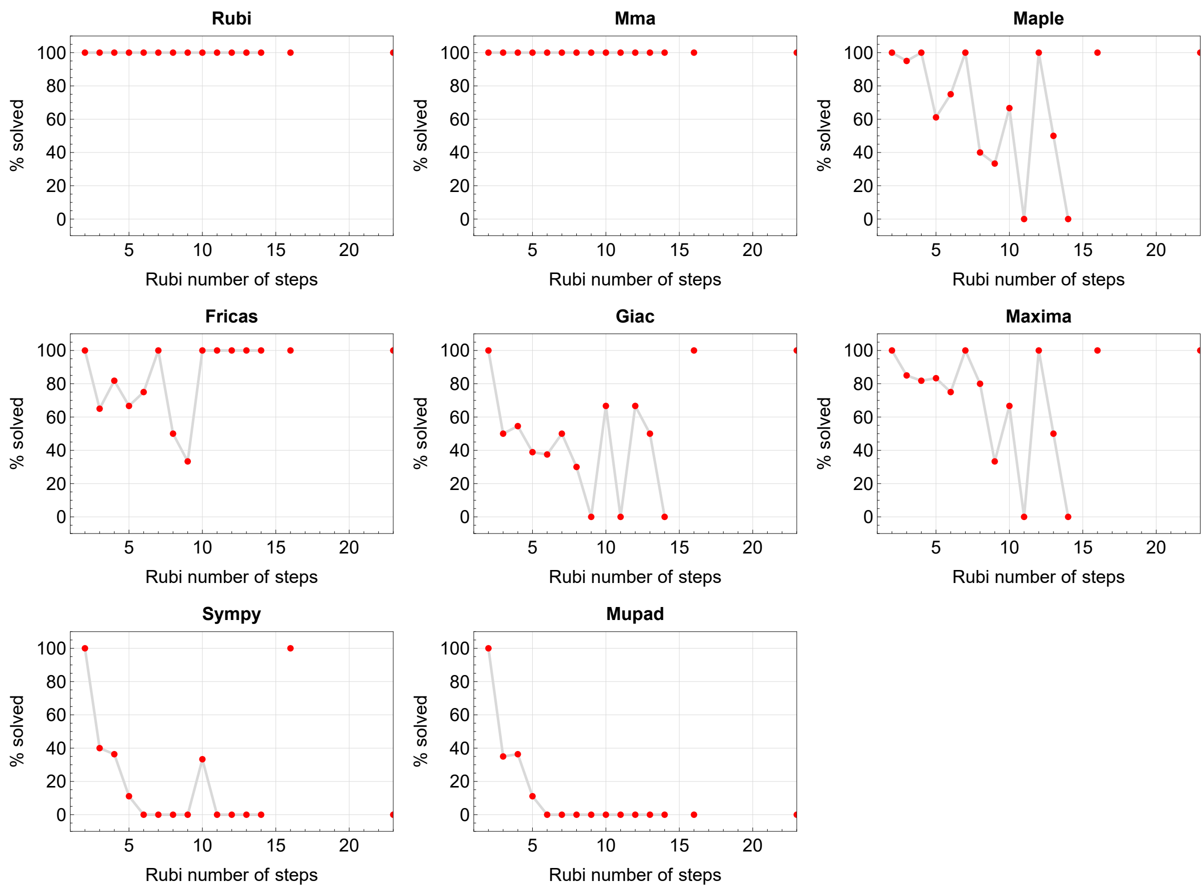


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

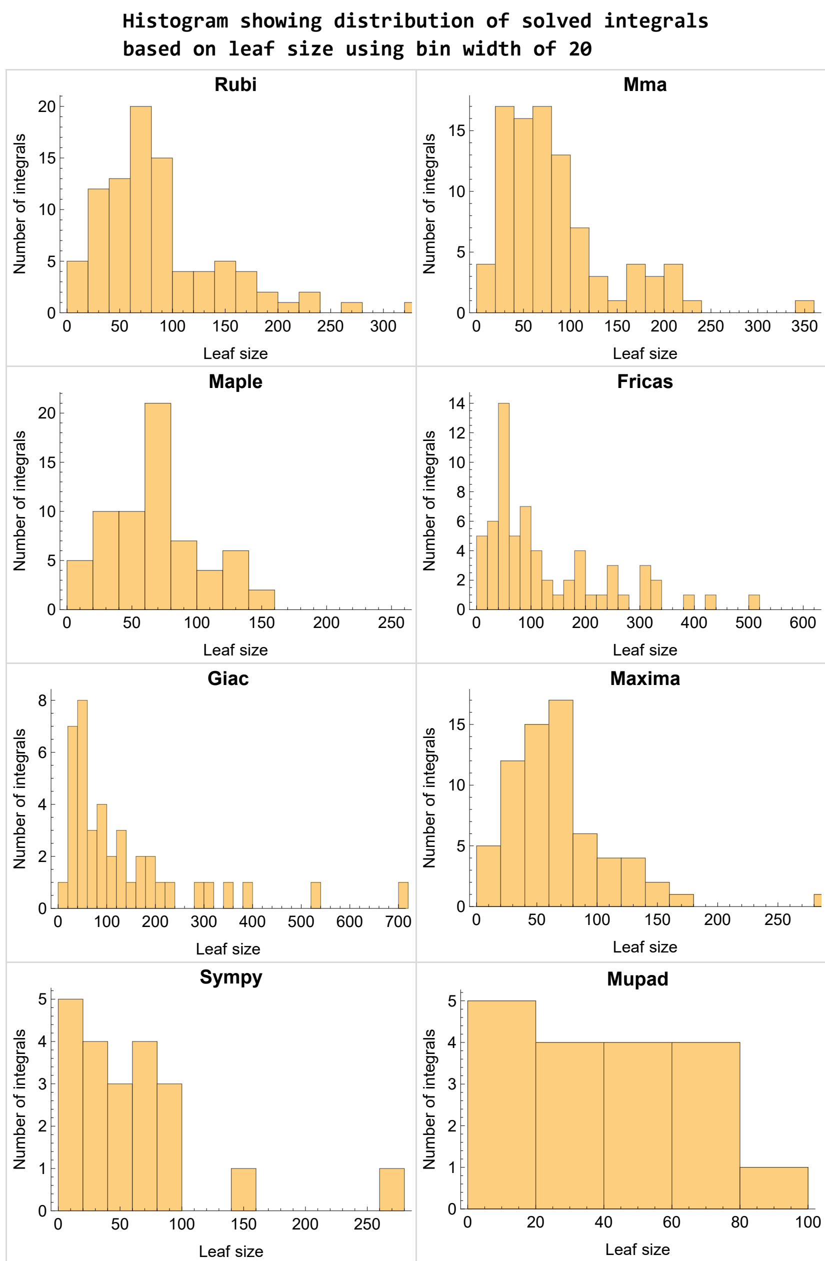


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

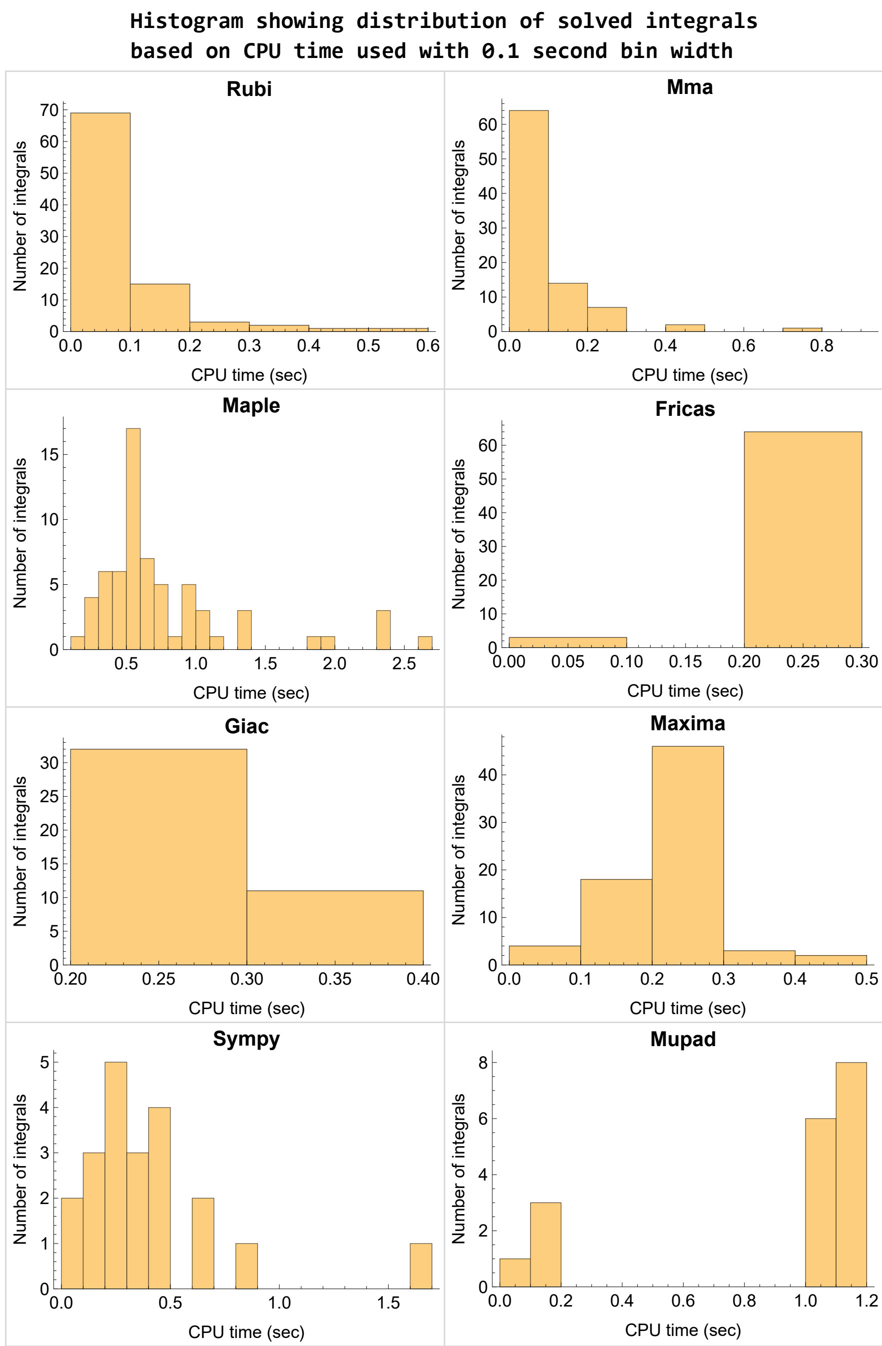


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

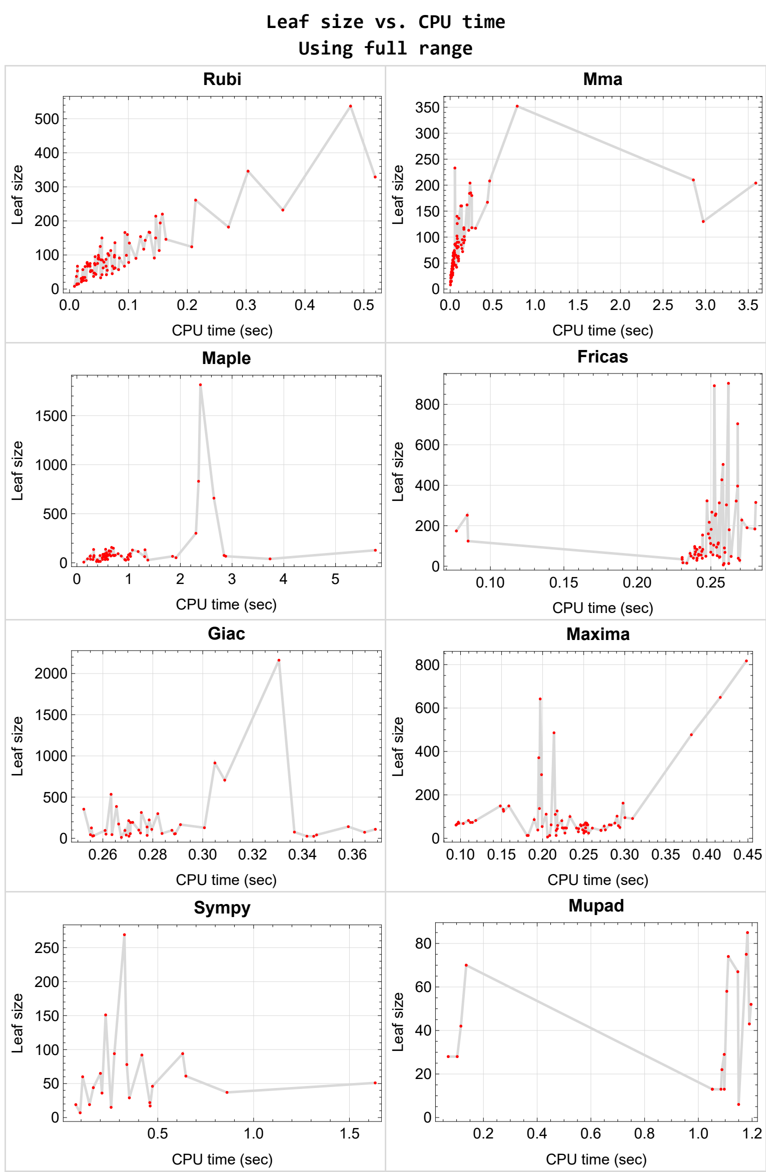


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{23, 27, 40, 56, 74, 75, 77, 79, 83, 91, 92\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

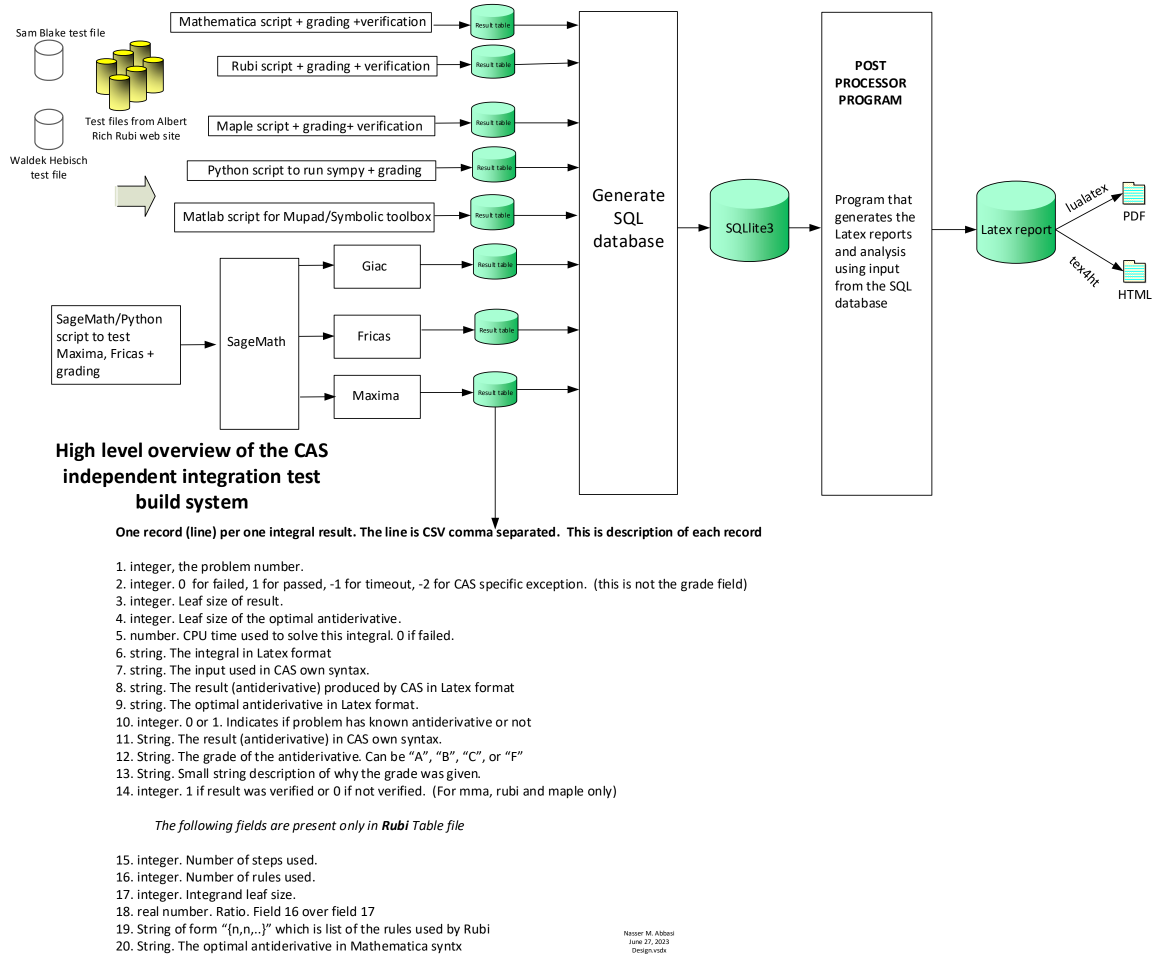
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 78, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 78, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100 }

B grade { 3 }

C grade { 101, 102 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 57, 61, 67, 72, 84, 85, 86, 87, 95, 100 }

B grade { 42, 93, 94, 98, 99 }

C grade { 26, 39, 55, 58, 59, 60, 62, 63, 82, 88, 89, 90 }

F normal fail { 24, 25, 37, 38, 53, 54, 64, 65, 66, 68, 69, 70, 71, 73, 76, 78, 80, 81, 96, 97, 101, 102 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 3, 4, 5, 7, 8, 10, 11, 12, 14, 15, 17, 18, 19, 21, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 42, 44, 46, 47, 48, 50, 52, 57, 67, 72, 87, 90, 93, 94, 95, 98, 99, 100 }

B grade { 2, 6, 9, 13, 16, 20, 22, 41, 43, 45, 49, 51, 61, 84, 85, 86, 88, 89, 96, 97, 101, 102 }

C grade { }

F normal fail { 37, 38, 39, 53, 54, 55, 58, 59, 60, 62, 63, 64, 65, 66, 68, 69, 70, 71, 73, 76, 78, 80, 81, 82 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 28, 29, 30, 31, 32, 33, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 84, 85, 86, 87, 93, 94, 98, 99, 100 }

B grade { 1, 2, 4, 17, 22, 88, 89, 90, 95 }

C grade { 34, 35, 36, 50, 52 }

F normal fail { 24, 25, 26, 37, 38, 39, 53, 54, 55, 76, 78, 80, 81, 82, 96, 97, 101, 102 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 2, 3, 4, 5, 9, 11, 12, 16, 17, 18, 19, 22, 28, 50, 57, 87, 95, 100 }

B grade { 1, 7, 8, 10, 14, 15, 21, 29, 30, 31, 32, 33, 34, 35, 36, 42, 44, 48, 93, 94, 98, 99 }

C grade { 88, 89, 90 }

F normal fail { 6, 13, 20, 24, 25, 26, 37, 38, 39, 41, 43, 45, 46, 47, 49, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 78, 80, 81, 82, 84, 85, 86, 96, 97, 101, 102 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 3, 8, 10, 15, 17, 22, 28, 33, 34, 35, 36, 48, 50, 52, 57, 95, 100 }

C grade { }

F normal fail { }

F(-1) timedout fail { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 24, 25, 26, 29, 30, 31, 32, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 51, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 78, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 93, 94, 96, 97, 98, 99, 101, 102 }

F(-2) exception fail { }

Sympy

A grade { 1, 3, 8, 15, 17, 22, 28, 32, 33, 34, 35, 36, 48, 50, 52, 57, 93, 94, 95, 100 }

B grade { 10 }

C grade { }

F normal fail { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 24, 25, 26, 29, 30, 31, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 51, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 78, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 96, 97, 98, 99, 101, 102 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	30	81	29	36	73	28
N.S.	1	1.00	0.91	0.88	2.38	0.85	1.06	2.15	0.82
time (sec)	N/A	0.026	0.030	0.395	0.223	0.270	0.209	0.365	0.102

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	67	74	110	190	0	75	0
N.S.	1	1.00	0.97	1.07	1.59	2.75	0.00	1.09	0.00
time (sec)	N/A	0.031	0.052	0.330	0.215	0.275	0.000	0.337	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	31	14	13	13	19	25	13
N.S.	1	1.00	2.07	0.93	0.87	0.87	1.27	1.67	0.87
time (sec)	N/A	0.012	0.015	0.379	0.209	0.259	0.072	0.342	1.052

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	45	40	86	48	0	41	0
N.S.	1	1.00	0.85	0.75	1.62	0.91	0.00	0.77	0.00
time (sec)	N/A	0.013	0.029	0.205	0.190	0.264	0.000	0.346	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	33	24	39	0	24	0
N.S.	1	1.00	0.92	1.32	0.96	1.56	0.00	0.96	0.00
time (sec)	N/A	0.024	0.012	0.310	0.250	0.269	0.000	0.344	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	70	54	184	0	0	0
N.S.	1	1.00	1.06	1.06	0.82	2.79	0.00	0.00	0.00
time (sec)	N/A	0.026	0.047	0.261	0.199	0.280	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	38	59	39	71	0	109	0
N.S.	1	1.00	0.90	1.40	0.93	1.69	0.00	2.60	0.00
time (sec)	N/A	0.063	0.030	0.323	0.276	0.240	0.000	0.369	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	42	55	59	56	78	141	42
N.S.	1	1.00	0.82	1.08	1.16	1.10	1.53	2.76	0.82
time (sec)	N/A	0.035	0.076	0.542	0.218	0.251	0.339	0.358	0.116

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	101	90	95	427	0	97	0
N.S.	1	1.00	1.02	0.91	0.96	4.31	0.00	0.98	0.00
time (sec)	N/A	0.067	0.165	0.533	0.300	0.257	0.000	0.288	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	38	29	60	56	22
N.S.	1	1.00	0.87	1.10	1.23	0.94	1.94	1.81	0.71
time (sec)	N/A	0.020	0.030	0.595	0.194	0.240	0.108	0.289	1.088

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	86	51	56	73	0	58	0
N.S.	1	1.00	1.10	0.65	0.72	0.94	0.00	0.74	0.00
time (sec)	N/A	0.030	0.055	0.490	0.275	0.244	0.000	0.284	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	34	31	49	0	35	0
N.S.	1	1.00	0.89	0.92	0.84	1.32	0.00	0.95	0.00
time (sec)	N/A	0.041	0.016	0.496	0.253	0.247	0.000	0.278	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	94	86	61	396	0	0	0
N.S.	1	1.00	1.07	0.98	0.69	4.50	0.00	0.00	0.00
time (sec)	N/A	0.049	0.169	0.502	0.283	0.268	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	66	36	90	0	126	0
N.S.	1	1.00	0.81	1.16	0.63	1.58	0.00	2.21	0.00
time (sec)	N/A	0.083	0.064	0.510	0.271	0.259	0.000	0.255	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	58	93	100	94	92	192	70
N.S.	1	1.00	0.73	1.18	1.27	1.19	1.16	2.43	0.89
time (sec)	N/A	0.058	0.099	0.796	0.233	0.254	0.417	0.271	0.136

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	184	157	162	904	0	166	0
N.S.	1	1.00	1.15	0.98	1.01	5.65	0.00	1.04	0.00
time (sec)	N/A	0.098	0.227	0.671	0.298	0.262	0.000	0.291	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	62	46	44	56	28
N.S.	1	1.00	1.00	0.85	1.88	1.39	1.33	1.70	0.85
time (sec)	N/A	0.023	0.021	1.371	0.210	0.256	0.164	0.271	0.069

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	136	86	91	112	0	95	0
N.S.	1	1.00	1.09	0.69	0.73	0.90	0.00	0.76	0.00
time (sec)	N/A	0.052	0.102	0.631	0.310	0.250	0.000	0.261	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	69	50	83	0	50	0
N.S.	1	1.00	0.89	1.25	0.91	1.51	0.00	0.91	0.00
time (sec)	N/A	0.064	0.027	0.858	0.294	0.241	0.000	0.261	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	204	149	102	892	0	0	0
N.S.	1	1.00	1.50	1.10	0.75	6.56	0.00	0.00	0.00
time (sec)	N/A	0.077	0.233	0.705	0.291	0.252	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	90	123	58	160	0	223	0
N.S.	1	1.00	0.99	1.35	0.64	1.76	0.00	2.45	0.00
time (sec)	N/A	0.144	0.083	0.621	0.292	0.248	0.000	0.279	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	52	126	154	94	108	52
N.S.	1	1.00	1.00	0.78	1.88	2.30	1.40	1.61	0.78
time (sec)	N/A	0.035	0.047	1.916	0.218	0.244	0.630	0.280	1.196

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12
time (sec)	N/A	0.014	1.780	0.197	0.345	0.246	3.187	0.364	1.151

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	180	0	0	252	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.146	0.254	0.000	0.000	0.084	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	113	0	0	174	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.102	0.218	0.000	0.000	0.077	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	82	77	0	124	0	0	0
N.S.	1	1.00	0.86	0.81	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.043	0.089	0.701	0.000	0.085	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	16	18	18	14	18	18
N.S.	1	1.00	1.14	1.14	1.29	1.29	1.00	1.29	1.29
time (sec)	N/A	0.019	2.259	0.312	0.413	0.238	0.466	0.297	1.104

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	19	25	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.27	1.67	0.87
time (sec)	N/A	0.015	0.015	0.414	0.182	0.262	0.144	0.270	1.084

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	130	47	93	0	534	0
N.S.	1	1.00	0.90	1.67	0.60	1.19	0.00	6.85	0.00
time (sec)	N/A	0.100	0.047	1.074	0.229	0.243	0.000	0.263	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	93	44	83	0	313	0
N.S.	1	1.00	0.90	1.55	0.73	1.38	0.00	5.22	0.00
time (sec)	N/A	0.075	0.033	1.030	0.218	0.244	0.000	0.275	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	56	36	58	0	173	0
N.S.	1	1.00	1.00	1.70	1.09	1.76	0.00	5.24	0.00
time (sec)	N/A	0.053	0.016	0.977	0.217	0.242	0.000	0.266	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	27	24	39	17	44	0
N.S.	1	1.00	1.00	1.29	1.14	1.86	0.81	2.10	0.00
time (sec)	N/A	0.020	0.008	0.944	0.227	0.244	0.460	0.264	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	15	27	13
N.S.	1	1.00	1.00	1.08	1.00	1.15	1.15	2.08	1.00
time (sec)	N/A	0.012	0.005	0.452	0.181	0.234	0.257	0.256	1.097

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	34	48	34	29	95	29
N.S.	1	1.00	1.00	1.17	1.66	1.17	1.00	3.28	1.00
time (sec)	N/A	0.021	0.018	0.995	0.224	0.230	0.352	0.269	1.096

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	39	65	47	43	46	214	67
N.S.	1	1.00	0.85	1.41	1.02	0.93	1.00	4.65	1.46
time (sec)	N/A	0.043	0.035	1.037	0.225	0.230	0.472	0.270	1.147

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	48	81	48	53	61	386	85
N.S.	1	1.00	0.77	1.31	0.77	0.85	0.98	6.23	1.37
time (sec)	N/A	0.057	0.041	0.999	0.227	0.242	0.647	0.265	1.183

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	146	115	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	0.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	78	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.138	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	61	70	0	0	0	0	0
N.S.	1	1.00	0.91	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.084	0.657	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	16	18	20	12	18	18
N.S.	1	1.00	1.14	1.14	1.29	1.43	0.86	1.29	1.29
time (sec)	N/A	0.018	2.566	0.357	0.367	0.234	0.612	0.281	1.125

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	102	138	62	323	0	0	0
N.S.	1	1.00	0.98	1.33	0.60	3.11	0.00	0.00	0.00
time (sec)	N/A	0.065	0.080	0.555	0.255	0.247	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	117	44	89	0	353	0
N.S.	1	1.00	0.90	1.89	0.71	1.44	0.00	5.69	0.00
time (sec)	N/A	0.077	0.031	0.653	0.244	0.239	0.000	0.252	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	84	103	58	267	0	0	0
N.S.	1	1.00	0.98	1.20	0.67	3.10	0.00	0.00	0.00
time (sec)	N/A	0.053	0.063	0.572	0.247	0.251	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	39	73	39	63	0	193	0
N.S.	1	1.00	0.93	1.74	0.93	1.50	0.00	4.60	0.00
time (sec)	N/A	0.055	0.019	0.593	0.249	0.236	0.000	0.272	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	70	70	71	228	0	0	0
N.S.	1	1.00	1.04	1.04	1.06	3.40	0.00	0.00	0.00
time (sec)	N/A	0.031	0.049	0.557	0.252	0.271	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	33	24	39	0	0	0
N.S.	1	1.00	1.00	1.32	0.96	1.56	0.00	0.00	0.00
time (sec)	N/A	0.021	0.012	0.546	0.256	0.241	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	50	44	62	52	0	0	0
N.S.	1	1.00	0.88	0.77	1.09	0.91	0.00	0.00	0.00
time (sec)	N/A	0.023	0.026	0.573	0.253	0.254	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	22	31	13
N.S.	1	1.00	1.00	0.93	0.87	1.13	1.47	2.07	0.87
time (sec)	N/A	0.014	0.005	0.426	0.216	0.231	0.458	0.256	1.052

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	82	62	251	0	0	0
N.S.	1	1.00	0.99	1.09	0.83	3.35	0.00	0.00	0.00
time (sec)	N/A	0.035	0.049	0.598	0.250	0.253	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	41	48	40	37	43	58
N.S.	1	1.00	1.00	1.21	1.41	1.18	1.09	1.26	1.71
time (sec)	N/A	0.023	0.018	0.571	0.252	0.238	0.861	0.255	1.106

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	97	117	62	313	0	0	0
N.S.	1	1.00	1.04	1.26	0.67	3.37	0.00	0.00	0.00
time (sec)	N/A	0.049	0.084	0.622	0.246	0.256	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	73	47	50	51	0	74
N.S.	1	1.00	0.94	1.55	1.00	1.06	1.09	0.00	1.57
time (sec)	N/A	0.041	0.032	0.722	0.241	0.236	1.634	0.000	1.112

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	162	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	0.198	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	118	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	82	77	0	0	0	0	0
N.S.	1	1.00	0.94	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.076	0.741	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	16	18	22	14	18	18
N.S.	1	1.00	1.14	1.14	1.29	1.57	1.00	1.29	1.29
time (sec)	N/A	0.018	2.619	0.336	0.437	0.260	1.076	0.297	1.121

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	11	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	1.38	0.75
time (sec)	N/A	0.008	0.004	0.133	0.206	0.258	0.095	0.267	1.150

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	70	77	73	0	0	0	0
N.S.	1	1.00	0.93	1.03	0.97	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.051	0.607	0.114	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	70	69	73	0	0	0	0
N.S.	1	1.00	0.93	0.92	0.97	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.038	0.552	0.112	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	74	61	0	0	0	0
N.S.	1	1.00	0.94	1.10	0.91	0.00	0.00	0.00	0.00
time (sec)	N/A	0.013	0.045	0.447	0.095	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	33	30	55	0	0	0
N.S.	1	1.00	0.92	1.32	1.20	2.20	0.00	0.00	0.00
time (sec)	N/A	0.028	0.017	0.926	0.244	0.243	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	64	77	65	0	0	0	0
N.S.	1	1.00	0.90	1.08	0.92	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.033	0.580	0.096	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	68	77	69	0	0	0	0
N.S.	1	1.00	0.91	1.03	0.92	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.039	0.589	0.098	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	89	0	82	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.83	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	0.159	0.000	0.119	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	82	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.83	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.130	0.000	0.110	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	81	0	68	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.095	0.000	0.103	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	40	37	69	0	0	0
N.S.	1	1.00	0.91	0.93	0.86	1.60	0.00	0.00	0.00
time (sec)	N/A	0.045	0.022	3.734	0.271	0.250	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	79	0	74	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.81	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.081	0.000	0.097	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	160	0	149	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.90	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.134	0.000	0.148	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	160	0	149	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.90	0.00	0.00	0.00	0.00
time (sec)	N/A	0.094	0.126	0.000	0.159	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	140	0	125	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.83	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.080	0.000	0.152	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	52	67	62	115	0	0	0
N.S.	1	1.00	0.78	1.00	0.93	1.72	0.00	0.00	0.00
time (sec)	N/A	0.073	0.041	2.877	0.281	0.255	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	126	0	133	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	0.083	0.000	0.152	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.016	3.991	0.342	0.442	0.249	10.446	1.389	1.133

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	0.017	6.098	0.318	0.408	0.255	37.287	7.849	1.172

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	90	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	20	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.91	1.09	1.09
time (sec)	N/A	0.038	4.377	0.691	0.467	0.267	9.131	1.402	1.087

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	167	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	0.438	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08
time (sec)	N/A	0.041	6.402	0.445	0.423	0.266	28.859	7.752	1.109

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	220	185	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.241	0.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	117	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.129	0.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	87	115	0	0	0	0	0
N.S.	1	1.00	0.88	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.062	1.184	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	123	18	15	18	18
N.S.	1	1.00	1.12	1.00	7.69	1.12	0.94	1.12	1.12
time (sec)	N/A	0.029	18.351	0.694	1.096	0.246	1.181	0.330	1.127

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	66	34	139	0	0	0
N.S.	1	1.00	1.02	1.47	0.76	3.09	0.00	0.00	0.00
time (sec)	N/A	0.072	0.049	1.849	0.250	0.249	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	75	47	182	0	0	0
N.S.	1	1.00	0.81	1.12	0.70	2.72	0.00	0.00	0.00
time (sec)	N/A	0.093	0.098	2.845	0.261	0.250	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	95	128	70	303	0	0	0
N.S.	1	1.00	0.84	1.13	0.62	2.68	0.00	0.00	0.00
time (sec)	N/A	0.152	0.143	5.770	0.288	0.261	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	59	54	69	97	0	53	0
N.S.	1	1.00	0.83	0.76	0.97	1.37	0.00	0.75	0.00
time (sec)	N/A	0.033	0.046	0.564	0.254	0.239	0.000	0.289	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	63	136	817	322	0	137	0
N.S.	1	1.00	0.56	1.20	7.23	2.85	0.00	1.21	0.00
time (sec)	N/A	0.070	0.096	0.323	0.448	0.267	0.000	0.278	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	44	66	649	258	0	99	0
N.S.	1	1.00	0.81	1.22	12.02	4.78	0.00	1.83	0.00
time (sec)	N/A	0.038	0.023	0.255	0.416	0.253	0.000	0.274	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	27	36	477	58	0	39	0
N.S.	1	1.00	0.73	0.97	12.89	1.57	0.00	1.05	0.00
time (sec)	N/A	0.012	0.007	0.263	0.381	0.239	0.000	0.269	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	23	20	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.92	1.67	1.17	1.17
time (sec)	N/A	0.026	5.127	0.207	0.850	0.237	2.553	0.281	1.190

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	23	22	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.92	1.83	1.17	1.17
time (sec)	N/A	0.029	6.942	0.189	0.603	0.228	2.940	0.284	1.142

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	208	831	486	104	269	914	0
N.S.	1	1.00	0.60	2.40	1.40	0.30	0.78	2.64	0.00
time (sec)	N/A	0.303	0.464	2.352	0.214	0.252	0.326	0.305	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	72	301	293	68	151	299	0
N.S.	1	1.00	0.43	1.80	1.75	0.41	0.90	1.79	0.00
time (sec)	N/A	0.135	0.146	2.301	0.199	0.260	0.228	0.282	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	63	111	44	65	64	43
N.S.	1	1.00	0.93	1.17	2.06	0.81	1.20	1.19	0.80
time (sec)	N/A	0.036	0.042	1.309	0.204	0.256	0.201	0.275	1.190

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	130	0	0	217	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.208	2.972	0.000	0.000	0.249	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	182	204	0	0	315	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.270	3.589	0.000	0.000	0.281	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	537	352	1815	642	180	0	2162	0
N.S.	1	1.00	0.66	3.38	1.20	0.34	0.00	4.03	0.00
time (sec)	N/A	0.477	0.787	2.388	0.197	0.262	0.000	0.331	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	118	659	371	109	0	706	0
N.S.	1	1.00	0.45	2.52	1.42	0.42	0.00	2.70	0.00
time (sec)	N/A	0.214	0.255	2.649	0.195	0.255	0.000	0.309	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	133	137	58	94	128	75
N.S.	1	1.00	0.76	1.56	1.61	0.68	1.11	1.51	0.88
time (sec)	N/A	0.056	0.067	1.316	0.196	0.242	0.274	0.301	1.178

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	232	233	0	0	503	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.362	0.056	0.000	0.000	0.258	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	329	329	210	0	0	704	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.519	2.856	0.000	0.000	0.268	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [88] had the largest ratio of [.7500000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	12	0.250
2	A	4	4	1.00	12	0.333
3	A	2	2	1.00	10	0.200
4	A	3	3	1.00	8	0.375
5	A	3	3	1.00	12	0.250
6	A	4	4	1.00	12	0.333
7	A	5	5	1.00	12	0.417
8	A	3	3	1.00	14	0.214
9	A	6	5	1.00	14	0.357
10	A	3	3	1.00	12	0.250
11	A	5	4	1.00	10	0.400
12	A	5	4	1.00	14	0.286
13	A	6	6	1.00	14	0.429
14	A	7	6	1.00	14	0.429
15	A	4	4	1.00	14	0.286
16	A	10	5	1.00	14	0.357
17	A	3	2	1.00	12	0.167
18	A	8	4	1.00	10	0.400
19	A	8	4	1.00	14	0.286
20	A	9	5	1.00	14	0.357
21	A	12	6	1.00	14	0.429
22	A	3	2	1.00	12	0.167
23	N/A	0	0	1.00	16	0.000
24	A	8	3	1.00	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	3	1.00	16	0.188
26	A	3	2	1.00	14	0.143
27	N/A	0	0	1.00	14	0.000
28	A	2	2	1.00	12	0.167
29	A	7	5	1.00	12	0.417
30	A	6	5	1.00	10	0.500
31	A	5	5	1.00	8	0.625
32	A	3	3	1.00	12	0.250
33	A	2	2	1.00	12	0.167
34	A	3	3	1.00	12	0.250
35	A	4	3	1.00	12	0.250
36	A	5	3	1.00	12	0.250
37	A	9	4	1.00	16	0.250
38	A	6	4	1.00	16	0.250
39	A	4	3	1.00	14	0.214
40	N/A	0	0	1.00	14	0.000
41	A	7	6	1.00	12	0.500
42	A	6	5	1.00	12	0.417
43	A	6	6	1.00	12	0.500
44	A	5	5	1.00	10	0.500
45	A	5	5	1.00	8	0.625
46	A	3	3	1.00	12	0.250
47	A	4	4	1.00	12	0.333
48	A	2	2	1.00	12	0.167
49	A	5	5	1.00	12	0.417
50	A	3	3	1.00	12	0.250
51	A	6	6	1.00	12	0.500
52	A	4	3	1.00	12	0.250
53	A	9	4	1.00	16	0.250
54	A	6	4	1.00	16	0.250
55	A	4	3	1.00	14	0.214
56	N/A	0	0	1.00	14	0.000
57	A	2	2	1.00	12	0.167
58	A	3	2	1.00	12	0.167
59	A	3	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	3	2	1.00	8	0.250
61	A	3	3	1.00	12	0.250
62	A	3	2	1.00	12	0.167
63	A	3	2	1.00	12	0.167
64	A	5	3	1.00	14	0.214
65	A	5	3	1.00	12	0.250
66	A	5	3	1.00	10	0.300
67	A	5	4	1.00	14	0.286
68	A	5	3	1.00	14	0.214
69	A	8	3	1.00	14	0.214
70	A	8	3	1.00	12	0.250
71	A	8	3	1.00	10	0.300
72	A	8	4	1.00	14	0.286
73	A	8	3	1.00	14	0.214
74	N/A	0	0	1.00	18	0.000
75	N/A	0	0	1.00	20	0.000
76	A	3	3	1.00	20	0.150
77	N/A	0	0	1.00	22	0.000
78	A	5	5	1.00	22	0.227
79	N/A	0	0	1.00	24	0.000
80	A	8	3	1.00	16	0.188
81	A	5	3	1.00	16	0.188
82	A	3	2	1.00	14	0.143
83	N/A	0	0	1.00	16	0.000
84	A	5	5	1.00	16	0.312
85	A	7	6	1.00	18	0.333
86	A	12	6	1.00	18	0.333
87	A	4	4	1.00	18	0.222
88	A	12	9	1.00	12	0.750
89	A	8	7	1.00	10	0.700
90	A	4	4	1.00	8	0.500
91	N/A	0	0	1.00	12	0.000
92	N/A	0	0	1.00	12	0.000
93	A	16	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	10	4	1.00	16	0.250
95	A	4	4	1.00	14	0.286
96	A	10	5	1.00	18	0.278
97	A	11	6	1.00	18	0.333
98	A	23	6	1.00	18	0.333
99	A	13	5	1.00	16	0.312
100	A	5	4	1.00	14	0.286
101	A	13	5	1.00	18	0.278
102	A	14	6	1.00	18	0.333

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3 \sinh(a+bx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int x^3 \sinh\left(a+bx^2\right)\,dx = \frac{x^2\cosh\left(a+bx^2\right)}{2b} - \frac{\sinh\left(a+bx^2\right)}{2b^2}$$

```
[Out] 1/2*x^2*cosh(b*x^2+a)/b-1/2*sinh(b*x^2+a)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5428, 3377, 2717}

$$\int x^3 \sinh\left(a+bx^2\right)\,dx = \frac{x^2\cosh\left(a+bx^2\right)}{2b} - \frac{\sinh\left(a+bx^2\right)}{2b^2}$$

```
[In] Int[x^3*Sinh[a + b*x^2],x]
```

```
[Out] (x^2*Cosh[a + b*x^2])/(2*b) - Sinh[a + b*x^2]/(2*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\sinh(a+bx)\,dx,x,x^2\right)\\
&= \frac{x^2\cosh\left(a+bx^2\right)}{2b}-\frac{\text{Subst}\left(\int\cosh(a+bx)\,dx,x,x^2\right)}{2b}\\
&= \frac{x^2\cosh\left(a+bx^2\right)}{2b}-\frac{\sinh\left(a+bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int x^3\sinh\left(a+bx^2\right)\,dx=\frac{bx^2\cosh\left(a+bx^2\right)-\sinh\left(a+bx^2\right)}{2b^2}$$

```
[In] Integrate[x^3*Sinh[a + b*x^2],x]
[Out] (b*x^2*Cosh[a + b*x^2] - Sinh[a + b*x^2])/(2*b^2)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{\cosh\left(x^2b+a\right)b\,x^2-\sinh\left(x^2b+a\right)}{2b^2}$	30
risch	$\frac{\left(x^2b-1\right)\mathrm{e}^{x^2b+a}}{4b^2}+\frac{\left(x^2b+1\right)\mathrm{e}^{-x^2b-a}}{4b^2}$	45
meijerg	$-\frac{\sinh(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh\left(x^2b\right)}{2\sqrt{\pi}}-\frac{x^2b\sinh\left(x^2b\right)}{2\sqrt{\pi}}\right)}{b^2}+\frac{\cosh(a)\left(\cosh\left(x^2b\right)x^2b-\sinh\left(x^2b\right)\right)}{2b^2}$	71

```
[In] int(x^3*sinh(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(cosh(b*x^2+a)*b*x^2-sinh(b*x^2+a))/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^3 \sinh(a+bx^2)\, dx = \frac{bx^2 \cosh(bx^2+a) - \sinh(bx^2+a)}{2\,b^2}$$

```
[In] integrate(x^3*sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*cosh(b*x^2 + a) - sinh(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x^3 \sinh(a+bx^2)\, dx = \begin{cases} \frac{x^2 \cosh(a+bx^2)}{2b} - \frac{\sinh(a+bx^2)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^4 \sinh(a)}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*sinh(b*x**2+a),x)
```

```
[Out] Piecewise((x**2*cosh(a + b*x**2)/(2*b) - sinh(a + b*x**2)/(2*b**2), Ne(b, 0
)), (x**4*sinh(a)/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(30) = 60.

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.38

$$\int x^3 \sinh(a+bx^2)\, dx = \frac{1}{4}\,x^4 \sinh(bx^2+a) - \frac{1}{8}\,b\left(\frac{(b^2x^4e^a - 2\,bx^2e^a + 2\,e^a)e^{(bx^2)}}{b^3} - \frac{(b^2x^4 + 2\,bx^2 + 2)e^{(-bx^2-a)}}{b^3}\right)$$

```
[In] integrate(x^3*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*sinh(b*x^2 + a) - 1/8*b*((b^2*x^4*e^a - 2*b*x^2*e^a + 2*e^a)*e^(b*x
^2)/b^3 - (b^2*x^4 + 2*b*x^2 + 2)*e^(-b*x^2 - a)/b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(30) = 60$.

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.15

$$\int x^3 \sinh \left(a + bx^2\right) \, dx = \frac{(bx^2 + a - 1)e^{(bx^2+a)} + (bx^2 + a + 1)e^{(-bx^2-a)}}{4 \, b^2} - \frac{ae^{(bx^2+a)} + ae^{(-bx^2-a)}}{4 \, b^2}$$

```
[In] integrate(x^3*sinh(b*x^2+a),x, algorithm="giac")
[Out] 1/4*((b*x^2 + a - 1)*e^(b*x^2 + a) + (b*x^2 + a + 1)*e^(-b*x^2 - a))/b^2 -
1/4*(a*e^(b*x^2 + a) + a*e^(-b*x^2 - a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int x^3 \sinh \left(a + bx^2\right) \, dx = -\frac{\sinh(b \, x^2 + a) - b \, x^2 \cosh(b \, x^2 + a)}{2 \, b^2}$$

```
[In] int(x^3*sinh(a + b*x^2),x)
[Out] -(sinh(a + b*x^2) - b*x^2*cosh(a + b*x^2))/(2*b^2)
```

3.2 $\int x^2 \sinh(a+bx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int x^2 \sinh(a+bx^2)\,dx = \frac{x\cosh(a+bx^2)}{2b} - \frac{e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)}{8b^{3/2}} - \frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x\right)}{8b^{3/2}}$$

```
[Out] 1/2*x*cosh(b*x^2+a)/b-1/8*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)-1/8*exp(a)
*erfi(x*b^(1/2))*Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5432, 5407, 2235, 2236}

$$\int x^2 \sinh(a+bx^2)\,dx = -\frac{\sqrt{\pi}e^{-a}\,\mathrm{erf}\left(\sqrt{b}x\right)}{8b^{3/2}} - \frac{\sqrt{\pi}e^{a}\,\mathrm{erfi}\left(\sqrt{b}x\right)}{8b^{3/2}} + \frac{x\cosh(a+bx^2)}{2b}$$

```
[In] Int[x^2*Sinh[a + b*x^2],x]
```

```
[Out] (x*Cosh[a + b*x^2])/(2*b) - (Sqrt[Pi]*Erf[Sqrt[b]*x])/(8*b^(3/2)*E^a) - (E^
a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(8*b^(3/2))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\cosh\left(a+bx^2\right)}{2b} - \frac{\int \cosh\left(a+bx^2\right)\,dx}{2b}\\
&= \frac{x\cosh\left(a+bx^2\right)}{2b} - \frac{\int e^{-a-bx^2}\,dx}{4b} - \frac{\int e^{a+bx^2}\,dx}{4b}\\
&= \frac{x\cosh\left(a+bx^2\right)}{2b} - \frac{e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)}{8b^{3/2}} - \frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x\right)}{8b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$
\int x^2\sinh\left(a+bx^2\right)\,dx
$$

$$
= \frac{4\sqrt{b}x\cosh\left(a+bx^2\right)+\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)\left(-\cosh(a)+\sinh(a)\right)-\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x\right)\left(\cosh(a)+\sinh(a)\right)}{8b^{3/2}}
$$

```
[In] Integrate[x^2*Sinh[a + b*x^2],x]
```

```
[Out] (4*Sqrt[b]*x*Cosh[a + b*x^2] + Sqrt[Pi]*Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a])
 - Sqrt[Pi]*Erfi[Sqrt[b]*x]*(Cosh[a] + Sinh[a]))/(8*b^(3/2))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

method	result
risch	$\frac{e^{-a} x e^{-x^2 b}}{4b} - \frac{\operatorname{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\, e^{-a}}{8 b^{\frac{3}{2}}} + \frac{e^{a} e^{x^2 b} x}{4b} - \frac{e^{a}\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b}\, x\right)}{8b\sqrt{-b}}$
meijerg	$-\frac{i\sinh(a)\sqrt{\pi}\,\sqrt{2}\left(\frac{x\sqrt{2}\,(ib)^{\frac{3}{2}} e^{x^2 b}}{4\sqrt{\pi}\, b} - \frac{x\sqrt{2}\,(ib)^{\frac{3}{2}} e^{-x^2 b}}{4\sqrt{\pi}\, b} + \frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erf}\left(x\sqrt{b}\right)}{8b^{\frac{3}{2}}} - \frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erfi}\left(x\sqrt{b}\right)}{8b^{\frac{3}{2}}}\right)}{2b\sqrt{ib}} - \frac{\cosh(a)\sqrt{\pi}\,\sqrt{2}\left(\frac{x\sqrt{2}\,(ib)^{\frac{5}{2}} e^{-x^2 b}}{4\sqrt{\pi}\, b^2}\right.}{}$

```
[In] int(x^2*sinh(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/exp(a)/b*x*exp(-x^2*b)-1/8*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)+1/4*e
xp(a)*exp(x^2*b)*x/b-1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(49) = 98$.

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.75

$$\int x^2 \sinh\left(a + bx^2\right)\, dx$$
$$= \frac{2\,bx\cosh\left(bx^2+a\right)^2 + 4\,bx\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right) + 2\,bx\sinh\left(bx^2+a\right)^2 + \sqrt{\pi}(\cosh\left(bx^2+a\right)\cosh(}{}$$

```
[In] integrate(x^2*sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*x*cosh(b*x^2 + a)^2 + 4*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a) + 2*b*
x*sinh(b*x^2 + a)^2 + sqrt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) + sinh(a
))*sinh(b*x^2 + a) + cosh(b*x^2 + a)*sinh(a))*sqrt(-b)*erf(sqrt(-b)*x) - sq
rt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a) - cos
h(b*x^2 + a)*sinh(a))*sqrt(b)*erf(sqrt(b)*x) + 2*b*x)/(b^2*cosh(b*x^2 + a)
+ b^2*sinh(b*x^2 + a))
```

Sympy [F]

$$\int x^2 \sinh\left(a + bx^2\right)\, dx = \int x^2 \sinh\left(a + bx^2\right) dx$$

```
[In] integrate(x**2*sinh(b*x**2+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(49) = 98$.

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int x^2 \sinh\left(a + bx^2\right) dx = \frac{1}{3} x^3 \sinh\left(bx^2 + a\right) - \frac{1}{24} b \left(\frac{2\left(2bx^3e^a - 3xe^a\right)e^{\left(bx^2\right)}}{b^2} - \frac{2\left(2bx^3 + 3x\right)e^{\left(-bx^2 - a\right)}}{b^2} + \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)e^{(-a)}}{b^{\frac{5}{2}}} + \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{-b}x\right)e^a}{\sqrt{-b}b^2}$$

```
[In] integrate(x^2*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sinh(b*x^2 + a) - 1/24*b*(2*(2*b*x^3*e^a - 3*x*e^a)*e^(b*x^2)/b^2 -
 2*(2*b*x^3 + 3*x)*e^(-b*x^2 - a)/b^2 + 3*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^
(5/2) + 3*sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b^2))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int x^2 \sinh\left(a+bx^2\right) dx = \frac{xe^{\left(bx^2+a\right)}}{4b} + \frac{xe^{\left(-bx^2-a\right)}}{4b} + \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{8b^{\frac{3}{2}}} + \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-b}x\right)e^a}{8\sqrt{-b}b}$$

```
[In] integrate(x^2*sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*x*e^(b*x^2 + a)/b + 1/4*x*e^(-b*x^2 - a)/b + 1/8*sqrt(pi)*erf(-sqrt(b)*
x)*e^(-a)/b^(3/2) + 1/8*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh\left(a + bx^2\right) dx = \int x^2 \sinh\left(b\,x^2 + a\right) dx$$

```
[In] int(x^2*sinh(a + b*x^2),x)
```

```
[Out] int(x^2*sinh(a + b*x^2), x)
```

3.3 $\int x \sinh(a+bx^2)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int x \sinh\left(a+bx^2\right)\, dx = \frac{\cosh(a+bx^2)}{2b}$$

```
[Out] 1/2*cosh(b*x^2+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5428, 2718}

$$\int x \sinh\left(a+bx^2\right)\, dx = \frac{\cosh(a+bx^2)}{2b}$$

```
[In] Int[x*Sinh[a + b*x^2],x]
```

```
[Out] Cosh[a + b*x^2]/(2*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
```

```
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sinh(a+bx)\,dx,x,x^2\right) \\ &= \frac{\cosh(a+bx^2)}{2b} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int x\sinh\left(a+bx^2\right)\,dx = \frac{\cosh(a)\cosh\left(bx^2\right)}{2b} + \frac{\sinh(a)\sinh\left(bx^2\right)}{2b}$$

```
[In] Integrate[x*Sinh[a + b*x^2],x]
[Out] (Cosh[a]*Cosh[b*x^2])/(2*b) + (Sinh[a]*Sinh[b*x^2])/(2*b)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh(x^2b+a)}{2b}$	14
default	$\frac{\cosh(x^2b+a)}{2b}$	14
parallelrisch	$\frac{1+\cosh(x^2b+a)}{2b}$	16
risch	$\frac{e^{x^2b+a}}{4b}+\frac{e^{-x^2b-a}}{4b}$	31
meijerg	$\frac{\sinh(a)\sinh(x^2b)}{2b}-\frac{\cosh(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(x^2b\right)}{\sqrt{\pi}}\right)}{2b}$	40

```
[In] int(x*sinh(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*cosh(b*x^2+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \sinh\left(a+bx^2\right) \, dx = \frac{\cosh\left(bx^2+a\right)}{2\,b}$$

```
[In] integrate(x*sinh(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*cosh(b*x^2 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x \sinh\left(a+bx^2\right) \, dx = \begin{cases} \frac{\cosh\left(a+bx^2\right)}{2b} & \text{for } b \neq 0 \\ \frac{x^2 \sinh\left(a\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*sinh(b*x**2+a),x)
[Out] Piecewise((cosh(a + b*x**2)/(2*b), Ne(b, 0)), (x**2*sinh(a)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \sinh\left(a+bx^2\right) \, dx = \frac{\cosh\left(bx^2+a\right)}{2\,b}$$

```
[In] integrate(x*sinh(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*cosh(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int x \sinh\left(a+bx^2\right) \, dx = \frac{e^{\left(bx^2+a\right)} + e^{\left(-bx^2-a\right)}}{4\,b}$$

```
[In] integrate(x*sinh(b*x^2+a),x, algorithm="giac")
[Out] 1/4*(e^(b*x^2 + a) + e^(-b*x^2 - a))/b
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \sinh\left(a+bx^2\right)\,dx=\frac{\cosh(b\,x^2+a)}{2\,b}$$

```
[In] int(x*sinh(a + b*x^2),x)
```

```
[Out] cosh(a + b*x^2)/(2*b)
```

3.4 $\int \sinh(a + bx^2)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 53

$$\int \sinh\left(a+bx^2\right)\, dx = -\frac{e^{-a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)}{4\sqrt{b}} + \frac{e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}}$$

```
[Out] -1/4*erf(x*b^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+1/4*exp(a)*erfi(x*b^(1/2))*Pi^(
1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5406, 2235, 2236}

$$\int \sinh\left(a+bx^2\right)\, dx = \frac{\sqrt{\pi}e^{a}\operatorname{erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}} - \frac{\sqrt{\pi}e^{-a}\operatorname{erf}\left(\sqrt{b}x\right)}{4\sqrt{b}}$$

```
[In] Int[Sinh[a + b*x^2],x]
```

```
[Out] -1/4*(Sqrt[Pi]*Erf[Sqrt[b]*x])/(Sqrt[b]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x
])/(4*Sqrt[b])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int e^{-a-bx^2}\,dx\right)+\frac{1}{2}\int e^{a+bx^2}\,dx\\ &= -\frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\sqrt{b}x\right)}{4\sqrt{b}}+\frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \sinh\left(a+bx^2\right)\,dx = \frac{\sqrt{\pi}\left(\text{erf}\left(\sqrt{b}x\right)(-\cosh(a)+\sinh(a))+\text{erfi}\left(\sqrt{b}x\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{b}}$$

```
[In] Integrate[Sinh[a + b*x^2],x]
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a]) + Erfi[Sqrt[b]*x]*(Cosh[a] +
 Sinh[a])))/(4*Sqrt[b])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\operatorname{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\,\mathrm{e}^{-a}}{4\sqrt{b}}+\frac{\mathrm{e}^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)}{4\sqrt{-b}}$	40
meijerg	$\frac{\sinh(a)\sqrt{\pi}\,\sqrt{2}\left(\frac{\sqrt{ib}\,\sqrt{2}\,\operatorname{erf}\left(x\sqrt{b}\right)}{2\sqrt{b}}+\frac{\sqrt{ib}\,\sqrt{2}\,\operatorname{erfi}\left(x\sqrt{b}\right)}{2\sqrt{b}}\right)}{4\sqrt{ib}}-\frac{i\cosh(a)\sqrt{\pi}\,\sqrt{2}\left(-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erf}\left(x\sqrt{b}\right)}{2b^{\frac{3}{2}}}+\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erfi}\left(x\sqrt{b}\right)}{2b^{\frac{3}{2}}}\right)}{4\sqrt{ib}}$	117

```
[In] int(sinh(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*erf(x*b^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)*
erf((-b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \sinh \left(a+bx^2\right)\,dx = -\frac{\sqrt{\pi}\sqrt{-b}(\cosh(a)+\sinh(a))\operatorname{erf}\left(\sqrt{-b}x\right)+\sqrt{\pi}\sqrt{b}(\cosh(a)-\sinh(a))\operatorname{erf}\left(\sqrt{b}x\right)}{4\,b}$$

```
[In] integrate(sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) + sqrt(pi)*sqrt
(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b
```

Sympy [F]

$$\int \sinh \left(a+bx^2\right)\,dx = \int \sinh \left(a+bx^2\right)dx$$

```
[In] integrate(sinh(b*x**2+a),x)
```

```
[Out] Integral(sinh(a + b*x**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(35) = 70.

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int \sinh \left(a+bx^2\right)\,dx = -\frac{1}{4}b\left(\frac{2\,xe^{\left(bx^2+a\right)}}{b}-\frac{2\,xe^{\left(-bx^2-a\right)}}{b}+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)e^{(-a)}}{b^{\frac{3}{2}}}-\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{-b}x\right)e^{a}}{\sqrt{-b}b}\right) + x\sinh\left(bx^2+a\right)$$

```
[In] integrate(sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*x*e^(b*x^2 + a)/b - 2*x*e^(-b*x^2 - a)/b + sqrt(pi)*erf(sqrt(b)*x
)*e^(-a)/b^(3/2) - sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b)) + x*sinh(b*x^
2 + a)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \sinh \left(a+bx^2\right)\,dx=\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{4\sqrt{b}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-b}x\right)e^{a}}{4\sqrt{-b}}$$

```
[In] integrate(sinh(b*x^2+a),x, algorithm="giac")
[Out] 1/4*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/sqrt(b) - 1/4*sqrt(pi)*erf(-sqrt(-b)*x)
*e^a/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh \left(a+bx^2\right)\,dx=\int \sinh \left(b\,x^2+a\right)\,dx$$

```
[In] int(sinh(a + b*x^2),x)
[Out] int(sinh(a + b*x^2), x)
```

3.5 $\int \frac{\sinh(a+bx^2)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\sinh(a+bx^2)}{x}\,dx = \frac{1}{2}\text{Chi}(bx^2)\sinh(a) + \frac{1}{2}\cosh(a)\text{Shi}(bx^2)$$

```
[Out] 1/2*cosh(a)*Shi(b*x^2)+1/2*Chi(b*x^2)*sinh(a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5426, 5425, 5424}

$$\int \frac{\sinh(a+bx^2)}{x}\,dx = \frac{1}{2}\sinh(a)\text{Chi}(bx^2) + \frac{1}{2}\cosh(a)\text{Shi}(bx^2)$$

```
[In] Int[Sinh[a + b*x^2]/x,x]
[Out] (CoshIntegral[b*x^2]*Sinh[a])/2 + (Cosh[a]*SinhIntegral[b*x^2])/2
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5426

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\text{integral} = \cosh(a)\int \frac{\sinh(bx^2)}{x}\,dx + \sinh(a)\int \frac{\cosh(bx^2)}{x}\,dx$$
$$= \frac{1}{2}\text{Chi}(bx^2)\sinh(a) + \frac{1}{2}\cosh(a)\text{Shi}(bx^2)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sinh(a+bx^2)}{x}\,dx = \frac{1}{2}\left(\text{Chi}(bx^2)\sinh(a) + \cosh(a)\text{Shi}(bx^2)\right)$$

```
[In] Integrate[Sinh[a + b*x^2]/x,x]
[Out] (CoshIntegral[b*x^2]*Sinh[a] + Cosh[a]*SinhIntegral[b*x^2])/2
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{e^{-a}\,\text{Ei}_1(-x^2b)e^{2a}}{4}+\frac{e^{-a}\,\text{Ei}_1(x^2b)}{4}$	33
meijerg	$\frac{\sinh(a)\sqrt{\pi}\left(\frac{2\ \text{Chi}(x^2b)-2\ln(x^2b)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+4\ln(x)+2\ln(ib)}{\sqrt{\pi}}\right)}{4}+\frac{\cosh(a)\ \text{Shi}(x^2b)}{2}$	62

```
[In] int(sinh(b*x^2+a)/x,x,method=_RETURNVERBOSE)
[Out] -1/4*exp(-a)*Ei(1,-x^2*b)*exp(2*a)+1/4*exp(-a)*Ei(1,x^2*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\sinh(a+bx^2)}{x}\,dx = \frac{1}{4}\left(\mathrm{Ei}(bx^2) - \mathrm{Ei}(-bx^2)\right)\cosh(a) + \frac{1}{4}\left(\mathrm{Ei}(bx^2) + \mathrm{Ei}(-bx^2)\right)\sinh(a)$$

```
[In] integrate(sinh(b*x^2+a)/x,x, algorithm="fricas")
[Out] 1/4*(Ei(b*x^2) - Ei(-b*x^2))*cosh(a) + 1/4*(Ei(b*x^2) + Ei(-b*x^2))*sinh(a)
```

Sympy [F]

$$\int \frac{\sinh(a+bx^2)}{x}\,dx = \int \frac{\sinh(a+bx^2)}{x}\,dx$$

```
[In] integrate(sinh(b*x**2+a)/x,x)
[Out] Integral(sinh(a + b*x**2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sinh(a+bx^2)}{x}\,dx = -\frac{1}{4}\,\mathrm{Ei}(-bx^2)\,e^{(-a)} + \frac{1}{4}\,\mathrm{Ei}(bx^2)\,e^{a}$$

```
[In] integrate(sinh(b*x^2+a)/x,x, algorithm="maxima")
[Out] -1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sinh(a+bx^2)}{x}\,dx = -\frac{1}{4}\,\mathrm{Ei}(-bx^2)\,e^{(-a)} + \frac{1}{4}\,\mathrm{Ei}(bx^2)\,e^{a}$$

```
[In] integrate(sinh(b*x^2+a)/x,x, algorithm="giac")
[Out] -1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx^2)}{x}\,dx = \frac{\sinh(a)\ \mathrm{coshint}(b\,x^2)}{2} + \frac{\cosh(a)\ \mathrm{sinhint}(b\,x^2)}{2}$$

```
[In] int(sinh(a + b*x^2)/x,x)
[Out] (sinh(a)*coshint(b*x^2))/2 + (cosh(a)*sinhint(b*x^2))/2
```

3.6 $\int \frac{\sinh(a+bx^2)}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int \frac{\sinh(a+bx^2)}{x^2} dx = \frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right) + \frac{1}{2}\sqrt{b}e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\right) - \frac{\sinh(a+bx^2)}{x}$$

```
[Out] -sinh(b*x^2+a)/x+1/2*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)+1/2*exp(a)*erfi
(x*b^(1/2))*b^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5434, 5407, 2235, 2236}

$$\int \frac{\sinh(a+bx^2)}{x^2} dx = \frac{1}{2}\sqrt{\pi}e^{-a}\sqrt{b}\operatorname{erf}\left(\sqrt{b}x\right) + \frac{1}{2}\sqrt{\pi}e^{a}\sqrt{b}\operatorname{erfi}\left(\sqrt{b}x\right) - \frac{\sinh(a+bx^2)}{x}$$

```
[In] Int[Sinh[a + b*x^2]/x^2,x]
```

```
[Out] (Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*x])/(2*E^a) + (Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqrt
[b]*x])/2 - Sinh[a + b*x^2]/x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5434

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x
)^(m + 1)*(Sinh[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sinh(a+bx^2)}{x} + (2b)\int \cosh(a+bx^2)\,dx \\
&= -\frac{\sinh(a+bx^2)}{x} + b\int e^{-a-bx^2}\,dx + b\int e^{a+bx^2}\,dx \\
&= \frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right) + \frac{1}{2}\sqrt{b}e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\right) - \frac{\sinh(a+bx^2)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(a+bx^2)}{x^2}\,dx$$
$$= \frac{\sqrt{b}\sqrt{\pi}x\operatorname{erf}\left(\sqrt{b}x\right)(\cosh(a)-\sinh(a)) + \sqrt{b}\sqrt{\pi}x\operatorname{erfi}\left(\sqrt{b}x\right)(\cosh(a)+\sinh(a)) - 2\sinh(a+bx^2)}{2x}$$

```
[In] Integrate[Sinh[a + b*x^2]/x^2,x]
```

```
[Out] (Sqrt[b]*Sqrt[Pi]*x*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + Sqrt[b]*Sqrt[Pi]*x
*Erfi[Sqrt[b]*x]*(Cosh[a] + Sinh[a]) - 2*Sinh[a + b*x^2])/(2*x)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result
risch	$\frac{e^{-a}e^{-x^2 b}}{2x}+\frac{\operatorname{erf}\left(x\sqrt{b}\right)\sqrt{b}\,\sqrt{\pi}\,e^{-a}}{2}-\frac{e^{a}e^{x^2 b}}{2x}+\frac{e^{a}b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)}{2\sqrt{-b}}$
meijerg	$\frac{i\sinh(a)\sqrt{\pi}\,b\sqrt{2}\left(-\frac{2\sqrt{2}\,e^{x^2 b}}{\sqrt{\pi}\,x\sqrt{ib}}-\frac{2\sqrt{2}\,e^{-x^2 b}}{\sqrt{\pi}\,x\sqrt{ib}}-\frac{2\sqrt{2}\,\sqrt{b}\,\operatorname{erf}\left(x\sqrt{b}\right)}{\sqrt{ib}}+\frac{2\sqrt{2}\,\sqrt{b}\,\operatorname{erfi}\left(x\sqrt{b}\right)}{\sqrt{ib}}\right)}{8\sqrt{ib}}+\frac{\cosh(a)\sqrt{\pi}\,b\sqrt{2}\left(\frac{2\sqrt{2}\,\sqrt{ib}\,e^{-x^2 b}}{\sqrt{\pi}\,xb}-\frac{2\sqrt{2}\,\sqrt{ib}\,e^{x^2 b}}{\sqrt{\pi}\,xb}\right.}{8\sqrt{ib}}$

```
[In] int(sinh(b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/2/exp(a)/x*exp(-x^2*b)+1/2*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)-1/2*exp
(a)*exp(x^2*b)/x+1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(48) = 96.

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.79

$$\int \frac{\sinh(a+bx^2)}{x^2}\,dx =$$
$$-\frac{\sqrt{\pi}(x\cosh(bx^2+a)\cosh(a)+x\cosh(bx^2+a)\sinh(a)+(x\cosh(a)+x\sinh(a))\sinh(bx^2+a))\sqrt{-b}}{}$$

```
[In] integrate(sinh(b*x^2+a)/x^2,x, algorithm="fricas")
[Out] -1/2*(sqrt(pi)*(x*cosh(b*x^2 + a)*cosh(a) + x*cosh(b*x^2 + a)*sinh(a) + (x*
cosh(a) + x*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) - sqrt(pi)*(
x*cosh(b*x^2 + a)*cosh(a) - x*cosh(b*x^2 + a)*sinh(a) + (x*cosh(a) - x*sinh
(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + cosh(b*x^2 + a)^2 + 2*cosh(b
*x^2 + a)*sinh(b*x^2 + a) + sinh(b*x^2 + a)^2 - 1)/(x*cosh(b*x^2 + a) + x*s
inh(b*x^2 + a))
```

Sympy [F]

$$\int \frac{\sinh(a+bx^2)}{x^2}\,dx = \int \frac{\sinh(a+bx^2)}{x^2}\,dx$$

```
[In] integrate(sinh(b*x**2+a)/x**2,x)
[Out] Integral(sinh(a + b*x**2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{\sinh (a + bx^2)}{x^2} \, dx = \frac{1}{2} \left(\frac{\sqrt{\pi} \operatorname{erf} \left(\sqrt{b}x\right) e^{(-a)}}{\sqrt{b}} + \frac{\sqrt{\pi} \operatorname{erf} \left(\sqrt{-b}x\right) e^{a}}{\sqrt{-b}} \right) b - \frac{\sinh (bx^2 + a)}{x}$$

```
[In] integrate(sinh(b*x^2+a)/x^2,x, algorithm="maxima")
[Out] 1/2*(sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/sqrt(b) + sqrt(pi)*erf(sqrt(-b)*x)*e^a/
sqrt(-b))*b - sinh(b*x^2 + a)/x
```

Giac [F]

$$\int \frac{\sinh (a + bx^2)}{x^2} \, dx = \int \frac{\sinh (bx^2 + a)}{x^2} \, dx$$

```
[In] integrate(sinh(b*x^2+a)/x^2,x, algorithm="giac")
[Out] integrate(sinh(b*x^2 + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh (a + bx^2)}{x^2} \, dx = \int \frac{\sinh(b\,x^2 + a)}{x^2} \, dx$$

```
[In] int(sinh(a + b*x^2)/x^2,x)
[Out] int(sinh(a + b*x^2)/x^2, x)
```

3.7 $\int \frac{\sinh(a+bx^2)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \frac{\sinh(a+bx^2)}{x^3}\,dx = \frac{1}{2}b\cosh(a)\mathrm{Chi}(bx^2) - \frac{\sinh(a+bx^2)}{2x^2} + \frac{1}{2}b\sinh(a)\mathrm{Shi}(bx^2)$$

```
[Out] 1/2*b*Chi(b*x^2)*cosh(a)+1/2*b*Shi(b*x^2)*sinh(a)-1/2*sinh(b*x^2+a)/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5428, 3378, 3384, 3379, 3382}

$$\int \frac{\sinh(a+bx^2)}{x^3}\,dx = \frac{1}{2}b\cosh(a)\mathrm{Chi}(bx^2) + \frac{1}{2}b\sinh(a)\mathrm{Shi}(bx^2) - \frac{\sinh(a+bx^2)}{2x^2}$$

```
[In] Int[Sinh[a + b*x^2]/x^3,x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^2])/2 - Sinh[a + b*x^2]/(2*x^2) + (b*Sinh[a]*Si
nhIntegral[b*x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sinh\left(a+bx^2\right)}{2x^2} + \frac{1}{2}b\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx, x, x^2\right)\\
&= -\frac{\sinh\left(a+bx^2\right)}{2x^2} + \frac{1}{2}(b\cosh(a))\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, x^2\right)\\
&\quad + \frac{1}{2}(b\sinh(a))\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}b\cosh(a)\text{Chi}\left(bx^2\right) - \frac{\sinh\left(a+bx^2\right)}{2x^2} + \frac{1}{2}b\sinh(a)\text{Shi}\left(bx^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\sinh\left(a+bx^2\right)}{x^3}\,dx=\frac{1}{2}\left(b\cosh(a)\mathrm{Chi}\left(bx^2\right)-\frac{\sinh\left(a+bx^2\right)}{x^2}+b\sinh(a)\mathrm{Shi}\left(bx^2\right)\right)$$

```
[In] Integrate[Sinh[a + b*x^2]/x^3,x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^2] - Sinh[a + b*x^2]/x^2 + b*Sinh[a]*SinhIntegr
al[b*x^2])/2
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

method	result
risch	$-\frac{\mathrm{e}^{a}\,\mathrm{Ei}_{1}\left(-x^{2} b\right) b\,x^{2}+\mathrm{Ei}_{1}\left(x^{2} b\right) \mathrm{e}^{-a} b\,x^{2}+\mathrm{e}^{x^{2} b+a}-\mathrm{e}^{-x^{2} b-a}}{4 x^{2}}$
meijerg	$\frac{i\sinh(a)\sqrt{\pi}\,b\left(\frac{4 i \cosh\left(x^{2} b\right)}{b\,x^{2} \sqrt{\pi}}-\frac{4 i\,\mathrm{Shi}\left(x^{2} b\right)}{\sqrt{\pi}}\right)}{8}+\frac{\cosh(a)\sqrt{\pi}\,b\left(\frac{4}{\sqrt{\pi}}-\frac{4\sinh\left(x^{2} b\right)}{\sqrt{\pi}\,x^{2} b}+\frac{4\ \mathrm{Chi}\left(x^{2} b\right)-4\ln\left(x^{2} b\right)-4\gamma}{\sqrt{\pi}}+\frac{4\gamma-4+8\ln(x)+4\ln(ib)}{\sqrt{\pi}}\right)}{8}$

```
[In] int(sinh(b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(exp(a)*Ei(1,-x^2*b)*b*x^2+Ei(1,x^2*b)*exp(-a)*b*x^2+exp(b*x^2+a)-exp(
-b*x^2-a))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \frac{\sinh\left(a+bx^2\right)}{x^3}\,dx$$
$$=\frac{\left(bx^2\mathrm{Ei}\left(bx^2\right)+bx^2\mathrm{Ei}\left(-bx^2\right)\right)\cosh\left(a\right)+\left(bx^2\mathrm{Ei}\left(bx^2\right)-bx^2\mathrm{Ei}\left(-bx^2\right)\right)\sinh\left(a\right)-2\,\sinh\left(bx^2+a\right)}{4\,x^2}$$

```
[In] integrate(sinh(b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^2*Ei(b*x^2) + b*x^2*Ei(-b*x^2))*cosh(a) + (b*x^2*Ei(b*x^2) - b*x^
2*Ei(-b*x^2))*sinh(a) - 2*sinh(b*x^2 + a))/x^2
```

Sympy [F]

$$\int \frac{\sinh(a+bx^2)}{x^3}\,dx = \int \frac{\sinh(a+bx^2)}{x^3}\,dx$$

```
[In] integrate(sinh(b*x**2+a)/x**3,x)
[Out] Integral(sinh(a + b*x**2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{\sinh(a+bx^2)}{x^3}\,dx = \frac{1}{4}\left(\mathrm{Ei}\left(-bx^2\right)e^{(-a)} + \mathrm{Ei}\left(bx^2\right)e^{a}\right)b - \frac{\sinh(bx^2+a)}{2\,x^2}$$

```
[In] integrate(sinh(b*x^2+a)/x^3,x, algorithm="maxima")
[Out] 1/4*(Ei(-b*x^2)*e^(-a) + Ei(b*x^2)*e^a)*b - 1/2*sinh(b*x^2 + a)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(36) = 72.

Time = 0.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.60

$$\int \frac{\sinh(a+bx^2)}{x^3}\,dx = \frac{(bx^2+a)b^2\mathrm{Ei}(-bx^2)\,e^{(-a)} - ab^2\mathrm{Ei}(-bx^2)\,e^{(-a)} + (bx^2+a)b^2\mathrm{Ei}(bx^2)\,e^{a} - ab^2\mathrm{Ei}(bx^2)\,e^{a} - b^2e^{(bx^2+a)} + b^2e^{(-}}{4\,b^2x^2}$$

```
[In] integrate(sinh(b*x^2+a)/x^3,x, algorithm="giac")
[Out] 1/4*((b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) - a*b^2*Ei(-b*x^2)*e^(-a) + (b*x^2 +
 a)*b^2*Ei(b*x^2)*e^a - a*b^2*Ei(b*x^2)*e^a - b^2*e^(b*x^2 + a) + b^2*e^(-b
*x^2 - a))/(b^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx^2)}{x^3}\,dx = \int \frac{\sinh(b\,x^2+a)}{x^3}\,dx$$

```
[In] int(sinh(a + b*x^2)/x^3,x)
[Out] int(sinh(a + b*x^2)/x^3, x)
```

3.8 $\int x^3 \sinh^2(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int x^3 \sinh^2\left(a+bx^2\right)\,dx=-\frac{x^4}{8}+\frac{x^2\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}-\frac{\sinh^2\left(a+bx^2\right)}{8b^2}$$

```
[Out] -1/8*x^4+1/4*x^2*cosh(b*x^2+a)*sinh(b*x^2+a)/b-1/8*sinh(b*x^2+a)^2/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5428, 3391, 30}

$$\int x^3 \sinh^2\left(a+bx^2\right)\,dx=-\frac{\sinh^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{x^4}{8}$$

```
[In] Int[x^3*Sinh[a + b*x^2]^2,x]
```

```
[Out] -1/8*x^4 + (x^2*Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b) - Sinh[a + b*x^2]^2/
(8*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
```

```
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\sinh^2(a+bx)\,dx,x,x^2\right)\\
&= \frac{x^2\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}-\frac{\sinh^2\left(a+bx^2\right)}{8b^2}-\frac{1}{4}\text{Subst}\left(\int x\,dx,x,x^2\right)\\
&= -\frac{x^4}{8}+\frac{x^2\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}-\frac{\sinh^2\left(a+bx^2\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int x^3\sinh^2\left(a+bx^2\right)\,dx=-\frac{\cosh\left(2(a+bx^2)\right)+2bx^2(bx^2-\sinh\left(2(a+bx^2)\right))}{16b^2}$$

```
[In] Integrate[x^3*Sinh[a + b*x^2]^2,x]
[Out] -1/16*(Cosh[2*(a + b*x^2)] + 2*b*x^2*(b*x^2 - Sinh[2*(a + b*x^2)]))/b^2
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{x^4}{8}+\frac{(2x^2b-1)\,\mathrm{e}^{2x^2b+2a}}{32b^2}-\frac{(2x^2b+1)\,\mathrm{e}^{-2x^2b-2a}}{32b^2}$	55

```
[In] int(x^3*sinh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/8*x^4+1/32*(2*b*x^2-1)/b^2*exp(2*b*x^2+2*a)-1/32*(2*b*x^2+1)/b^2*exp(-2*
b*x^2-2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int x^3 \sinh^2\left(a + bx^2\right) dx = -\frac{2 b^2 x^4 - 4 b x^2 \cosh\left(bx^2 + a\right) \sinh\left(bx^2 + a\right) + \cosh\left(bx^2 + a\right)^2 + \sinh\left(bx^2 + a\right)^2}{16 b^2}$$

```
[In] integrate(x^3*sinh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^2*x^4 - 4*b*x^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) + cosh(b*x^2 + a
)^2 + sinh(b*x^2 + a)^2)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int x^3 \sinh^2\left(a + bx^2\right) dx = \begin{cases} \frac{x^4 \sinh^2(a+bx^2)}{8} - \frac{x^4 \cosh^2(a+bx^2)}{8} + \frac{x^2 \sinh(a+bx^2)\cosh(a+bx^2)}{4b} - \frac{\cosh^2(a+bx^2)}{8b^2} & \text{for } b \neq 0 \\ \frac{x^4 \sinh^2(a)}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*sinh(b*x**2+a)**2,x)
```

```
[Out] Piecewise((x**4*sinh(a + b*x**2)**2/8 - x**4*cosh(a + b*x**2)**2/8 + x**2*s
inh(a + b*x**2)*cosh(a + b*x**2)/(4*b) - cosh(a + b*x**2)**2/(8*b**2), Ne(b
, 0)), (x**4*sinh(a)**2/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x^3 \sinh^2\left(a + bx^2\right) dx = -\frac{1}{8} x^4 + \frac{\left(2 b x^2 e^{(2 a)} - e^{(2 a)}\right) e^{\left(2 b x^2\right)}}{32 b^2} - \frac{\left(2 b x^2 + 1\right) e^{\left(-2 b x^2 - 2 a\right)}}{32 b^2}$$

```
[In] integrate(x^3*sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*x^4 + 1/32*(2*b*x^2*e^(2*a) - e^(2*a))*e^(2*b*x^2)/b^2 - 1/32*(2*b*x^2
 + 1)*e^(-2*b*x^2 - 2*a)/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(45) = 90$.

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.76

$$\int x^3 \sinh^2\left(a+bx^2\right)\,dx = -\frac{4\,\left(bx^2+a\right)^2 - 2\,\left(bx^2+a\right)e^{\left(2\,bx^2+2\,a\right)} + 2\,\left(bx^2+a\right)e^{\left(-2\,bx^2-2\,a\right)} + e^{\left(2\,bx^2+2\,a\right)} + e^{\left(-2\,bx^2-2\,a\right)}}{32\,b^2} + \frac{4\,\left(bx^2+a\right)a - ae^{\left(2\,bx^2+2\,a\right)} - \left(2\,ae^{\left(2\,bx^2+2\,a\right)} - a\right)e^{\left(-2\,bx^2-2\,a\right)}}{16\,b^2}$$

```
[In] integrate(x^3*sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/32*(4*(b*x^2 + a)^2 - 2*(b*x^2 + a)*e^(2*b*x^2 + 2*a) + 2*(b*x^2 + a)*e^
(-2*b*x^2 - 2*a) + e^(2*b*x^2 + 2*a) + e^(-2*b*x^2 - 2*a))/b^2 + 1/16*(4*(b
*x^2 + a)*a - a*e^(2*b*x^2 + 2*a) - (2*a*e^(2*b*x^2 + 2*a) - a)*e^(-2*b*x^2
 - 2*a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int x^3 \sinh^2\left(a+bx^2\right)\,dx = -\frac{\frac{\cosh\left(2\,b\,x^2+2\,a\right)}{16} - \frac{b\,x^2\,\sinh\left(2\,b\,x^2+2\,a\right)}{8}}{b^2} - \frac{x^4}{8}$$

```
[In] int(x^3*sinh(a + b*x^2)^2,x)
```

```
[Out] - (cosh(2*a + 2*b*x^2)/16 - (b*x^2*sinh(2*a + 2*b*x^2))/8)/b^2 - x^4/8
```

3.9 $\int x^2 \sinh^2(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int x^2 \sinh^2(a+bx^2)\,dx = -\frac{x^3}{6}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}} - \frac{e^{2a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}+\frac{x\sinh(2a+2bx^2)}{8b}$$

```
[Out] -1/6*x^3+1/8*x*sinh(2*b*x^2+2*a)/b+1/64*erf(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/exp(2*a)-1/64*exp(2*a)*erfi(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2
)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5448, 5433, 5406, 2235, 2236}

$$\int x^2 \sinh^2(a+bx^2)\,dx = \frac{\sqrt{\frac{\pi}{2}}e^{-2a}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}e^{2a}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}+\frac{x\sinh(2a+2bx^2)}{8b}-\frac{x^3}{6}$$

```
[In] Int[x^2*Sinh[a + b*x^2]^2,x]
```

```
[Out] -1/6*x^3 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)*E^(2*a)) - (E^(2
*a)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)) + (x*Sinh[2*a + 2*b*x^
2])/(8*b)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5448

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{x^2}{2}+\frac{1}{2}x^2\cosh\left(2a+2bx^2\right)\right)\,dx\\
&= -\frac{x^3}{6}+\frac{1}{2}\int x^2\cosh\left(2a+2bx^2\right)\,dx\\
&= -\frac{x^3}{6}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}-\frac{\int \sinh\left(2a+2bx^2\right)\,dx}{8b}\\
&= -\frac{x^3}{6}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}+\frac{\int e^{-2a-2bx^2}\,dx}{16b}-\frac{\int e^{2a+2bx^2}\,dx}{16b}\\
&= -\frac{x^3}{6}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}-\frac{e^{2a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int x^2 \sinh^2\left(a+bx^2\right) dx$$
$$= \frac{3\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{b}x\right)(\cosh(2a)-\sinh(2a))-3\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{b}x\right)(\cosh(2a)+\sinh(2a))+8\sqrt{b}x(-4bx^2+3}{192b^{3/2}}$$

```
[In] Integrate[x^2*Sinh[a + b*x^2]^2,x]
[Out] (3*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2*a]) - 3*Sqrt[2*Pi]
*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]) + 8*Sqrt[b]*x*(-4*b*x^2 +
3*Sinh[2*(a + b*x^2)]))/(192*b^(3/2))
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{x^3}{6}-\frac{e^{-2a}x\,e^{-2x^2b}}{16b}+\frac{e^{-2a}\sqrt{\pi}\,\sqrt{2}\,\text{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{64b^{\frac{3}{2}}}+\frac{e^{2a}x\,e^{2x^2b}}{16b}-\frac{e^{2a}\sqrt{\pi}\,\text{erf}\left(\sqrt{-2b}\,x\right)}{32b\sqrt{-2b}}$	90

```
[In] int(x^2*sinh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/6*x^3-1/16*exp(-2*a)/b*x*exp(-2*x^2*b)+1/64*exp(-2*a)/b^(3/2)*Pi^(1/2)*2
^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/16*exp(2*a)/b*x*exp(2*x^2*b)-1/32*exp(2*a)/
b*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(71) = 142.

Time = 0.26 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.31

$$\int x^2 \sinh^2\left(a+bx^2\right) dx =$$
$$-\frac{32\,b^2x^3\cosh\left(bx^2+a\right)^2-12\,bx\cosh\left(bx^2+a\right)^4-48\,bx\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^3-12\,bx\sinh\left(bx^2+\right.}{}$$

```
[In] integrate(x^2*sinh(b*x^2+a)^2,x, algorithm="fricas")
[Out] -1/192*(32*b^2*x^3*cosh(b*x^2 + a)^2 - 12*b*x*cosh(b*x^2 + a)^4 - 48*b*x*co
sh(b*x^2 + a)*sinh(b*x^2 + a)^3 - 12*b*x*sinh(b*x^2 + a)^4 - 3*sqrt(2)*sqrt
(pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) + sinh(2*a))*sinh(b*x^2 + a)
```

```
^2 + cosh(b*x^2 + a)^2*sinh(2*a) + 2*(cosh(b*x^2 + a)*cosh(2*a) + cosh(b*x^
2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(2)*sqrt(-b)*x) - 3*sqr
t(2)*sqrt(pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) - sinh(2*a))*sinh(b
*x^2 + a)^2 - cosh(b*x^2 + a)^2*sinh(2*a) + 2*(cosh(b*x^2 + a)*cosh(2*a) -
cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt(b)*x)
+ 8*(4*b^2*x^3 - 9*b*x*cosh(b*x^2 + a)^2)*sinh(b*x^2 + a)^2 + 12*b*x + 16*(
4*b^2*x^3*cosh(b*x^2 + a) - 3*b*x*cosh(b*x^2 + a)^3)*sinh(b*x^2 + a))/(b^2*
cosh(b*x^2 + a)^2 + 2*b^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) + b^2*sinh(b*x^2
+ a)^2)
```

Sympy [F]

$$\int x^2 \sinh^2\left(a+bx^2\right)\,dx=\int x^2 \sinh^2\left(a+bx^2\right)dx$$

```
[In] integrate(x**2*sinh(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int x^2 \sinh^2\left(a+bx^2\right)\,dx=-\frac{1}{6}\,x^3-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{-b}x\right)e^{(2\,a)}}{64\,\sqrt{-b}b}+\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b}x\right)e^{(-2\,a)}}{64\,b^{\frac{3}{2}}}+\frac{xe^{(2\,bx^2+2\,a)}}{16\,b}-\frac{xe^{(-2\,bx^2-2\,a)}}{16\,b}$$

```
[In] integrate(x^2*sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*x^3 - 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)*
b) + 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16*x
*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int x^2 \sinh^2\left(a+bx^2\right)\,dx = -\frac{1}{6}\,x^3 + \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{2}\sqrt{-b}x\right)e^{(2\,a)}}{64\,\sqrt{-b}b} - \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{2}\sqrt{b}x\right)e^{(-2\,a)}}{64\,b^{\frac{3}{2}}} + \frac{xe^{\left(2\,bx^2+2\,a\right)}}{16\,b} - \frac{xe^{\left(-2\,bx^2-2\,a\right)}}{16\,b}$$

```
[In] integrate(x^2*sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*x^3 + 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)
*b) - 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16
*x*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh^2\left(a+bx^2\right)\,dx = \int x^2\,{\mathrm{sinh}\left(b\,x^2+a\right)}^2\,dx$$

```
[In] int(x^2*sinh(a + b*x^2)^2,x)
```

```
[Out] int(x^2*sinh(a + b*x^2)^2, x)
```

3.10 $\int x \sinh^2\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int x \sinh^2\left(a+bx^2\right)\,dx=-\frac{x^2}{4}+\frac{\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}$$

```
[Out] -1/4*x^2+1/4*cosh(b*x^2+a)*sinh(b*x^2+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5428, 2715, 8}

$$\int x \sinh^2\left(a+bx^2\right)\,dx=\frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{x^2}{4}$$

```
[In] Int[x*Sinh[a + b*x^2]^2,x]
```

```
[Out] -1/4*x^2 + (Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sinh^2(a+bx)\,dx, x, x^2\right) \\
&= \frac{\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b} - \frac{1}{4}\text{Subst}\left(\int 1\,dx, x, x^2\right) \\
&= -\frac{x^2}{4} + \frac{\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int x\sinh^2\left(a+bx^2\right)\,dx = \frac{-2(a+bx^2)+\sinh\left(2(a+bx^2)\right)}{8b}$$

```
[In] Integrate[x*Sinh[a + b*x^2]^2,x]
[Out] (-2*(a + b*x^2) + Sinh[2*(a + b*x^2)])/(8*b)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{\cosh\left(x^2b+a\right)\sinh\left(x^2b+a\right)}{2}-\frac{x^2b}{2}-\frac{a}{2}}{2b}$	34
default	$\frac{\frac{\cosh\left(x^2b+a\right)\sinh\left(x^2b+a\right)}{2}-\frac{x^2b}{2}-\frac{a}{2}}{2b}$	34
risch	$-\frac{x^2}{4}+\frac{e^{2x^2b+2a}}{16b}-\frac{e^{-2x^2b-2a}}{16b}$	39
parallelrisch	$\frac{2\ln\left(1-\tanh\left(\frac{x^2b}{2}+\frac{a}{2}\right)\right)-2\ln\left(\tanh\left(\frac{x^2b}{2}+\frac{a}{2}\right)+1\right)+\sinh\left(2x^2b+2a\right)}{8b}$	52

```
[In] int(x*sinh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(1/2*cosh(b*x^2+a)*sinh(b*x^2+a)-1/2*x^2*b-1/2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int x\sinh^2\left(a+bx^2\right)\,dx=-\frac{bx^2-\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)}{4\,b}$$

```
[In] integrate(x*sinh(b*x^2+a)^2,x, algorithm="fricas")
[Out] -1/4*(b*x^2 - cosh(b*x^2 + a)*sinh(b*x^2 + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(24) = 48.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int x\sinh^2\left(a+bx^2\right)\,dx=\begin{cases}\frac{x^2\sinh^2\left(a+bx^2\right)}{4}-\frac{x^2\cosh^2\left(a+bx^2\right)}{4}+\frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b} & \text{for } b\neq 0\\ \frac{x^2\sinh^2\left(a\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*sinh(b*x**2+a)**2,x)
[Out] Piecewise((x**2*sinh(a + b*x**2)**2/4 - x**2*cosh(a + b*x**2)**2/4 + sinh(a
 + b*x**2)*cosh(a + b*x**2)/(4*b), Ne(b, 0)), (x**2*sinh(a)**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int x\sinh^2\left(a+bx^2\right)\,dx=-\frac{1}{4}\,x^2+\frac{e^{\left(2\,bx^2+2\,a\right)}}{16\,b}-\frac{e^{\left(-2\,bx^2-2\,a\right)}}{16\,b}$$

```
[In] integrate(x*sinh(b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/4*x^2 + 1/16*e^(2*b*x^2 + 2*a)/b - 1/16*e^(-2*b*x^2 - 2*a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(27) = 54$.

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int x \sinh^2\left(a + bx^2\right) dx = -\frac{4\,bx^2 - \left(2\,e^{\left(2\,bx^2+2\,a\right)} - 1\right)e^{\left(-2\,bx^2-2\,a\right)} + 4\,a - e^{\left(2\,bx^2+2\,a\right)}}{16\,b}$$

```
[In] integrate(x*sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*(4*b*x^2 - (2*e^(2*b*x^2 + 2*a) - 1)*e^(-2*b*x^2 - 2*a) + 4*a - e^(2*
b*x^2 + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x \sinh^2\left(a + bx^2\right) dx = \frac{\sinh(2\,b\,x^2 + 2\,a)}{8\,b} - \frac{x^2}{4}$$

```
[In] int(x*sinh(a + b*x^2)^2,x)
```

```
[Out] sinh(2*a + 2*b*x^2)/(8*b) - x^2/4
```

3.11 $\int \sinh^2(a+bx^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \sinh^2\left(a+bx^2\right)\,dx=-\frac{x}{2}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}+\frac{e^{2a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}$$

```
[Out] -1/2*x+1/16*erf(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)/exp(2*a)/b^(1/2)+1/16*e
xp(2*a)*erfi(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5408, 5407, 2235, 2236}

$$\int \sinh^2\left(a+bx^2\right)\,dx=\frac{\sqrt{\frac{\pi}{2}}e^{-2a}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}-\frac{x}{2}$$

```
[In] Int[Sinh[a + b*x^2]^2,x]
```

```
[Out] -1/2*x + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b]*E^(2*a)) + (E^(2*a)
*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5408

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}+\frac{1}{2}\cosh\left(2a+2bx^2\right)\right)dx \\
&= -\frac{x}{2}+\frac{1}{2}\int \cosh\left(2a+2bx^2\right)dx \\
&= -\frac{x}{2}+\frac{1}{4}\int e^{-2a-2bx^2}\,dx+\frac{1}{4}\int e^{2a+2bx^2}\,dx \\
&= -\frac{x}{2}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{b}\,x\right)}{8\sqrt{b}}+\frac{e^{2a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}\,x\right)}{8\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \sinh^2\left(a+bx^2\right)dx = \frac{-4\sqrt{2}\sqrt{b}\,x+\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{b}\,x\right)(\cosh(2a)-\sinh(2a))+\sqrt{\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}\,x\right)(\cosh(2a)+\sinh(2a))}{8\sqrt{2}\sqrt{b}}$$

```
[In] Integrate[Sinh[a + b*x^2]^2,x]
[Out] (-4*Sqrt[2]*Sqrt[b]*x + Sqrt[Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2
*a]) + Sqrt[Pi]*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]))/(8*Sqrt[2]
*Sqrt[b])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{x}{2}+\frac{\mathrm{e}^{-2a}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{16\sqrt{b}}+\frac{\mathrm{e}^{2a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2b}\,x\right)}{8\sqrt{-2b}}$	51

[In] int(sinh(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*x+1/16*exp(-2*a)*Pi^(1/2)*2^(1/2)/b^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/8*e
xp(2*a)*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \sinh^2\left(a+bx^2\right)\,dx = -\frac{\sqrt{2}\sqrt{\pi}\sqrt{-b}(\cosh(2a)+\sinh(2a))\,\mathrm{erf}\left(\sqrt{2}\sqrt{-b}x\right)-\sqrt{2}\sqrt{\pi}\sqrt{b}(\cosh(2a)-\sinh(2a))\,\mathrm{erf}\left(\sqrt{2}\sqrt{b}x\right)+}{16\,b}$$

[In] integrate(sinh(b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/16*(sqrt(2)*sqrt(pi)*sqrt(-b)*(cosh(2*a) + sinh(2*a))*erf(sqrt(2)*sqrt(-
b)*x) - sqrt(2)*sqrt(pi)*sqrt(b)*(cosh(2*a) - sinh(2*a))*erf(sqrt(2)*sqrt(b
)*x) + 8*b*x)/b

Sympy [F]

$$\int \sinh^2\left(a+bx^2\right)\,dx = \int \sinh^2\left(a+bx^2\right)dx$$

[In] integrate(sinh(b*x**2+a)**2,x)

[Out] Integral(sinh(a + b*x**2)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \sinh^2\left(a+bx^2\right)\,dx = \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{-b}x\right)e^{(2\,a)}}{16\,\sqrt{-b}} + \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b}x\right)e^{(-2\,a)}}{16\,\sqrt{b}} - \frac{1}{2}\,x$$

```
[In] integrate(sinh(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/16*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) + 1/16*sqrt(
2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) - 1/2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \sinh^2\left(a+bx^2\right)\,dx = -\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{2}\sqrt{-b}x\right)e^{(2\,a)}}{16\,\sqrt{-b}} - \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{2}\sqrt{b}x\right)e^{(-2\,a)}}{16\,\sqrt{b}} - \frac{1}{2}\,x$$

```
[In] integrate(sinh(b*x^2+a)^2,x, algorithm="giac")
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) - 1/16*sqr
t(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) - 1/2*x
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2\left(a+bx^2\right)\,dx = \int {\mathrm{sinh}\left(b\,x^2+a\right)}^2\,dx$$

```
[In] int(sinh(a + b*x^2)^2,x)
[Out] int(sinh(a + b*x^2)^2, x)
```

3.12 $\int \frac{\sinh^2(a+bx^2)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{\sinh^2(a+bx^2)}{x} dx = \frac{1}{4}\cosh(2a)\text{Chi}(2bx^2) - \frac{\log(x)}{2} + \frac{1}{4}\sinh(2a)\text{Shi}(2bx^2)$$

```
[Out] 1/4*Chi(2*b*x^2)*cosh(2*a)-1/2*ln(x)+1/4*Shi(2*b*x^2)*sinh(2*a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5448, 5427, 5425, 5424}

$$\int \frac{\sinh^2(a+bx^2)}{x} dx = \frac{1}{4}\cosh(2a)\text{Chi}(2bx^2) + \frac{1}{4}\sinh(2a)\text{Shi}(2bx^2) - \frac{\log(x)}{2}$$

```
[In] Int[Sinh[a + b*x^2]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^2])/4 - Log[x]/2 + (Sinh[2*a]*SinhIntegral[2*
b*x^2])/4
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5448

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2x} + \frac{\cosh(2a + 2bx^2)}{2x}\right) dx \\
&= -\frac{\log(x)}{2} + \frac{1}{2}\int \frac{\cosh(2a + 2bx^2)}{x}\, dx \\
&= -\frac{\log(x)}{2} + \frac{1}{2}\cosh(2a)\int \frac{\cosh(2bx^2)}{x}\, dx + \frac{1}{2}\sinh(2a)\int \frac{\sinh(2bx^2)}{x}\, dx \\
&= \frac{1}{4}\cosh(2a)\text{Chi}(2bx^2) - \frac{\log(x)}{2} + \frac{1}{4}\sinh(2a)\text{Shi}(2bx^2)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\sinh^2(a + bx^2)}{x}\, dx = \frac{1}{4}\left(\cosh(2a)\text{Chi}(2bx^2) - 2\log(x) + \sinh(2a)\text{Shi}(2bx^2)\right)$$

```
[In] Integrate[Sinh[a + b*x^2]^2/x,x]
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^2] - 2*Log[x] + Sinh[2*a]*SinhIntegral[2*b*x^
2])/4
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{\ln(x)}{2} - \frac{e^{-2a}\,\text{Ei}_1(2x^2b)}{8} - \frac{e^{2a}\,\text{Ei}_1(-2x^2b)}{8}$	34

```
[In] int(sinh(b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x)-1/8*exp(-2*a)*Ei(1,2*x^2*b)-1/8*exp(2*a)*Ei(1,-2*x^2*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{\sinh^2(a+bx^2)}{x}\,dx = \frac{1}{8}\left(\mathrm{Ei}\left(2\,bx^2\right)+\mathrm{Ei}\left(-2\,bx^2\right)\right)\cosh(2\,a) + \frac{1}{8}\left(\mathrm{Ei}\left(2\,bx^2\right)-\mathrm{Ei}\left(-2\,bx^2\right)\right)\sinh(2\,a) - \frac{1}{2}\log(x)$$

```
[In] integrate(sinh(b*x^2+a)^2/x,x, algorithm="fricas")
[Out] 1/8*(Ei(2*b*x^2) + Ei(-2*b*x^2))*cosh(2*a) + 1/8*(Ei(2*b*x^2) - Ei(-2*b*x^2
))*sinh(2*a) - 1/2*log(x)
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx^2)}{x}\,dx = \int \frac{\sinh^2(a+bx^2)}{x}\,dx$$

```
[In] integrate(sinh(b*x**2+a)**2/x,x)
[Out] Integral(sinh(a + b*x**2)**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\sinh^2(a+bx^2)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)} - \frac{1}{2}\log(x)$$

```
[In] integrate(sinh(b*x^2+a)^2/x,x, algorithm="maxima")
[Out] 1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) - 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\sinh^2(a+bx^2)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)} - \frac{1}{4}\,\log\left(bx^2\right)$$

```
[In] integrate(sinh(b*x^2+a)^2/x,x, algorithm="giac")
[Out] 1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) - 1/4*log(b*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx^2)}{x}\,dx = \int \frac{\sinh(b\,x^2+a)^2}{x}\,dx$$

```
[In] int(sinh(a + b*x^2)^2/x,x)
[Out] int(sinh(a + b*x^2)^2/x, x)
```

3.13 $\int \frac{\sinh^2(a+bx^2)}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{\sinh^2(a+bx^2)}{x^2} dx = -\frac{1}{2}\sqrt{b}e^{-2a}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{b}x\right) + \frac{1}{2}\sqrt{b}e^{2a}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{b}x\right) - \frac{\sinh^2(a+bx^2)}{x}$$

```
[Out] -sinh(b*x^2+a)^2/x-1/4*erf(x*2^(1/2)*b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/exp(
2*a)+1/4*exp(2*a)*erfi(x*2^(1/2)*b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5438, 5736, 5422, 5406, 2235, 2236}

$$\int \frac{\sinh^2(a+bx^2)}{x^2} dx = -\frac{1}{2}\sqrt{\frac{\pi}{2}}e^{-2a}\sqrt{b}\text{erf}\left(\sqrt{2}\sqrt{b}x\right) + \frac{1}{2}\sqrt{\frac{\pi}{2}}e^{2a}\sqrt{b}\text{erfi}\left(\sqrt{2}\sqrt{b}x\right) - \frac{\sinh^2(a+bx^2)}{x}$$

```
[In] Int[Sinh[a + b*x^2]^2/x^2,x]
```

```
[Out] -1/2*(Sqrt[b]*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/E^(2*a) + (Sqrt[b]*E^(2*a)
*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/2 - Sinh[a + b*x^2]^2/x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5422

```
Int[((a_.) + (b_.)*Sinh[u_])^(p_.), x_Symbol] :> Int[(a + b*Sinh[ExpandToSu
m[u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatc
hQ[u, x]
```

Rule 5438

```
Int[(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[-Sinh[
a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Dist[b*n*(p/(n - 1)), Int[Sinh[a + b
*x^n]^(p - 1)*Cosh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n, p
] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]
```

Rule 5736

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sinh^2\left(a+bx^2\right)}{x}+(4b)\int \cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)\,dx\\
&= -\frac{\sinh^2\left(a+bx^2\right)}{x}+(2b)\int \sinh\left(2\left(a+bx^2\right)\right)\,dx\\
&= -\frac{\sinh^2\left(a+bx^2\right)}{x}+(2b)\int \sinh\left(2a+2bx^2\right)\,dx\\
&= -\frac{\sinh^2\left(a+bx^2\right)}{x}-b\int e^{-2a-2bx^2}\,dx+b\int e^{2a+2bx^2}\,dx\\
&= -\frac{1}{2}\sqrt{b}e^{-2a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)+\frac{1}{2}\sqrt{b}e^{2a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}x\right)-\frac{\sinh^2\left(a+bx^2\right)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(a+bx^2)}{x^2}\,dx = \frac{\sqrt{b}\sqrt{2\pi}x\text{erf}\left(\sqrt{2}\sqrt{b}x\right)(-\cosh(2a)+\sinh(2a))+\sqrt{b}\sqrt{2\pi}x\text{erfi}\left(\sqrt{2}\sqrt{b}x\right)(\cosh(2a)+\sinh(2a))-4\sinh^2}{4x}$$

```
[In] Integrate[Sinh[a + b*x^2]^2/x^2,x]
```

```
[Out] (Sqrt[b]*Sqrt[2*Pi]*x*Erf[Sqrt[2]*Sqrt[b]*x]*(-Cosh[2*a] + Sinh[2*a]) + Sqr
t[b]*Sqrt[2*Pi]*x*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]) - 4*Sinh[
a + b*x^2]^2)/(4*x)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{1}{2x}-\frac{e^{-2a}e^{-2x^2b}}{4x}-\frac{e^{-2a}\sqrt{b}\sqrt{\pi}\sqrt{2}\,\text{erf}\left(x\sqrt{2}\sqrt{b}\right)}{4}-\frac{e^{2a}e^{2x^2b}}{4x}+\frac{e^{2a}b\sqrt{\pi}\,\text{erf}\left(\sqrt{-2b}\,x\right)}{2\sqrt{-2b}}$	86

```
[In] int(sinh(b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x-1/4*exp(-2*a)/x*exp(-2*x^2*b)-1/4*exp(-2*a)*b^(1/2)*Pi^(1/2)*2^(1/2)*
erf(x*2^(1/2)*b^(1/2))-1/4*exp(2*a)/x*exp(2*x^2*b)+1/2*exp(2*a)*b*Pi^(1/2)/
(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(64) = 128.

Time = 0.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.50

$$\int \frac{\sinh^2(a+bx^2)}{x^2}\,dx = -\frac{\cosh(bx^2+a)^4+4\cosh(bx^2+a)\sinh(bx^2+a)^3+\sinh(bx^2+a)^4+\sqrt{2}\sqrt{\pi}\Big(x\cosh(bx^2+a)^2\cosh(2}{}$$

```
[In] integrate(sinh(b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cosh(b*x^2 + a)^4 + 4*cosh(b*x^2 + a)*sinh(b*x^2 + a)^3 + sinh(b*x^2
+ a)^4 + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh(2*a) + x*cosh(b*x^2 + a
```

```
)^2*sinh(2*a) + (x*cosh(2*a) + x*sinh(2*a))*sinh(b*x^2 + a)^2 + 2*(x*cosh(b
*x^2 + a)*cosh(2*a) + x*cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(-b
)*erf(sqrt(2)*sqrt(-b)*x) + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh(2*a)
 - x*cosh(b*x^2 + a)^2*sinh(2*a) + (x*cosh(2*a) - x*sinh(2*a))*sinh(b*x^2 +
 a)^2 + 2*(x*cosh(b*x^2 + a)*cosh(2*a) - x*cosh(b*x^2 + a)*sinh(2*a))*sinh(
b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt(b)*x) + 2*(3*cosh(b*x^2 + a)^2 - 1)*si
nh(b*x^2 + a)^2 - 2*cosh(b*x^2 + a)^2 + 4*(cosh(b*x^2 + a)^3 - cosh(b*x^2 +
 a))*sinh(b*x^2 + a) + 1)/(x*cosh(b*x^2 + a)^2 + 2*x*cosh(b*x^2 + a)*sinh(b
*x^2 + a) + x*sinh(b*x^2 + a)^2)
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx^2)}{x^2}\,dx = \int \frac{\sinh^2(a+bx^2)}{x^2}\,dx$$

```
[In] integrate(sinh(b*x**2+a)**2/x**2,x)
[Out] Integral(sinh(a + b*x**2)**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \frac{\sinh^2(a+bx^2)}{x^2}\,dx = -\frac{\sqrt{2}\sqrt{bx^2}e^{(-2a)}\Gamma\left(-\frac{1}{2},2bx^2\right)}{8x} - \frac{\sqrt{2}\sqrt{-bx^2}e^{(2a)}\Gamma\left(-\frac{1}{2},-2bx^2\right)}{8x} + \frac{1}{2x}$$

```
[In] integrate(sinh(b*x^2+a)^2/x^2,x, algorithm="maxima")
[Out] -1/8*sqrt(2)*sqrt(b*x^2)*e^(-2*a)*gamma(-1/2, 2*b*x^2)/x - 1/8*sqrt(2)*sqrt
(-b*x^2)*e^(2*a)*gamma(-1/2, -2*b*x^2)/x + 1/2/x
```

Giac [F]

$$\int \frac{\sinh^2(a+bx^2)}{x^2}\,dx = \int \frac{\sinh(bx^2+a)^2}{x^2}\,dx$$

```
[In] integrate(sinh(b*x^2+a)^2/x^2,x, algorithm="giac")
[Out] integrate(sinh(b*x^2 + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx^2)}{x^2}\,dx = \int \frac{\sinh(b\,x^2+a)^2}{x^2}\,dx$$

```
[In] int(sinh(a + b*x^2)^2/x^2,x)
```

```
[Out] int(sinh(a + b*x^2)^2/x^2, x)
```

3.14 $\int \frac{\sinh^2(a+bx^2)}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{\sinh^2(a+bx^2)}{x^3} dx = \frac{1}{4x^2} - \frac{\cosh(2(a+bx^2))}{4x^2} + \frac{1}{2}b\text{Chi}(2bx^2)\sinh(2a) + \frac{1}{2}b\cosh(2a)\text{Shi}(2bx^2)$$

```
[Out] 1/4/x^2-1/4*cosh(2*b*x^2+2*a)/x^2+1/2*b*cosh(2*a)*Shi(2*b*x^2)+1/2*b*Chi(2*
b*x^2)*sinh(2*a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5448, 5429, 3378, 3384, 3379, 3382}

$$\int \frac{\sinh^2(a+bx^2)}{x^3} dx = \frac{1}{2}b\sinh(2a)\text{Chi}(2bx^2) + \frac{1}{2}b\cosh(2a)\text{Shi}(2bx^2) - \frac{\cosh(2(a+bx^2))}{4x^2} + \frac{1}{4x^2}$$

```
[In] Int[Sinh[a + b*x^2]^2/x^3,x]
```

```
[Out] 1/(4*x^2) - Cosh[2*(a + b*x^2)]/(4*x^2) + (b*CoshIntegral[2*b*x^2]*Sinh[2*a
])/2 + (b*Cosh[2*a]*SinhIntegral[2*b*x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5448

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{1}{2x^3}+\frac{\cosh\left(2a+2bx^2\right)}{2x^3}\right) dx\\ &= \frac{1}{4x^2}+\frac{1}{2}\int \frac{\cosh\left(2a+2bx^2\right)}{x^3}\,dx\\ &= \frac{1}{4x^2}+\frac{1}{4}\text{Subst}\left(\int \frac{\cosh(2a+2bx)}{x^2}\,dx,x,x^2\right)\end{aligned}$$

$$= \frac{1}{4x^2} - \frac{\cosh\left(2(a+bx^2)\right)}{4x^2} + \frac{1}{2} b\text{Subst}\left(\int \frac{\sinh(2a+2bx)}{x} dx, x, x^2\right)$$
$$= \frac{1}{4x^2} - \frac{\cosh\left(2(a+bx^2)\right)}{4x^2} + \frac{1}{2}(b\cosh(2a))\text{Subst}\left(\int \frac{\sinh(2bx)}{x} dx, x, x^2\right) + \frac{1}{2}(b\sinh(2a))\text{Subst}\left(\int \frac{\cosh(2bx)}{x} dx, x, x^2\right)$$
$$= \frac{1}{4x^2} - \frac{\cosh\left(2(a+bx^2)\right)}{4x^2} + \frac{1}{2} b\text{Chi}\left(2bx^2\right)\sinh(2a) + \frac{1}{2} b\cosh(2a)\text{Shi}\left(2bx^2\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\sinh^2\left(a+bx^2\right)}{x^3} dx = \frac{1}{2}\left(b\text{Chi}\left(2bx^2\right)\sinh(2a) - \frac{\sinh^2\left(a+bx^2\right)}{x^2} + b\cosh(2a)\text{Shi}\left(2bx^2\right)\right)$$

```
[In] Integrate[Sinh[a + b*x^2]^2/x^3,x]
```

```
[Out] (b*CoshIntegral[2*b*x^2]*Sinh[2*a] - Sinh[a + b*x^2]^2/x^2 + b*Cosh[2*a]*Si
nhIntegral[2*b*x^2])/2
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{2\ \text{Ei}_1\left(-2x^2 b\right)\text{e}^{2a} b\, x^2 - 2\ \text{Ei}_1\left(2x^2 b\right)\text{e}^{-2a} b\, x^2 + \text{e}^{2x^2 b+2a} + \text{e}^{-2x^2 b-2a} - 2}{8x^2}$	66

```
[In] int(sinh(b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(2*Ei(1,-2*x^2*b)*exp(2*a)*b*x^2-2*Ei(1,2*x^2*b)*exp(-2*a)*b*x^2+exp(2
*b*x^2+2*a)+exp(-2*b*x^2-2*a)-2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\sinh^2(a+bx^2)}{x^3}\,dx = -\frac{\cosh(bx^2+a)^2 - (bx^2\mathrm{Ei}(2\,bx^2) - bx^2\mathrm{Ei}(-2\,bx^2))\cosh(2\,a) + \sinh(bx^2+a)^2 - (bx^2\mathrm{Ei}(2\,bx^2) + bx^2\mathrm{Ei}(}{4\,x^2}$$

```
[In] integrate(sinh(b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(cosh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) - b*x^2*Ei(-2*b*x^2))*cosh(2*a
) + sinh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) + b*x^2*Ei(-2*b*x^2))*sinh(2*a)
- 1)/x^2
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx^2)}{x^3}\,dx = \int \frac{\sinh^2(a+bx^2)}{x^3}\,dx$$

```
[In] integrate(sinh(b*x**2+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x**2)**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{\sinh^2(a+bx^2)}{x^3}\,dx = -\frac{1}{4}\,be^{(-2\,a)}\Gamma(-1,2\,bx^2) + \frac{1}{4}\,be^{(2\,a)}\Gamma(-1,-2\,bx^2) + \frac{1}{4\,x^2}$$

```
[In] integrate(sinh(b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^(-2*a)*gamma(-1, 2*b*x^2) + 1/4*b*e^(2*a)*gamma(-1, -2*b*x^2) + 1/
4/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(50) = 100$.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.21

$$\int \frac{\sinh^2(a+bx^2)}{x^3}\,dx = \frac{2\,(bx^2+a)b^2\mathrm{Ei}(2\,bx^2)\,e^{(2\,a)} - 2\,ab^2\mathrm{Ei}(2\,bx^2)\,e^{(2\,a)} - 2\,(bx^2+a)b^2\mathrm{Ei}(-2\,bx^2)\,e^{(-2\,a)} + 2\,ab^2\mathrm{Ei}(-2\,bx^2)\,e^{(-2\,a)}}{8\,b^2x^2}$$

```
[In] integrate(sinh(b*x^2+a)^2/x^3,x, algorithm="giac")
[Out] 1/8*(2*(b*x^2 + a)*b^2*Ei(2*b*x^2)*e^(2*a) - 2*a*b^2*Ei(2*b*x^2)*e^(2*a) -
2*(b*x^2 + a)*b^2*Ei(-2*b*x^2)*e^(-2*a) + 2*a*b^2*Ei(-2*b*x^2)*e^(-2*a) - b
^2*e^(2*b*x^2 + 2*a) - b^2*e^(-2*b*x^2 - 2*a) + 2*b^2)/(b^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx^2)}{x^3}\,dx = \int \frac{\sinh(b\,x^2+a)^2}{x^3}\,dx$$

```
[In] int(sinh(a + b*x^2)^2/x^3,x)
[Out] int(sinh(a + b*x^2)^2/x^3, x)
```

3.15 $\int x^3 \sinh^3(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int x^3 \sinh^3\left(a+bx^2\right)\,dx = -\frac{x^2\cosh(a+bx^2)}{3b}+\frac{\sinh(a+bx^2)}{3b^2}+\frac{x^2\cosh(a+bx^2)\sinh^2(a+bx^2)}{6b}-\frac{\sinh^3(a+bx^2)}{18b^2}$$

```
[Out] -1/3*x^2*cosh(b*x^2+a)/b+1/3*sinh(b*x^2+a)/b^2+1/6*x^2*cosh(b*x^2+a)*sinh(b
*x^2+a)^2/b-1/18*sinh(b*x^2+a)^3/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5428, 3391, 3377, 2717}

$$\int x^3 \sinh^3\left(a+bx^2\right)\,dx = -\frac{\sinh^3(a+bx^2)}{18b^2}+\frac{\sinh(a+bx^2)}{3b^2}-\frac{x^2\cosh(a+bx^2)}{3b}+\frac{x^2\sinh^2(a+bx^2)\cosh(a+bx^2)}{6b}$$

```
[In] Int[x^3*Sinh[a + b*x^2]^3,x]
```

```
[Out] -1/3*(x^2*Cosh[a + b*x^2])/b + Sinh[a + b*x^2]/(3*b^2) + (x^2*Cosh[a + b*x^
2]*Sinh[a + b*x^2]^2)/(6*b) - Sinh[a + b*x^2]^3/(18*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\sinh^3(a+bx)\,dx, x, x^2\right)\\
&= \frac{x^2\cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)}{6b}-\frac{\sinh^3\left(a+bx^2\right)}{18b^2}-\frac{1}{3}\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, x^2\right)\\
&= -\frac{x^2\cosh\left(a+bx^2\right)}{3b}+\frac{x^2\cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)}{6b}\\
&\quad -\frac{\sinh^3\left(a+bx^2\right)}{18b^2}+\frac{\text{Subst}\left(\int\cosh(a+bx)\,dx, x, x^2\right)}{3b}\\
&= -\frac{x^2\cosh\left(a+bx^2\right)}{3b}+\frac{\sinh\left(a+bx^2\right)}{3b^2}+\frac{x^2\cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)}{6b}-\frac{\sinh^3\left(a+bx^2\right)}{18b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int x^3 \sinh^3\left(a+bx^2\right)\,dx \\ = -\frac{27bx^2\cosh\left(a+bx^2\right)-3bx^2\cosh\left(3(a+bx^2)\right)-27\sinh\left(a+bx^2\right)+\sinh\left(3(a+bx^2)\right)}{72b^2}$$

```
[In] Integrate[x^3*Sinh[a + b*x^2]^3,x]
```

```
[Out] -1/72*(27*b*x^2*Cosh[a + b*x^2] - 3*b*x^2*Cosh[3*(a + b*x^2)] - 27*Sinh[a +
 b*x^2] + Sinh[3*(a + b*x^2)])/b^2
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18

method	result
risch	$\frac{(3x^2b-1)\mathrm{e}^{3x^2b+3a}}{144b^2}-\frac{3(x^2b-1)\mathrm{e}^{x^2b+a}}{16b^2}-\frac{3(x^2b+1)\mathrm{e}^{-x^2b-a}}{16b^2}+\frac{(3x^2b+1)\mathrm{e}^{-3x^2b-3a}}{144b^2}$
parallelrisch	$\frac{-27\cosh(x^2b+a)b\,x^2+3x^2b\cosh(3x^2b+3a)-24x^2b+27\sinh(x^2b+a)-24\ln\left(1-\tanh\left(\frac{x^2b}{2}+\frac{a}{2}\right)\right)+24\ln\left(\tanh\left(\frac{x^2b}{2}+\frac{a}{2}\right)+1\right)-}{72b^2}$

```
[In] int(x^3*sinh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/144*(3*b*x^2-1)/b^2*exp(3*b*x^2+3*a)-3/16*(b*x^2-1)/b^2*exp(b*x^2+a)-3/16
*(b*x^2+1)/b^2*exp(-b*x^2-a)+1/144*(3*b*x^2+1)/b^2*exp(-3*b*x^2-3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int x^3 \sinh^3\left(a+bx^2\right)\,dx \\ = \frac{3\,bx^2\cosh\left(bx^2+a\right)^3+9\,bx^2\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^2-27\,bx^2\cosh\left(bx^2+a\right)-\sinh\left(bx^2+a\right)^3-3\,(}{72\,b^2}$$

```
[In] integrate(x^3*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72*(3*b*x^2*cosh(b*x^2 + a)^3 + 9*b*x^2*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2
 - 27*b*x^2*cosh(b*x^2 + a) - sinh(b*x^2 + a)^3 - 3*(cosh(b*x^2 + a)^2 - 9)
*sinh(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int x^3 \sinh^3\left(a+bx^2\right)\,dx = \begin{cases} \frac{x^2\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{2b} - \frac{x^2\cosh^3\left(a+bx^2\right)}{3b} - \frac{7\sinh^3\left(a+bx^2\right)}{18b^2} + \frac{\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{3b^2} & \text{for } b \neq 0 \\ \frac{x^4\sinh^3\left(a\right)}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*sinh(b*x**2+a)**3,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x**2)**2*cosh(a + b*x**2)/(2*b) - x**2*cosh(a +
b*x**2)**3/(3*b) - 7*sinh(a + b*x**2)**3/(18*b**2) + sinh(a + b*x**2)*cosh(
a + b*x**2)**2/(3*b**2), Ne(b, 0)), (x**4*sinh(a)**3/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.27

$$\int x^3 \sinh^3\left(a+bx^2\right)\,dx = \frac{\left(3\,bx^2e^{(3\,a)} - e^{(3\,a)}\right)e^{(3\,bx^2)}}{144\,b^2} - \frac{3\,\left(bx^2e^a - e^a\right)e^{(bx^2)}}{16\,b^2} - \frac{3\,\left(bx^2+1\right)e^{(-bx^2-a)}}{16\,b^2} + \frac{\left(3\,bx^2+1\right)e^{(-3\,bx^2-3\,a)}}{144\,b^2}$$

```
[In] integrate(x^3*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/144*(3*b*x^2*e^(3*a) - e^(3*a))*e^(3*b*x^2)/b^2 - 3/16*(b*x^2*e^a - e^a)*
e^(b*x^2)/b^2 - 3/16*(b*x^2 + 1)*e^(-b*x^2 - a)/b^2 + 1/144*(3*b*x^2 + 1)*e
^(-3*b*x^2 - 3*a)/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.43

$$\int x^3 \sinh^3\left(a+bx^2\right)\,dx = \frac{3\,\left(bx^2+a\right)e^{(3\,bx^2+3\,a)} - 27\,\left(bx^2+a\right)e^{(bx^2+a)} - 27\,\left(bx^2+a\right)e^{(-bx^2-a)} + 3\,\left(bx^2+a\right)e^{(-3\,bx^2-3\,a)} - e^{(3\,bx^2+3\,a)} +}{144\,b^2} - \frac{ae^{(3\,bx^2+3\,a)} - 9\,ae^{(bx^2+a)} - \left(9\,ae^{(2\,bx^2+2\,a)} - a\right)e^{(-3\,bx^2-3\,a)}}{48\,b^2}$$

```
[In] integrate(x^3*sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/144*(3*(b*x^2 + a)*e^(3*b*x^2 + 3*a) - 27*(b*x^2 + a)*e^(b*x^2 + a) - 27*
(b*x^2 + a)*e^(-b*x^2 - a) + 3*(b*x^2 + a)*e^(-3*b*x^2 - 3*a) - e^(3*b*x^2
+ 3*a) + 27*e^(b*x^2 + a) - 27*e^(-b*x^2 - a) + e^(-3*b*x^2 - 3*a))/b^2 - 1
/48*(a*e^(3*b*x^2 + 3*a) - 9*a*e^(b*x^2 + a) - (9*a*e^(2*b*x^2 + 2*a) - a)*
e^(-3*b*x^2 - 3*a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x^3 \sinh^3 \left(a + bx^2\right) \, dx = \frac{\frac{x^2 \cosh\left(b\,x^2+a\right)^3}{6} - \frac{x^2 \cosh\left(b\,x^2+a\right)}{2}}{b} + \frac{7\sinh(b\,x^2+a)}{18\,b^2} - \frac{\cosh(b\,x^2+a)^2\sinh(b\,x^2+a)}{18\,b^2}$$

```
[In] int(x^3*sinh(a + b*x^2)^3,x)
```

```
[Out] ((x^2*cosh(a + b*x^2)^3)/6 - (x^2*cosh(a + b*x^2))/2)/b + (7*sinh(a + b*x^2
))/(18*b^2) - (cosh(a + b*x^2)^2*sinh(a + b*x^2))/(18*b^2)
```

3.16 $\int x^2 \sinh^3\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 160

$$\int x^2\sinh^3\left(a+bx^2\right)\,dx=-\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(3a+3bx^2\right)}{24b}+\frac{3e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}+\frac{3e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{e^{3a}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}$$

```
[Out] -3/8*x*cosh(b*x^2+a)/b+1/24*x*cosh(3*b*x^2+3*a)/b-1/288*erf(x*3^(1/2)*b^(1/
2))*3^(1/2)*Pi^(1/2)/b^(3/2)/exp(3*a)-1/288*exp(3*a)*erfi(x*3^(1/2)*b^(1/2)
)*3^(1/2)*Pi^(1/2)/b^(3/2)+3/32*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)+3/32
*exp(a)*erfi(x*b^(1/2))*Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5448, 5432, 5407, 2235, 2236}

$$\int x^2\sinh^3\left(a+bx^2\right)\,dx=\frac{3\sqrt{\pi}e^{-a}\mathrm{erf}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}e^{-3a}\mathrm{erf}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}+\frac{3\sqrt{\pi}e^{a}\mathrm{erfi}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}e^{3a}\mathrm{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}-\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(3a+3bx^2\right)}{24b}$$

```
[In] Int[x^2*Sinh[a + b*x^2]^3,x]
```

```
[Out] (-3*x*Cosh[a + b*x^2])/(8*b) + (x*Cosh[3*a + 3*b*x^2])/(24*b) + (3*Sqrt[Pi]
*Erf[Sqrt[b]*x])/(32*b^(3/2)*E^a) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[b]*x])/(96
*b^(3/2)*E^(3*a)) + (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(32*b^(3/2)) - (E^(3*a
)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(96*b^(3/2))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5448

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{4}x^2\sinh\left(a+bx^2\right)+\frac{1}{4}x^2\sinh\left(3a+3bx^2\right)\right)\,dx\\
&= \frac{1}{4}\int x^2\sinh\left(3a+3bx^2\right)\,dx-\frac{3}{4}\int x^2\sinh\left(a+bx^2\right)\,dx\\
&= -\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(3a+3bx^2\right)}{24b}-\frac{\int\cosh\left(3a+3bx^2\right)\,dx}{24b}+\frac{3\int\cosh\left(a+bx^2\right)\,dx}{8b}
\end{aligned}$$

$$= -\frac{3x\cosh(a+bx^2)}{8b} + \frac{x\cosh(3a+3bx^2)}{24b} - \frac{\int e^{-3a-3bx^2}\,dx}{48b} - \frac{\int e^{3a+3bx^2}\,dx}{48b} + \frac{3\int e^{-a-bx^2}\,dx}{16b} + \frac{3\int e^{a+bx^2}\,dx}{16b}$$

$$= -\frac{3x\cosh(a+bx^2)}{8b} + \frac{x\cosh(3a+3bx^2)}{24b} + \frac{3e^{-a}\sqrt{\pi}\text{erf}\left(\sqrt{b}x\right)}{32b^{3/2}} - \frac{e^{-3a}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}} + \frac{3e^{a}\sqrt{\pi}\text{erfi}\left(\sqrt{b}x\right)}{32b^{3/2}} - \frac{e^{3a}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.15

$$\int x^2\sinh^3\left(a+bx^2\right)\,dx$$

$$= \frac{-108\sqrt{b}x\cosh(a+bx^2) + 12\sqrt{b}x\cosh(3(a+bx^2)) + 27\sqrt{\pi}\cosh(a)\text{erfi}\left(\sqrt{b}x\right) - \sqrt{3\pi}\cosh(3a)\text{erfi}\left(\sqrt{3}\sqrt{b}\right.}{}$$

```
[In] Integrate[x^2*Sinh[a + b*x^2]^3,x]
```

```
[Out] (-108*Sqrt[b]*x*Cosh[a + b*x^2] + 12*Sqrt[b]*x*Cosh[3*(a + b*x^2)] + 27*Sqr
t[Pi]*Cosh[a]*Erfi[Sqrt[b]*x] - Sqrt[3*Pi]*Cosh[3*a]*Erfi[Sqrt[3]*Sqrt[b]*x
] + 27*Sqrt[Pi]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + 27*Sqrt[Pi]*Erfi[Sqrt[
b]*x]*Sinh[a] - Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + Sqrt[3*Pi]*E
rf[Sqrt[3]*Sqrt[b]*x]*(-Cosh[3*a] + Sinh[3*a]))/(288*b^(3/2))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

method	result
risch	$\frac{e^{-3a}x\,e^{-3x^2b}}{48b} - \frac{e^{-3a}\sqrt{\pi}\sqrt{3}\,\text{erf}\left(x\sqrt{3}\sqrt{b}\right)}{288b^{\frac{3}{2}}} - \frac{3\,e^{-a}x\,e^{-x^2b}}{16b} + \frac{3\,\text{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\,e^{-a}}{32b^{\frac{3}{2}}} - \frac{3\,e^{a}e^{x^2b}x}{16b} + \frac{3\,e^{a}\sqrt{\pi}\,\text{erf}\left(\sqrt{-b}\,x\right)}{32b\sqrt{-b}} + \frac{e^{3a}}{}$

```
[In] int(x^2*sinh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/exp(a)^3/b*x*exp(-3*x^2*b)-1/288/exp(a)^3/b^(3/2)*Pi^(1/2)*3^(1/2)*erf
(x*3^(1/2)*b^(1/2))-3/16/exp(a)/b*x*exp(-x^2*b)+3/32*erf(x*b^(1/2))*Pi^(1/2
)/b^(3/2)/exp(a)-3/16*exp(a)*exp(x^2*b)*x/b+3/32*exp(a)/b*Pi^(1/2)/(-b)^(1/
2)*erf((-b)^(1/2)*x)+1/48*exp(a)^3/b*x*exp(3*x^2*b)-1/96*exp(a)^3/b*Pi^(1/2
)/(-3*b)^(1/2)*erf((-3*b)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 904 vs. $2(114) = 228$.

Time = 0.26 (sec) , antiderivative size = 904, normalized size of antiderivative = 5.65

$$\int x^2 \sinh^3\left(a + bx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/288*(6*b*x*cosh(b*x^2 + a)^6 + 36*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5 +
 6*b*x*sinh(b*x^2 + a)^6 - 54*b*x*cosh(b*x^2 + a)^4 + 18*(5*b*x*cosh(b*x^2
+ a)^2 - 3*b*x)*sinh(b*x^2 + a)^4 - 54*b*x*cosh(b*x^2 + a)^2 + 24*(5*b*x*co
sh(b*x^2 + a)^3 - 9*b*x*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt(3)*sqrt(p
i)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) + sinh(3*a))*sinh(b*x^2 + a)^3
 + cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a) + cosh(b*x^2
+ a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(3*a) + cosh(b
*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(3)*sqrt(-b)*x) -
sqrt(3)*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) - sinh(3*a))*sin
h(b*x^2 + a)^3 - cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a)
 - cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh
(3*a) - cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(3)*s
qrt(b)*x) - 27*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) + sinh(a))*si
nh(b*x^2 + a)^3 + cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a) +
cosh(b*x^2 + a)*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(a) +
 cosh(b*x^2 + a)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + 27*
sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a)^3
 - cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a) - cosh(b*x^2 + a)
*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(a) - cosh(b*x^2 + a
)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + 18*(5*b*x*cosh(b*x^2
 + a)^4 - 18*b*x*cosh(b*x^2 + a)^2 - 3*b*x)*sinh(b*x^2 + a)^2 + 6*b*x + 36*
(b*x*cosh(b*x^2 + a)^5 - 6*b*x*cosh(b*x^2 + a)^3 - 3*b*x*cosh(b*x^2 + a))*s
inh(b*x^2 + a))/(b^2*cosh(b*x^2 + a)^3 + 3*b^2*cosh(b*x^2 + a)^2*sinh(b*x^2
 + a) + 3*b^2*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 + b^2*sinh(b*x^2 + a)^3)
```

Sympy [F]

$$\int x^2 \sinh^3\left(a + bx^2\right)\,dx = \int x^2 \sinh^3\left(a + bx^2\right)dx$$

```
[In] integrate(x**2*sinh(b*x**2+a)**3,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01

$$\int x^2 \sinh^3(a+bx^2)\,dx = -\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{-b}x\right)e^{(3a)}}{288\sqrt{-b}b} - \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{b}x\right)e^{(-3a)}}{288\,b^{\frac{3}{2}}} + \frac{xe^{(3bx^2+3a)}}{48\,b} - \frac{3\,xe^{(bx^2+a)}}{16\,b} - \frac{3\,xe^{(-bx^2-a)}}{16\,b} + \frac{xe^{(-3bx^2-3a)}}{48\,b} + \frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)e^{(-a)}}{32\,b^{\frac{3}{2}}} + \frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}x\right)e^{a}}{32\sqrt{-b}b}$$

```
[In] integrate(x^2*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/288*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/(sqrt(-b)*b) - 1/28
8*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/b^(3/2) + 1/48*x*e^(3*b*
x^2 + 3*a)/b - 3/16*x*e^(b*x^2 + a)/b - 3/16*x*e^(-b*x^2 - a)/b + 1/48*x*e^
(-3*b*x^2 - 3*a)/b + 3/32*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^(3/2) + 3/32*sqr
t(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04

$$\int x^2 \sinh^3(a+bx^2)\,dx = \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{3}\sqrt{-b}x\right)e^{(3a)}}{288\sqrt{-b}b} + \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{3}\sqrt{b}x\right)e^{(-3a)}}{288\,b^{\frac{3}{2}}} + \frac{xe^{(3bx^2+3a)}}{48\,b} - \frac{3\,xe^{(bx^2+a)}}{16\,b} - \frac{3\,xe^{(-bx^2-a)}}{16\,b} + \frac{xe^{(-3bx^2-3a)}}{48\,b} - \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{32\,b^{\frac{3}{2}}} - \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}x\right)e^{a}}{32\sqrt{-b}b}$$

```
[In] integrate(x^2*sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/288*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/(sqrt(-b)*b) + 1/28
8*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/b^(3/2) + 1/48*x*e^(3*b
*x^2 + 3*a)/b - 3/16*x*e^(b*x^2 + a)/b - 3/16*x*e^(-b*x^2 - a)/b + 1/48*x*e
^(-3*b*x^2 - 3*a)/b - 3/32*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/b^(3/2) - 3/32*s
qrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh^3\left(a+bx^2\right)\,dx=\int x^2\,\mathrm{sinh}{\left(b\,x^2+a\right)}^3\,dx$$

```
[In] int(x^2*sinh(a + b*x^2)^3,x)
[Out] int(x^2*sinh(a + b*x^2)^3, x)
```

3.17 $\int x \sinh^3(a+bx^2)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int x \sinh^3(a+bx^2)\, dx = -\frac{\cosh(a+bx^2)}{2b} + \frac{\cosh^3(a+bx^2)}{6b}$$

```
[Out] -1/2*cosh(b*x^2+a)/b+1/6*cosh(b*x^2+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5428, 2713}

$$\int x \sinh^3(a+bx^2)\, dx = \frac{\cosh^3(a+bx^2)}{6b} - \frac{\cosh(a+bx^2)}{2b}$$

```
[In] Int[x*Sinh[a + b*x^2]^3,x]
```

```
[Out] -1/2*Cosh[a + b*x^2]/b + Cosh[a + b*x^2]^3/(6*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
```

```
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sinh^3(a+bx)\,dx,x,x^2\right)\\ &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh\left(a+bx^2\right)\right)}{2b}\\ &= -\frac{\cosh\left(a+bx^2\right)}{2b}+\frac{\cosh^3\left(a+bx^2\right)}{6b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\sinh^3\left(a+bx^2\right)\,dx = -\frac{3\cosh\left(a+bx^2\right)}{8b}+\frac{\cosh\left(3(a+bx^2)\right)}{24b}$$

```
[In] Integrate[x*Sinh[a + b*x^2]^3,x]
[Out] (-3*Cosh[a + b*x^2])/(8*b) + Cosh[3*(a + b*x^2)]/(24*b)
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\left(-\frac{2}{3}+\frac{\sinh\left(x^2b+a\right)^2}{3}\right)\cosh\left(x^2b+a\right)}{2b}$	28
default	$\frac{\left(-\frac{2}{3}+\frac{\sinh\left(x^2b+a\right)^2}{3}\right)\cosh\left(x^2b+a\right)}{2b}$	28
parallelrisch	$\frac{-8+\cosh\left(3x^2b+3a\right)-9\cosh\left(x^2b+a\right)}{24b}$	29
risch	$\frac{e^{3x^2b+3a}}{48b}-\frac{3\,e^{x^2b+a}}{16b}-\frac{3\,e^{-x^2b-a}}{16b}+\frac{e^{-3x^2b-3a}}{48b}$	63

```
[In] int(x*sinh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/2/b*(-2/3+1/3*sinh(b*x^2+a)^2)*cosh(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int x\sinh^3\left(a+bx^2\right)\,dx = \frac{\cosh\left(bx^2+a\right)^3+3\,\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^2-9\,\cosh\left(bx^2+a\right)}{24\,b}$$

[In] integrate(x*sinh(b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/24*(cosh(b*x^2 + a)^3 + 3*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 - 9*cosh(b*x^2 + a))/b

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int x\sinh^3\left(a+bx^2\right)\,dx = \begin{cases} \frac{\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{2b}-\frac{\cosh^3\left(a+bx^2\right)}{3b} & \text{for}\ b\neq 0\\ \frac{x^2\sinh^3(a)}{2} & \text{otherwise}\end{cases}$$

[In] integrate(x*sinh(b*x**2+a)**3,x)

[Out] Piecewise((sinh(a + b*x**2)**2*cosh(a + b*x**2)/(2*b) - cosh(a + b*x**2)**3/(3*b), Ne(b, 0)), (x**2*sinh(a)**3/2, True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

$$\int x\sinh^3\left(a+bx^2\right)\,dx = \frac{e^{\left(3\,bx^2+3\,a\right)}}{48\,b}-\frac{3\,e^{\left(bx^2+a\right)}}{16\,b}-\frac{3\,e^{\left(-bx^2-a\right)}}{16\,b}+\frac{e^{\left(-3\,bx^2-3\,a\right)}}{48\,b}$$

[In] integrate(x*sinh(b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/48*e^(3*b*x^2 + 3*a)/b - 3/16*e^(b*x^2 + a)/b - 3/16*e^(-b*x^2 - a)/b + 1/48*e^(-3*b*x^2 - 3*a)/b

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int x\sinh^3\left(a+bx^2\right)\,dx=-\frac{\left(9\,e^{\left(2\,bx^2+2\,a\right)}-1\right)e^{\left(-3\,bx^2-3\,a\right)}-e^{\left(3\,bx^2+3\,a\right)}+9\,e^{\left(bx^2+a\right)}}{48\,b}$$

```
[In] integrate(x*sinh(b*x^2+a)^3,x, algorithm="giac")
[Out] -1/48*((9*e^(2*b*x^2 + 2*a) - 1)*e^(-3*b*x^2 - 3*a) - e^(3*b*x^2 + 3*a) + 9
*e^(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x\sinh^3\left(a+bx^2\right)\,dx=-\frac{3\cosh(b\,x^2+a)-\cosh(b\,x^2+a)^3}{6\,b}$$

```
[In] int(x*sinh(a + b*x^2)^3,x)
[Out] -(3*cosh(a + b*x^2) - cosh(a + b*x^2)^3)/(6*b)
```

3.18 $\int \sinh^3\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 125

$$\int \sinh^3\left(a+bx^2\right)\,dx=\frac{3e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)}{16\sqrt{b}}-\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}-\frac{3e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x\right)}{16\sqrt{b}}+\frac{e^{3a}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}$$

```
[Out] -1/48*erf(x*3^(1/2)*b^(1/2))*3^(1/2)*Pi^(1/2)/exp(3*a)/b^(1/2)+1/48*exp(3*a
)*erfi(x*3^(1/2)*b^(1/2))*3^(1/2)*Pi^(1/2)/b^(1/2)+3/16*erf(x*b^(1/2))*Pi^(
1/2)/exp(a)/b^(1/2)-3/16*exp(a)*erfi(x*b^(1/2))*Pi^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5408, 5406, 2235, 2236}

$$\int \sinh^3\left(a+bx^2\right)\,dx=\frac{3\sqrt{\pi}e^{-a}\mathrm{erf}\left(\sqrt{b}x\right)}{16\sqrt{b}}-\frac{\sqrt{\frac{\pi}{3}}e^{-3a}\mathrm{erf}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}-\frac{3\sqrt{\pi}e^{a}\mathrm{erfi}\left(\sqrt{b}x\right)}{16\sqrt{b}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a}\mathrm{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}$$

```
[In] Int[Sinh[a + b*x^2]^3,x]
```

```
[Out] (3*Sqrt[Pi]*Erf[Sqrt[b]*x])/(16*Sqrt[b]*E^a) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt
[b]*x])/(16*Sqrt[b]*E^(3*a)) - (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(16*Sqrt[b]
) + (E^(3*a)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(16*Sqrt[b])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5408

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{4}\sinh\left(a+bx^2\right)+\frac{1}{4}\sinh\left(3a+3bx^2\right)\right)\,dx\\
&= \frac{1}{4}\int \sinh\left(3a+3bx^2\right)\,dx-\frac{3}{4}\int \sinh\left(a+bx^2\right)\,dx\\
&= -\left(\frac{1}{8}\int e^{-3a-3bx^2}\,dx\right)+\frac{1}{8}\int e^{3a+3bx^2}\,dx+\frac{3}{8}\int e^{-a-bx^2}\,dx-\frac{3}{8}\int e^{a+bx^2}\,dx\\
&= \frac{3e^{-a}\sqrt{\pi}\,\text{erf}\left(\sqrt{b}x\right)}{16\sqrt{b}}-\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}-\frac{3e^{a}\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}x\right)}{16\sqrt{b}}+\frac{e^{3a}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09

$$\int \sinh^3(a+bx^2)\,dx$$
$$= \frac{\sqrt{\frac{\pi}{3}}\left(-3\sqrt{3}\cosh(a)\text{erfi}\left(\sqrt{b}x\right)+\cosh(3a)\text{erfi}\left(\sqrt{3}\sqrt{b}x\right)+3\sqrt{3}\text{erf}\left(\sqrt{b}x\right)(\cosh(a)-\sinh(a))-3\sqrt{3}\text{erfi}\left(\sqrt{\right.\right.}{16\sqrt{b}}$$

[In] Integrate[Sinh[a + b*x^2]^3,x]

[Out] (Sqrt[Pi/3]*(-3*Sqrt[3]*Cosh[a]*Erfi[Sqrt[b]*x] + Cosh[3*a]*Erfi[Sqrt[3]*Sqrt[b]*x] + 3*Sqrt[3]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) - 3*Sqrt[3]*Erfi[Sqrt[b]*x]*Sinh[a] + Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + Erf[Sqrt[3]*Sqrt[b]*x]*(-Cosh[3*a] + Sinh[3*a])))/(16*Sqrt[b])

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{e^{-3a}\sqrt{\pi}\sqrt{3}\,\text{erf}\left(x\sqrt{3}\sqrt{b}\right)}{48\sqrt{b}}+\frac{3\,\text{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\,e^{-a}}{16\sqrt{b}}+\frac{e^{3a}\sqrt{\pi}\,\text{erf}\left(\sqrt{-3b}\,x\right)}{16\sqrt{-3b}}-\frac{3\,e^{a}\sqrt{\pi}\,\text{erf}\left(\sqrt{-b}\,x\right)}{16\sqrt{-b}}$	86

[In] int(sinh(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] -1/48/exp(a)^3*Pi^(1/2)*3^(1/2)/b^(1/2)*erf(x*3^(1/2)*b^(1/2))+3/16*erf(x*b^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+1/16*exp(a)^3*Pi^(1/2)/(-3*b)^(1/2)*erf((-3*b)^(1/2)*x)-3/16*exp(a)*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \sinh^3(a+bx^2)\,dx =$$
$$-\frac{\sqrt{3}\sqrt{\pi}\sqrt{-b}(\cosh(3a)+\sinh(3a))\,\text{erf}\left(\sqrt{3}\sqrt{-b}x\right)+\sqrt{3}\sqrt{\pi}\sqrt{b}(\cosh(3a)-\sinh(3a))\,\text{erf}\left(\sqrt{3}\sqrt{b}x\right)-9}{48\,b}$$

[In] integrate(sinh(b*x^2+a)^3,x, algorithm="fricas")

[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-b)*(cosh(3*a) + sinh(3*a))*erf(sqrt(3)*sqrt(-b)*x) + sqrt(3)*sqrt(pi)*sqrt(b)*(cosh(3*a) - sinh(3*a))*erf(sqrt(3)*sqrt(b

```
)*x) - 9*sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) - 9*sqrt(pi)
*sqrt(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b
```

Sympy [F]

$$\int \sinh^3\left(a+bx^2\right)\,dx = \int \sinh^3\left(a+bx^2\right)dx$$

```
[In] integrate(sinh(b*x**2+a)**3,x)
[Out] Integral(sinh(a + b*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \sinh^3\left(a+bx^2\right)\,dx = \frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{-b}x\right)e^{(3a)}}{48\sqrt{-b}} - \frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{b}x\right)e^{(-3a)}}{48\sqrt{b}} + \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)e^{(-a)}}{16\sqrt{b}} - \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{-b}x\right)e^{a}}{16\sqrt{-b}}$$

```
[In] integrate(sinh(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/48*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) - 1/48*sqrt(
3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) + 3/16*sqrt(pi)*erf(sqr
t(b)*x)*e^(-a)/sqrt(b) - 3/16*sqrt(pi)*erf(sqrt(-b)*x)*e^a/sqrt(-b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.76

$$\int \sinh^3\left(a+bx^2\right)\,dx = -\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{3}\sqrt{-b}x\right)e^{(3a)}}{48\sqrt{-b}} + \frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{3}\sqrt{b}x\right)e^{(-3a)}}{48\sqrt{b}} - \frac{3\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{16\sqrt{b}} + \frac{3\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-b}x\right)e^{a}}{16\sqrt{-b}}$$

```
[In] integrate(sinh(b*x^2+a)^3,x, algorithm="giac")
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) + 1/48*sqr
t(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) - 3/16*sqrt(pi)*erf(
-sqrt(b)*x)*e^(-a)/sqrt(b) + 3/16*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3\left(a+bx^2\right)\,dx=\int \sinh\left(b\,x^2+a\right)^3\,dx$$

```
[In] int(sinh(a + b*x^2)^3,x)
[Out] int(sinh(a + b*x^2)^3, x)
```

3.19 $\int \frac{\sinh^3(a+bx^2)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{\sinh^3(a+bx^2)}{x} dx = -\frac{3}{8}\text{Chi}(bx^2)\sinh(a) + \frac{1}{8}\text{Chi}(3bx^2)\sinh(3a) - \frac{3}{8}\cosh(a)\text{Shi}(bx^2) + \frac{1}{8}\cosh(3a)\text{Shi}(3bx^2)$$

```
[Out] -3/8*cosh(a)*Shi(b*x^2)+1/8*cosh(3*a)*Shi(3*b*x^2)-3/8*Chi(b*x^2)*sinh(a)+1
/8*Chi(3*b*x^2)*sinh(3*a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5448, 5426, 5425, 5424}

$$\int \frac{\sinh^3(a+bx^2)}{x} dx = -\frac{3}{8}\sinh(a)\text{Chi}(bx^2) + \frac{1}{8}\sinh(3a)\text{Chi}(3bx^2) - \frac{3}{8}\cosh(a)\text{Shi}(bx^2) + \frac{1}{8}\cosh(3a)\text{Shi}(3bx^2)$$

```
[In] Int[Sinh[a + b*x^2]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^2]*Sinh[a])/8 + (CoshIntegral[3*b*x^2]*Sinh[3*a])/8 -
(3*Cosh[a]*SinhIntegral[b*x^2])/8 + (Cosh[3*a]*SinhIntegral[3*b*x^2])/8
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5426

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5448

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{3\sinh(a+bx^2)}{4x}+\frac{\sinh(3a+3bx^2)}{4x}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(3a+3bx^2)}{x}\,dx-\frac{3}{4}\int \frac{\sinh(a+bx^2)}{x}\,dx\\ &= -\left(\frac{1}{4}(3\cosh(a))\int \frac{\sinh(bx^2)}{x}\,dx\right)+\frac{1}{4}\cosh(3a)\int \frac{\sinh(3bx^2)}{x}\,dx\\ &\quad -\frac{1}{4}(3\sinh(a))\int \frac{\cosh(bx^2)}{x}\,dx+\frac{1}{4}\sinh(3a)\int \frac{\cosh(3bx^2)}{x}\,dx\\ &= -\frac{3}{8}\text{Chi}(bx^2)\sinh(a)+\frac{1}{8}\text{Chi}(3bx^2)\sinh(3a)-\frac{3}{8}\cosh(a)\text{Shi}(bx^2)+\frac{1}{8}\cosh(3a)\text{Shi}(3bx^2)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sinh^3(a+bx^2)}{x}\,dx = \frac{1}{8}\left(-3\text{Chi}(bx^2)\sinh(a)+\text{Chi}(3bx^2)\sinh(3a)-3\cosh(a)\text{Shi}(bx^2)+\cosh(3a)\text{Shi}(3bx^2)\right)$$

```
[In] Integrate[Sinh[a + b*x^2]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^2]*Sinh[a] + CoshIntegral[3*b*x^2]*Sinh[3*a] - 3*Cosh[
a]*SinhIntegral[b*x^2] + Cosh[3*a]*SinhIntegral[3*b*x^2])/8
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

method	result	size
risch	$-\frac{e^{6a} e^{-3a} \operatorname{Ei}_1(-3x^2 b)}{16}+\frac{3 e^{4a} e^{-3a} \operatorname{Ei}_1(-x^2 b)}{16}-\frac{3 e^{2a} e^{-3a} \operatorname{Ei}_1(x^2 b)}{16}+\frac{e^{-3a} \operatorname{Ei}_1(3x^2 b)}{16}$	69

```
[In] int(sinh(b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
[Out] -1/16*exp(6*a)*exp(-3*a)*Ei(1,-3*x^2*b)+3/16*exp(4*a)*exp(-3*a)*Ei(1,-x^2*b
)-3/16*exp(2*a)*exp(-3*a)*Ei(1,x^2*b)+1/16*exp(-3*a)*Ei(1,3*x^2*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{\sinh^3(a+bx^2)}{x}\,dx = \frac{1}{16}\left(\operatorname{Ei}(3\,bx^2)-\operatorname{Ei}(-3\,bx^2)\right)\cosh(3\,a) - \frac{3}{16}\left(\operatorname{Ei}(bx^2)-\operatorname{Ei}(-bx^2)\right)\cosh(a) + \frac{1}{16}\left(\operatorname{Ei}(3\,bx^2)+\operatorname{Ei}(-3\,bx^2)\right)\sinh(3\,a) - \frac{3}{16}\left(\operatorname{Ei}(bx^2)+\operatorname{Ei}(-bx^2)\right)\sinh(a)$$

```
[In] integrate(sinh(b*x^2+a)^3/x,x, algorithm="fricas")
[Out] 1/16*(Ei(3*b*x^2) - Ei(-3*b*x^2))*cosh(3*a) - 3/16*(Ei(b*x^2) - Ei(-b*x^2))
*cosh(a) + 1/16*(Ei(3*b*x^2) + Ei(-3*b*x^2))*sinh(3*a) - 3/16*(Ei(b*x^2) +
Ei(-b*x^2))*sinh(a)
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx^2)}{x}\,dx = \int \frac{\sinh^3(a+bx^2)}{x}\,dx$$

```
[In] integrate(sinh(b*x**2+a)**3/x,x)
[Out] Integral(sinh(a + b*x**2)**3/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^3(a+bx^2)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)} + \frac{3}{16}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)} - \frac{1}{16}\,\mathrm{Ei}\left(-3\,bx^2\right)e^{(-3\,a)} - \frac{3}{16}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

```
[In] integrate(sinh(b*x^2+a)^3/x,x, algorithm="maxima")
[Out] 1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) - 1/16*Ei(-3*b*x^2)*e^(-3
*a) - 3/16*Ei(b*x^2)*e^a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^3(a+bx^2)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)} + \frac{3}{16}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)} - \frac{1}{16}\,\mathrm{Ei}\left(-3\,bx^2\right)e^{(-3\,a)} - \frac{3}{16}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

```
[In] integrate(sinh(b*x^2+a)^3/x,x, algorithm="giac")
[Out] 1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) - 1/16*Ei(-3*b*x^2)*e^(-3
*a) - 3/16*Ei(b*x^2)*e^a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx^2)}{x}\,dx = \int \frac{\sinh\left(b\,x^2+a\right)^3}{x}\,dx$$

```
[In] int(sinh(a + b*x^2)^3/x,x)
[Out] int(sinh(a + b*x^2)^3/x, x)
```

3.20 $\int \frac{\sinh^3(a+bx^2)}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 136

$$\int \frac{\sinh^3(a+bx^2)}{x^2} dx = -\frac{3}{8}\sqrt{b}e^{-a}\sqrt{\pi}\text{erf}\left(\sqrt{b}x\right) + \frac{1}{8}\sqrt{b}e^{-3a}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{b}x\right) - \frac{3}{8}\sqrt{b}e^{a}\sqrt{\pi}\text{erfi}\left(\sqrt{b}x\right) + \frac{1}{8}\sqrt{b}e^{3a}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{b}x\right) - \frac{\sinh^3(a+bx^2)}{x}$$

```
[Out] -sinh(b*x^2+a)^3/x-3/8*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)-3/8*exp(a)*er
fi(x*b^(1/2))*b^(1/2)*Pi^(1/2)+1/8*erf(x*3^(1/2)*b^(1/2))*b^(1/2)*3^(1/2)*P
i^(1/2)/exp(3*a)+1/8*exp(3*a)*erfi(x*3^(1/2)*b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1
/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5438, 5737, 5407, 2235, 2236}

$$\int \frac{\sinh^3(a+bx^2)}{x^2} dx = -\frac{3}{8}\sqrt{\pi}e^{-a}\sqrt{b}\text{erf}\left(\sqrt{b}x\right) + \frac{1}{8}\sqrt{3\pi}e^{-3a}\sqrt{b}\text{erf}\left(\sqrt{3}\sqrt{b}x\right) - \frac{3}{8}\sqrt{\pi}e^{a}\sqrt{b}\text{erfi}\left(\sqrt{b}x\right) + \frac{1}{8}\sqrt{3\pi}e^{3a}\sqrt{b}\text{erfi}\left(\sqrt{3}\sqrt{b}x\right) - \frac{\sinh^3(a+bx^2)}{x}$$

```
[In] Int[Sinh[a + b*x^2]^3/x^2,x]
```

```
[Out] (-3*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*x])/(8*E^a) + (Sqrt[b]*Sqrt[3*Pi]*Erf[Sqrt
[3]*Sqrt[b]*x])/(8*E^(3*a)) - (3*Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/8 +
(Sqrt[b]*E^(3*a)*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[b]*x])/8 - Sinh[a + b*x^2]^3/
x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5438

```
Int[(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[-Sinh[
a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Dist[b*n*(p/(n - 1)), Int[Sinh[a + b
*x^n]^(p - 1)*Cosh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n, p
] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sinh^3\left(a+bx^2\right)}{x}+(6b)\int \cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)\,dx\\
&= -\frac{\sinh^3\left(a+bx^2\right)}{x}+(6b)\int \left(-\frac{1}{4}\cosh\left(a+bx^2\right)+\frac{1}{4}\cosh\left(3a+3bx^2\right)\right)dx\\
&= -\frac{\sinh^3\left(a+bx^2\right)}{x}-\frac{1}{2}(3b)\int \cosh\left(a+bx^2\right)\,dx+\frac{1}{2}(3b)\int \cosh\left(3a+3bx^2\right)\,dx\\
&= -\frac{\sinh^3\left(a+bx^2\right)}{x}+\frac{1}{4}(3b)\int e^{-3a-3bx^2}\,dx-\frac{1}{4}(3b)\int e^{-a-bx^2}\,dx\\
&\quad -\frac{1}{4}(3b)\int e^{a+bx^2}\,dx+\frac{1}{4}(3b)\int e^{3a+3bx^2}\,dx
\end{aligned}
$$

$$= -\frac{3}{8}\sqrt{b}e^{-a}\sqrt{\pi}\text{erf}\left(\sqrt{b}x\right) + \frac{1}{8}\sqrt{b}e^{-3a}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{b}x\right) - \frac{3}{8}\sqrt{b}e^{a}\sqrt{\pi}\text{erfi}\left(\sqrt{b}x\right) + \frac{1}{8}\sqrt{b}e^{3a}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{b}x\right) - \frac{\sinh^3(a+bx^2)}{x}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.50

$$\int \frac{\sinh^3(a+bx^2)}{x^2}\,dx$$
$$= \frac{-3\sqrt{b}\sqrt{\pi}x\cosh(a)\text{erfi}\left(\sqrt{b}x\right) + \sqrt{b}\sqrt{3\pi}x\cosh(3a)\text{erfi}\left(\sqrt{3}\sqrt{b}x\right) - 3\sqrt{b}\sqrt{\pi}x\text{erfi}\left(\sqrt{b}x\right)\sinh(a) + 3\sqrt{b}\sqrt{\pi}x}{}$$

```
[In] Integrate[Sinh[a + b*x^2]^3/x^2,x]
```

```
[Out] (-3*Sqrt[b]*Sqrt[Pi]*x*Cosh[a]*Erfi[Sqrt[b]*x] + Sqrt[b]*Sqrt[3*Pi]*x*Cosh[
3*a]*Erfi[Sqrt[3]*Sqrt[b]*x] - 3*Sqrt[b]*Sqrt[Pi]*x*Erfi[Sqrt[b]*x]*Sinh[a]
 + 3*Sqrt[b]*Sqrt[Pi]*x*Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a]) + Sqrt[b]*Sqrt[
3*Pi]*x*Erf[Sqrt[3]*Sqrt[b]*x]*(Cosh[3*a] - Sinh[3*a]) + Sqrt[b]*Sqrt[3*Pi]
*x*Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + 6*Sinh[a + b*x^2] - 2*Sinh[3*(a + b*
x^2)])/(8*x)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

method	result
risch	$\frac{e^{-3a}e^{-3x^2b}}{8x} + \frac{e^{-3a}\sqrt{b}\,\sqrt{\pi}\,\sqrt{3}\,\text{erf}\left(x\sqrt{3}\,\sqrt{b}\right)}{8} - \frac{3\,e^{-a}e^{-x^2b}}{8x} - \frac{3\,\text{erf}\left(x\sqrt{b}\right)\sqrt{b}\,\sqrt{\pi}\,e^{-a}}{8} + \frac{3\,e^{a}e^{x^2b}}{8x} - \frac{3\,e^{a}b\sqrt{\pi}\,\text{erf}(\sqrt{-b}\,x)}{8\sqrt{-b}} -$

```
[In] int(sinh(b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/exp(a)^3/x*exp(-3*x^2*b)+1/8/exp(a)^3*b^(1/2)*Pi^(1/2)*3^(1/2)*erf(x*3^
(1/2)*b^(1/2))-3/8/exp(a)/x*exp(-x^2*b)-3/8*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)
/exp(a)+3/8*exp(a)*exp(x^2*b)/x-3/8*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(
1/2)*x)-1/8*exp(a)^3/x*exp(3*x^2*b)+3/8*exp(a)^3*b*Pi^(1/2)/(-3*b)^(1/2)*er
f((-3*b)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(98) = 196$.

Time = 0.25 (sec) , antiderivative size = 892, normalized size of antiderivative = 6.56

$$\int \frac{\sinh^3(a+bx^2)}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(b*x^2 + a)^6 + 6*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5 + sinh(b*x^2
+ a)^6 + 3*(5*cosh(b*x^2 + a)^2 - 1)*sinh(b*x^2 + a)^4 - 3*cosh(b*x^2 + a)^
4 + 4*(5*cosh(b*x^2 + a)^3 - 3*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt(3)
*sqrt(pi)*(x*cosh(b*x^2 + a)^3*cosh(3*a) + x*cosh(b*x^2 + a)^3*sinh(3*a) +
(x*cosh(3*a) + x*sinh(3*a))*sinh(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(3
*a) + x*cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)
^2*cosh(3*a) + x*cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(-b)*erf
(sqrt(3)*sqrt(-b)*x) - sqrt(3)*sqrt(pi)*(x*cosh(b*x^2 + a)^3*cosh(3*a) - x*
cosh(b*x^2 + a)^3*sinh(3*a) + (x*cosh(3*a) - x*sinh(3*a))*sinh(b*x^2 + a)^3
 + 3*(x*cosh(b*x^2 + a)*cosh(3*a) - x*cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2
 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(3*a) - x*cosh(b*x^2 + a)^2*sinh(3*a))
*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(3)*sqrt(b)*x) - 3*sqrt(pi)*(x*cosh(b*x^2
 + a)^3*cosh(a) + x*cosh(b*x^2 + a)^3*sinh(a) + (x*cosh(a) + x*sinh(a))*sin
h(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(a) + x*cosh(b*x^2 + a)*sinh(a))*
sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(a) + x*cosh(b*x^2 + a)^2*si
nh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + 3*sqrt(pi)*(x*cosh(b*x^2
 + a)^3*cosh(a) - x*cosh(b*x^2 + a)^3*sinh(a) + (x*cosh(a) - x*sinh(a))*sin
h(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(a) - x*cosh(b*x^2 + a)*sinh(a))*
sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(a) - x*cosh(b*x^2 + a)^2*si
nh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + 3*(5*cosh(b*x^2 + a)^4 - 6
*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^2 + 3*cosh(b*x^2 + a)^2 + 6*(cosh(b
*x^2 + a)^5 - 2*cosh(b*x^2 + a)^3 + cosh(b*x^2 + a))*sinh(b*x^2 + a) - 1)/(
x*cosh(b*x^2 + a)^3 + 3*x*cosh(b*x^2 + a)^2*sinh(b*x^2 + a) + 3*x*cosh(b*x^
2 + a)*sinh(b*x^2 + a)^2 + x*sinh(b*x^2 + a)^3)
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx^2)}{x^2}\,dx = \int \frac{\sinh^3(a+bx^2)}{x^2}\,dx$$

```
[In] integrate(sinh(b*x**2+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**2)**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.75

$$\int \frac{\sinh^3(a+bx^2)}{x^2}\,dx = \frac{\sqrt{3}\sqrt{bx^2}e^{(-3\,a)}\Gamma\left(-\frac{1}{2},3\,bx^2\right)}{16\,x} - \frac{\sqrt{3}\sqrt{-bx^2}e^{(3\,a)}\Gamma\left(-\frac{1}{2},-3\,bx^2\right)}{16\,x} - \frac{3\,\sqrt{bx^2}e^{(-a)}\Gamma\left(-\frac{1}{2},bx^2\right)}{16\,x} + \frac{3\,\sqrt{-bx^2}e^{a}\Gamma\left(-\frac{1}{2},-bx^2\right)}{16\,x}$$

```
[In] integrate(sinh(b*x^2+a)^3/x^2,x, algorithm="maxima")
[Out] 1/16*sqrt(3)*sqrt(b*x^2)*e^(-3*a)*gamma(-1/2, 3*b*x^2)/x - 1/16*sqrt(3)*sqr
t(-b*x^2)*e^(3*a)*gamma(-1/2, -3*b*x^2)/x - 3/16*sqrt(b*x^2)*e^(-a)*gamma(-
1/2, b*x^2)/x + 3/16*sqrt(-b*x^2)*e^a*gamma(-1/2, -b*x^2)/x
```

Giac [F]

$$\int \frac{\sinh^3(a+bx^2)}{x^2}\,dx = \int \frac{\sinh\left(bx^2+a\right)^3}{x^2}\,dx$$

```
[In] integrate(sinh(b*x^2+a)^3/x^2,x, algorithm="giac")
[Out] integrate(sinh(b*x^2 + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx^2)}{x^2}\,dx = \int \frac{\sinh(b\,x^2+a)^3}{x^2}\,dx$$

```
[In] int(sinh(a + b*x^2)^3/x^2,x)
[Out] int(sinh(a + b*x^2)^3/x^2, x)
```

3.21 $\int \frac{\sinh^3(a+bx^2)}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{\sinh^3(a+bx^2)}{x^3}\,dx = -\frac{3}{8}b\cosh(a)\text{Chi}(bx^2) + \frac{3}{8}b\cosh(3a)\text{Chi}(3bx^2) + \frac{3\sinh(a+bx^2)}{8x^2} - \frac{\sinh(3(a+bx^2))}{8x^2} - \frac{3}{8}b\sinh(a)\text{Shi}(bx^2) + \frac{3}{8}b\sinh(3a)\text{Shi}(3bx^2)$$

```
[Out] -3/8*b*Chi(b*x^2)*cosh(a)+3/8*b*Chi(3*b*x^2)*cosh(3*a)-3/8*b*Shi(b*x^2)*sin
h(a)+3/8*b*Shi(3*b*x^2)*sinh(3*a)+3/8*sinh(b*x^2+a)/x^2-1/8*sinh(3*b*x^2+3*
a)/x^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5448, 5428, 3378, 3384, 3379, 3382}

$$\int \frac{\sinh^3(a+bx^2)}{x^3}\,dx = -\frac{3}{8}b\cosh(a)\text{Chi}(bx^2) + \frac{3}{8}b\cosh(3a)\text{Chi}(3bx^2) - \frac{3}{8}b\sinh(a)\text{Shi}(bx^2) + \frac{3}{8}b\sinh(3a)\text{Shi}(3bx^2) + \frac{3\sinh(a+bx^2)}{8x^2} - \frac{\sinh(3(a+bx^2))}{8x^2}$$

```
[In] Int[Sinh[a + b*x^2]^3/x^3,x]
```

```
[Out] (-3*b*Cosh[a]*CoshIntegral[b*x^2])/8 + (3*b*Cosh[3*a]*CoshIntegral[3*b*x^2]
)/8 + (3*Sinh[a + b*x^2])/(8*x^2) - Sinh[3*(a + b*x^2)]/(8*x^2) - (3*b*Sinh
[a]*SinhIntegral[b*x^2])/8 + (3*b*Sinh[3*a]*SinhIntegral[3*b*x^2])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5448

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{3\sinh\left(a+bx^2\right)}{4x^3}+\frac{\sinh\left(3a+3bx^2\right)}{4x^3}\right)\,dx\\ &= \frac{1}{4}\int \frac{\sinh\left(3a+3bx^2\right)}{x^3}\,dx-\frac{3}{4}\int \frac{\sinh\left(a+bx^2\right)}{x^3}\,dx\end{aligned}$$

$$= \frac{1}{8}\text{Subst}\left(\int \frac{\sinh(3a+3bx)}{x^2}\,dx, x, x^2\right) - \frac{3}{8}\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx, x, x^2\right)$$

$$= \frac{3\sinh(a+bx^2)}{8x^2} - \frac{\sinh(3(a+bx^2))}{8x^2} - \frac{1}{8}(3b)\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx, x, x^2\right) + \frac{1}{8}(3b)\text{Subst}\left(\int \frac{\cosh(3a+3bx)}{x}\,dx, x, x^2\right)$$

$$= \frac{3\sinh(a+bx^2)}{8x^2} - \frac{\sinh(3(a+bx^2))}{8x^2} - \frac{1}{8}(3b\cosh(a))\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, x^2\right) + \frac{1}{8}(3b\cosh(3a))\text{Subst}\left(\int \frac{\cosh(3bx)}{x}\,dx, x, x^2\right) - \frac{1}{8}(3b\sinh(a))\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, x^2\right) + \frac{1}{8}(3b\sinh(3a))\text{Subst}\left(\int \frac{\sinh(3bx)}{x}\,dx, x, x^2\right)$$

$$= -\frac{3}{8}b\cosh(a)\text{Chi}(bx^2) + \frac{3}{8}b\cosh(3a)\text{Chi}(3bx^2) + \frac{3\sinh(a+bx^2)}{8x^2} - \frac{\sinh(3(a+bx^2))}{8x^2} - \frac{3}{8}b\sinh(a)\text{Shi}(bx^2) + \frac{3}{8}b\sinh(3a)\text{Shi}(3bx^2)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\sinh^3(a+bx^2)}{x^3}\,dx = -\frac{3bx^2\cosh(a)\text{Chi}(bx^2) - 3bx^2\cosh(3a)\text{Chi}(3bx^2) - 3\sinh(a+bx^2) + \sinh(3(a+bx^2)) + 3bx^2\sinh(a)\text{Sh}}{8x^2}$$

```
[In] Integrate[Sinh[a + b*x^2]^3/x^3,x]
```

```
[Out] -1/8*(3*b*x^2*Cosh[a]*CoshIntegral[b*x^2] - 3*b*x^2*Cosh[3*a]*CoshIntegral[
3*b*x^2] - 3*Sinh[a + b*x^2] + Sinh[3*(a + b*x^2)] + 3*b*x^2*Sinh[a]*SinhIn
tegral[b*x^2] - 3*b*x^2*Sinh[3*a]*SinhIntegral[3*b*x^2])/x^2
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

method	result
risch	$\frac{-3\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1\left(3x^2b\right)b\,x^2+3\,\mathrm{Ei}_1\left(x^2b\right)\mathrm{e}^{-a}b\,x^2+3\,\mathrm{e}^{a}\,\mathrm{Ei}_1\left(-x^2b\right)b\,x^2-3\,\mathrm{e}^{3a}\,\mathrm{Ei}_1\left(-3x^2b\right)b\,x^2+\mathrm{e}^{-3x^2b-3a}-3\,\mathrm{e}^{-x^2b-a}+3\,\mathrm{e}^{x^2b+a}-\mathrm{e}^{3x^2b+3}}{16x^2}$

```
[In] int(sinh(b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(-3*exp(-3*a)*Ei(1,3*x^2*b)*b*x^2+3*Ei(1,x^2*b)*exp(-a)*b*x^2+3*exp(a)
*Ei(1,-x^2*b)*b*x^2-3*exp(3*a)*Ei(1,-3*x^2*b)*b*x^2+exp(-3*b*x^2-3*a)-3*exp
(-b*x^2-a)+3*exp(b*x^2+a)-exp(3*b*x^2+3*a))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.76

$$\int \frac{\sinh^3\left(a+bx^2\right)}{x^3}\,dx =$$
$$-\frac{2\,\sinh\left(bx^2+a\right)^3-3\left(bx^2\mathrm{Ei}(3\,bx^2)+bx^2\mathrm{Ei}(-3\,bx^2)\right)\cosh\left(3\,a\right)+3\left(bx^2\mathrm{Ei}(bx^2)+bx^2\mathrm{Ei}(-bx^2)\right)\cosh\left(a\right.}{}$$

```
[In] integrate(sinh(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*sinh(b*x^2 + a)^3 - 3*(b*x^2*Ei(3*b*x^2) + b*x^2*Ei(-3*b*x^2))*cos
h(3*a) + 3*(b*x^2*Ei(b*x^2) + b*x^2*Ei(-b*x^2))*cosh(a) + 6*(cosh(b*x^2 + a
)^2 - 1)*sinh(b*x^2 + a) - 3*(b*x^2*Ei(3*b*x^2) - b*x^2*Ei(-3*b*x^2))*sinh(
3*a) + 3*(b*x^2*Ei(b*x^2) - b*x^2*Ei(-b*x^2))*sinh(a))/x^2
```

Sympy [F]

$$\int \frac{\sinh^3\left(a+bx^2\right)}{x^3}\,dx = \int \frac{\sinh^3\left(a+bx^2\right)}{x^3}\,dx$$

```
[In] integrate(sinh(b*x**2+a)**3/x**3,x)
```

```
[Out] Integral(sinh(a + b*x**2)**3/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int \frac{\sinh^3(a+bx^2)}{x^3}\,dx = \frac{3}{16}\,be^{(-3\,a)}\Gamma\left(-1,3\,bx^2\right) - \frac{3}{16}\,be^{(-a)}\Gamma\left(-1,bx^2\right) - \frac{3}{16}\,be^{a}\Gamma\left(-1,-bx^2\right) + \frac{3}{16}\,be^{(3\,a)}\Gamma\left(-1,-3\,bx^2\right)$$

```
[In] integrate(sinh(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/16*b*e^(-3*a)*gamma(-1, 3*b*x^2) - 3/16*b*e^(-a)*gamma(-1, b*x^2) - 3/16*
b*e^a*gamma(-1, -b*x^2) + 3/16*b*e^(3*a)*gamma(-1, -3*b*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(80) = 160.

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.45

$$\int \frac{\sinh^3(a+bx^2)}{x^3}\,dx = \frac{3\,(bx^2+a)b^2\mathrm{Ei}(3\,bx^2)\,e^{(3\,a)} - 3\,ab^2\mathrm{Ei}(3\,bx^2)\,e^{(3\,a)} - 3\,(bx^2+a)b^2\mathrm{Ei}(-bx^2)\,e^{(-a)} + 3\,ab^2\mathrm{Ei}(-bx^2)\,e^{(-a)} + 3\,(\cdots}{\cdots}$$

```
[In] integrate(sinh(b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*(b*x^2 + a)*b^2*Ei(3*b*x^2)*e^(3*a) - 3*a*b^2*Ei(3*b*x^2)*e^(3*a) -
 3*(b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) + 3*a*b^2*Ei(-b*x^2)*e^(-a) + 3*(b*x^2
 + a)*b^2*Ei(-3*b*x^2)*e^(-3*a) - 3*a*b^2*Ei(-3*b*x^2)*e^(-3*a) - 3*(b*x^2
+ a)*b^2*Ei(b*x^2)*e^a + 3*a*b^2*Ei(b*x^2)*e^a - b^2*e^(3*b*x^2 + 3*a) + 3*
b^2*e^(b*x^2 + a) - 3*b^2*e^(-b*x^2 - a) + b^2*e^(-3*b*x^2 - 3*a))/(b^2*x^2
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx^2)}{x^3}\,dx = \int \frac{{\mathrm{sinh}\left(b\,x^2+a\right)}^3}{x^3}\,dx$$

```
[In] int(sinh(a + b*x^2)^3/x^3,x)
```

```
[Out] int(sinh(a + b*x^2)^3/x^3, x)
```

3.22 $\int x \sinh^7(a+bx^2)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 67

$$\int x \sinh^7(a+bx^2)\, dx = -\frac{\cosh(a+bx^2)}{2b} + \frac{\cosh^3(a+bx^2)}{2b} - \frac{3\cosh^5(a+bx^2)}{10b} + \frac{\cosh^7(a+bx^2)}{14b}$$

```
[Out] -1/2*cosh(b*x^2+a)/b+1/2*cosh(b*x^2+a)^3/b-3/10*cosh(b*x^2+a)^5/b+1/14*cosh
(b*x^2+a)^7/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5428, 2713}

$$\int x \sinh^7(a+bx^2)\, dx = \frac{\cosh^7(a+bx^2)}{14b} - \frac{3\cosh^5(a+bx^2)}{10b} + \frac{\cosh^3(a+bx^2)}{2b} - \frac{\cosh(a+bx^2)}{2b}$$

```
[In] Int[x*Sinh[a + b*x^2]^7,x]
```

```
[Out] -1/2*Cosh[a + b*x^2]/b + Cosh[a + b*x^2]^3/(2*b) - (3*Cosh[a + b*x^2]^5)/(1
0*b) + Cosh[a + b*x^2]^7/(14*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sinh^7(a+bx)\,dx, x, x^2\right)\\ &= -\frac{\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx, x, \cosh\left(a+bx^2\right)\right)}{2b}\\ &= -\frac{\cosh\left(a+bx^2\right)}{2b}+\frac{\cosh^3\left(a+bx^2\right)}{2b}-\frac{3\cosh^5\left(a+bx^2\right)}{10b}+\frac{\cosh^7\left(a+bx^2\right)}{14b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int x\sinh^7\left(a+bx^2\right)\,dx = -\frac{35\cosh\left(a+bx^2\right)}{128b}+\frac{7\cosh\left(3\left(a+bx^2\right)\right)}{128b}-\frac{7\cosh\left(5\left(a+bx^2\right)\right)}{640b}+\frac{\cosh\left(7\left(a+bx^2\right)\right)}{896b}$$

```
[In] Integrate[x*Sinh[a + b*x^2]^7,x]
[Out] (-35*Cosh[a + b*x^2])/(128*b) + (7*Cosh[3*(a + b*x^2)])/(128*b) - (7*Cosh[5
*(a + b*x^2)])/(640*b) + Cosh[7*(a + b*x^2)]/(896*b)
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\left(-\frac{16}{35}+\frac{\sinh\left(x^2b+a\right)^6}{7}-\frac{6\sinh\left(x^2b+a\right)^4}{35}+\frac{8\sinh\left(x^2b+a\right)^2}{35}\right)\cosh\left(x^2b+a\right)}{2b}$
default	$\frac{\left(-\frac{16}{35}+\frac{\sinh\left(x^2b+a\right)^6}{7}-\frac{6\sinh\left(x^2b+a\right)^4}{35}+\frac{8\sinh\left(x^2b+a\right)^2}{35}\right)\cosh\left(x^2b+a\right)}{2b}$
parallelrisch	$\frac{-1024+245\cosh\left(3x^2b+3a\right)-1225\cosh\left(x^2b+a\right)-49\cosh\left(5x^2b+5a\right)+5\cosh\left(7x^2b+7a\right)}{4480b}$
risch	$\frac{e^{7x^2b+7a}}{1792b}-\frac{7e^{5x^2b+5a}}{1280b}+\frac{7e^{3x^2b+3a}}{256b}-\frac{35e^{x^2b+a}}{256b}-\frac{35e^{-x^2b-a}}{256b}+\frac{7e^{-3x^2b-3a}}{256b}-\frac{7e^{-5x^2b-5a}}{1280b}+\frac{e^{-7x^2b-7}}{1792b}$

```
[In] int(x*sinh(b*x^2+a)^7,x,method=_RETURNVERBOSE)
[Out] 1/2/b*(-16/35+1/7*sinh(b*x^2+a)^6-6/35*sinh(b*x^2+a)^4+8/35*sinh(b*x^2+a)^2
)*cosh(b*x^2+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(59) = 118$.

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.30

$$\int x\sinh^7\left(a+bx^2\right)\,dx$$
$$= \frac{5\cosh\left(bx^2+a\right)^7+35\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^6-49\cosh\left(bx^2+a\right)^5+35\left(5\cosh\left(bx^2+a\right)^3-7\,\mathrm{co}\right.}{}$$

```
[In] integrate(x*sinh(b*x^2+a)^7,x, algorithm="fricas")
[Out] 1/4480*(5*cosh(b*x^2 + a)^7 + 35*cosh(b*x^2 + a)*sinh(b*x^2 + a)^6 - 49*cos
h(b*x^2 + a)^5 + 35*(5*cosh(b*x^2 + a)^3 - 7*cosh(b*x^2 + a))*sinh(b*x^2 +
a)^4 + 245*cosh(b*x^2 + a)^3 + 35*(3*cosh(b*x^2 + a)^5 - 14*cosh(b*x^2 + a)
^3 + 21*cosh(b*x^2 + a))*sinh(b*x^2 + a)^2 - 1225*cosh(b*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int x\sinh^7\left(a+bx^2\right)\,dx$$
$$= \begin{cases} \frac{\sinh^6\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{2b}-\frac{\sinh^4\left(a+bx^2\right)\cosh^3\left(a+bx^2\right)}{b}+\frac{4\sinh^2\left(a+bx^2\right)\cosh^5\left(a+bx^2\right)}{5b}-\frac{8\cosh^7\left(a+bx^2\right)}{35b} & \text{for } b\neq 0 \\ \frac{x^2\sinh^7\left(a\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*sinh(b*x**2+a)**7,x)
```

```
[Out] Piecewise((sinh(a + b*x**2)**6*cosh(a + b*x**2)/(2*b) - sinh(a + b*x**2)**4
*cosh(a + b*x**2)**3/b + 4*sinh(a + b*x**2)**2*cosh(a + b*x**2)**5/(5*b) -
8*cosh(a + b*x**2)**7/(35*b), Ne(b, 0)), (x**2*sinh(a)**7/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(59) = 118$.

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

$$\int x\sinh^7\left(a+bx^2\right)\,dx = \frac{e^{(7\,bx^2+7\,a)}}{1792\,b} - \frac{7\,e^{(5\,bx^2+5\,a)}}{1280\,b} + \frac{7\,e^{(3\,bx^2+3\,a)}}{256\,b} - \frac{35\,e^{(bx^2+a)}}{256\,b} - \frac{35\,e^{(-bx^2-a)}}{256\,b} + \frac{7\,e^{(-3\,bx^2-3\,a)}}{256\,b} - \frac{7\,e^{(-5\,bx^2-5\,a)}}{1280\,b} + \frac{e^{(-7\,bx^2-7\,a)}}{1792\,b}$$

```
[In] integrate(x*sinh(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/1792*e^(7*b*x^2 + 7*a)/b - 7/1280*e^(5*b*x^2 + 5*a)/b + 7/256*e^(3*b*x^2
+ 3*a)/b - 35/256*e^(b*x^2 + a)/b - 35/256*e^(-b*x^2 - a)/b + 7/256*e^(-3*b
*x^2 - 3*a)/b - 7/1280*e^(-5*b*x^2 - 5*a)/b + 1/1792*e^(-7*b*x^2 - 7*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.61

$$\int x\sinh^7\left(a+bx^2\right)\,dx = -\frac{\left(1225\,e^{(6\,bx^2+6\,a)} - 245\,e^{(4\,bx^2+4\,a)} + 49\,e^{(2\,bx^2+2\,a)} - 5\right)e^{(-7\,bx^2-7\,a)} - 5\,e^{(7\,bx^2+7\,a)} + 49\,e^{(5\,bx^2+5\,a)} - 245\,e^{(}}{8960\,b}$$

```
[In] integrate(x*sinh(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] -1/8960*((1225*e^(6*b*x^2 + 6*a) - 245*e^(4*b*x^2 + 4*a) + 49*e^(2*b*x^2 +
2*a) - 5)*e^(-7*b*x^2 - 7*a) - 5*e^(7*b*x^2 + 7*a) + 49*e^(5*b*x^2 + 5*a) -
 245*e^(3*b*x^2 + 3*a) + 1225*e^(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x\sinh^7\left(a+bx^2\right)\,dx$$
$$=-\frac{-5\cosh(b\,x^2+a)^7+21\cosh(b\,x^2+a)^5-35\cosh(b\,x^2+a)^3+35\cosh(b\,x^2+a)}{70\,b}$$

```
[In] int(x*sinh(a + b*x^2)^7,x)
[Out] -(35*cosh(a + b*x^2) - 35*cosh(a + b*x^2)^3 + 21*cosh(a + b*x^2)^5 - 5*cosh
(a + b*x^2)^7)/(70*b)
```

3.23 $\int (ex)^m \sinh^p(a+bx^2)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (ex)^m \sinh^p\left(a+bx^2\right)\,dx = \mathrm{Int}\left((ex)^m \sinh^p\left(a+bx^2\right),x\right)$$

```
[Out] Unintegrable((e*x)^m*sinh(b*x^2+a)^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m \sinh^p\left(a+bx^2\right)\,dx = \int (ex)^m \sinh^p\left(a+bx^2\right)\,dx$$

```
[In] Int[(e*x)^m*Sinh[a + b*x^2]^p,x]
```

```
[Out] Defer[Int][(e*x)^m*Sinh[a + b*x^2]^p, x]
```

Rubi steps

$$\text{integral} = \int (ex)^m \sinh^p\left(a+bx^2\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (ex)^m \sinh^p \left(a + bx^2\right) \, dx = \int (ex)^m \sinh^p \left(a + bx^2\right) \, dx$$

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2]^p,x]
[Out] Integrate[(e*x)^m*Sinh[a + b*x^2]^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (ex)^m \sinh \left(x^2 b + a\right)^p dx$$

```
[In] int((e*x)^m*sinh(b*x^2+a)^p,x)
[Out] int((e*x)^m*sinh(b*x^2+a)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (ex)^m \sinh^p \left(a + bx^2\right) \, dx = \int (ex)^m \sinh \left(bx^2 + a\right)^p \, dx$$

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x)^m*sinh(b*x^2 + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 3.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (ex)^m \sinh^p \left(a + bx^2\right) \, dx = \int (ex)^m \sinh^p \left(a + bx^2\right) dx$$

```
[In] integrate((e*x)**m*sinh(b*x**2+a)**p,x)
[Out] Integral((e*x)**m*sinh(a + b*x**2)**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (ex)^m \sinh^p\left(a+bx^2\right)\,dx = \int (ex)^m \sinh\left(bx^2+a\right)^p\,dx$$

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (ex)^m \sinh^p\left(a+bx^2\right)\,dx = \int (ex)^m \sinh\left(bx^2+a\right)^p\,dx$$

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (ex)^m \sinh^p\left(a+bx^2\right)\,dx = \int \sinh\left(b\,x^2+a\right)^p\,{\left(e\,x\right)}^m\,dx$$

```
[In] int(sinh(a + b*x^2)^p*(e*x)^m,x)
[Out] int(sinh(a + b*x^2)^p*(e*x)^m, x)
```

3.24 $\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 214

$$\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx = -\frac{3^{-\frac{1}{2}-\frac{m}{2}}e^{3a}(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-3bx^2\right)}{16e} + \frac{3e^a(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-bx^2\right)}{16e} - \frac{3e^{-a}(ex)^{1+m}\left(bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},bx^2\right)}{16e} + \frac{3^{-\frac{1}{2}-\frac{m}{2}}e^{-3a}(ex)^{1+m}\left(bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},3bx^2\right)}{16e}$$

```
[Out] -1/16*3^(-1/2-1/2*m)*exp(3*a)*(e*x)^(1+m)*(-b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1
/2*m,-3*b*x^2)/e+3/16*exp(a)*(e*x)^(1+m)*(-b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/
2*m,-b*x^2)/e-3/16*(e*x)^(1+m)*(b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,b*x^2)/
e/exp(a)+1/16*3^(-1/2-1/2*m)*(e*x)^(1+m)*(b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2
*m,3*b*x^2)/e/exp(3*a)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used

$= \{5448, 5436, 2250\}$

$$\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx = -\frac{e^{3a}3^{-\frac{m}{2}-\frac{1}{2}}(-bx^2)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-3bx^2\right)}{16e} + \frac{3e^a(-bx^2)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-bx^2\right)}{16e} - \frac{3e^{-a}(bx^2)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},bx^2\right)}{16e} + \frac{e^{-3a}3^{-\frac{m}{2}-\frac{1}{2}}(bx^2)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},3bx^2\right)}{16e}$$

```
[In] Int[(e*x)^m*Sinh[a + b*x^2]^3,x]
```

```
[Out] -1/16*(3^(-1/2 - m/2)*E^(3*a)*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(
1 + m)/2, -3*b*x^2])/e + (3*E^a*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma
[(1 + m)/2, -(b*x^2)])/(16*e) - (3*(e*x)^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma
[(1 + m)/2, b*x^2])/(16*e*E^a) + (3^(-1/2 - m/2)*(e*x)^(1 + m)*(b*x^2)^((-1
 - m)/2)*Gamma[(1 + m)/2, 3*b*x^2])/(16*e*E^(3*a))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5436

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 5448

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{3}{4}(ex)^m \sinh\left(a+bx^2\right) + \frac{1}{4}(ex)^m \sinh\left(3a+3bx^2\right)\right)\,dx$$

$$= \frac{1}{4}\int (ex)^m \sinh\left(3a+3bx^2\right)\,dx - \frac{3}{4}\int (ex)^m \sinh\left(a+bx^2\right)\,dx$$

$$= -\left(\frac{1}{8}\int e^{-3a-3bx^2}(ex)^m\,dx\right) + \frac{1}{8}\int e^{3a+3bx^2}(ex)^m\,dx$$
$$+ \frac{3}{8}\int e^{-a-bx^2}(ex)^m\,dx - \frac{3}{8}\int e^{a+bx^2}(ex)^m\,dx$$
$$= -\frac{3^{-\frac{1}{2}-\frac{m}{2}}e^{3a}(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-3bx^2\right)}{16e}$$
$$+ \frac{3e^{a}(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-bx^2\right)}{16e}$$
$$- \frac{3e^{-a}(ex)^{1+m}\left(bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},bx^2\right)}{16e}$$
$$+ \frac{3^{-\frac{1}{2}-\frac{m}{2}}e^{-3a}(ex)^{1+m}\left(bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},3bx^2\right)}{16e}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.84

$$\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx$$
$$= \frac{1}{16}3^{\frac{1}{2}(-1-m)}e^{-3a}x(ex)^m\left(-b^2x^4\right)^{\frac{1}{2}(-1-m)}\left(-e^{6a}\left(bx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},-3bx^2\right)\right.$$
$$+3^{\frac{3+m}{2}}e^{4a}\left(bx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},-bx^2\right)$$
$$\left.+\left(-bx^2\right)^{\frac{1+m}{2}}\left(-3^{\frac{3+m}{2}}e^{2a}\Gamma\left(\frac{1+m}{2},bx^2\right)+\Gamma\left(\frac{1+m}{2},3bx^2\right)\right)\right)$$

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2]^3,x]
```

```
[Out] (3^((-1 - m)/2)*x*(e*x)^m*(-(b^2*x^4))^((-1 - m)/2)*(-(E^(6*a)*(b*x^2)^((1
 + m)/2)*Gamma[(1 + m)/2, -3*b*x^2]) + 3^((3 + m)/2)*E^(4*a)*(b*x^2)^((1 + m
)/2)*Gamma[(1 + m)/2, -(b*x^2)] + (-(b*x^2))^((1 + m)/2)*(-(3^((3 + m)/2)*E
^(2*a)*Gamma[(1 + m)/2, b*x^2]) + Gamma[(1 + m)/2, 3*b*x^2])))/(16*E^(3*a))
```

Maple [F]

$$\int (ex)^m \sinh\left(x^2b+a\right)^3\,dx$$

```
[In] int((e*x)^m*sinh(b*x^2+a)^3,x)
```

```
[Out] int((e*x)^m*sinh(b*x^2+a)^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.18

$$\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx = \frac{e\cosh\left(\frac{1}{2}\,(m-1)\log\left(\frac{3\,b}{e^2}\right)+3\,a\right)\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},3\,bx^2\right)-9\,e\cosh\left(\frac{1}{2}\,(m-1)\log\left(\frac{b}{e^2}\right)+a\right)\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},bx^2\right)-9}{}$$

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(e*cosh(1/2*(m - 1)*log(3*b/e^2) + 3*a)*gamma(1/2*m + 1/2, 3*b*x^2) -
9*e*cosh(1/2*(m - 1)*log(b/e^2) + a)*gamma(1/2*m + 1/2, b*x^2) - 9*e*cosh(1
/2*(m - 1)*log(-b/e^2) - a)*gamma(1/2*m + 1/2, -b*x^2) + e*cosh(1/2*(m - 1)
*log(-3*b/e^2) - 3*a)*gamma(1/2*m + 1/2, -3*b*x^2) - e*gamma(1/2*m + 1/2, 3
*b*x^2)*sinh(1/2*(m - 1)*log(3*b/e^2) + 3*a) + 9*e*gamma(1/2*m + 1/2, b*x^2
)*sinh(1/2*(m - 1)*log(b/e^2) + a) + 9*e*gamma(1/2*m + 1/2, -b*x^2)*sinh(1/
2*(m - 1)*log(-b/e^2) - a) - e*gamma(1/2*m + 1/2, -3*b*x^2)*sinh(1/2*(m - 1
)*log(-3*b/e^2) - 3*a))/b
```

Sympy [F]

$$\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx = \int (ex)^m \sinh^3\left(a+bx^2\right)dx$$

```
[In] integrate((e*x)**m*sinh(b*x**2+a)**3,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**2)**3, x)
```

Maxima [F]

$$\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx = \int (ex)^m \sinh\left(bx^2+a\right)^3\,dx$$

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^3, x)
```

Giac [F]

$$\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx=\int (ex)^m \sinh\left(bx^2+a\right)^3\,dx$$

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx=\int \sinh\left(b\,x^2+a\right)^3\,(e\,x)^m\,dx$$

```
[In] int(sinh(a + b*x^2)^3*(e*x)^m,x)
[Out] int(sinh(a + b*x^2)^3*(e*x)^m, x)
```

3.25 $\int (ex)^m \sinh^2(a+bx^2)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int (ex)^m \sinh^2(a+bx^2)\,dx = -\frac{(ex)^{1+m}}{2e(1+m)} - \frac{2^{-\frac{7}{2}-\frac{m}{2}}e^{2a}(ex)^{1+m}(-bx^2)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-2bx^2\right)}{e} - \frac{2^{-\frac{7}{2}-\frac{m}{2}}e^{-2a}(ex)^{1+m}(bx^2)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},2bx^2\right)}{e}$$

```
[Out] -1/2*(e*x)^(1+m)/e/(1+m)-2^(-7/2-1/2*m)*exp(2*a)*(e*x)^(1+m)*(-b*x^2)^(-1/2
-1/2*m)*GAMMA(1/2+1/2*m,-2*b*x^2)/e-2^(-7/2-1/2*m)*(e*x)^(1+m)*(b*x^2)^(-1/
2-1/2*m)*GAMMA(1/2+1/2*m,2*b*x^2)/e/exp(2*a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5448, 5437, 2250}

$$\int (ex)^m \sinh^2(a+bx^2)\,dx = -\frac{e^{2a}2^{-\frac{m}{2}-\frac{7}{2}}(-bx^2)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-2bx^2\right)}{e} - \frac{e^{-2a}2^{-\frac{m}{2}-\frac{7}{2}}(bx^2)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},2bx^2\right)}{e} - \frac{(ex)^{m+1}}{2e(m+1)}$$

```
[In] Int[(e*x)^m*Sinh[a + b*x^2]^2,x]
```

```
[Out] -1/2*(e*x)^(1 + m)/(e*(1 + m)) - (2^(-7/2 - m/2)*E^(2*a)*(e*x)^(1 + m)*(-(b
*x^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -2*b*x^2])/e - (2^(-7/2 - m/2)*(e*x)^(
1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, 2*b*x^2])/(e*E^(2*a))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5437

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 5448

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}(ex)^m + \frac{1}{2}(ex)^m \cosh\left(2a+2bx^2\right)\right) dx \\
&= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{2}\int (ex)^m \cosh\left(2a+2bx^2\right) dx \\
&= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{4}\int e^{-2a-2bx^2}(ex)^m\,dx + \frac{1}{4}\int e^{2a+2bx^2}(ex)^m\,dx \\
&= -\frac{(ex)^{1+m}}{2e(1+m)} - \frac{2^{-\frac{7}{2}-\frac{m}{2}}e^{2a}(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-2bx^2\right)}{e} \\
&\quad - \frac{2^{-\frac{7}{2}-\frac{m}{2}}e^{-2a}(ex)^{1+m}\left(bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},2bx^2\right)}{e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int (ex)^m \sinh^2\left(a+bx^2\right) dx = \frac{1}{8}x(ex)^m\left(-\frac{4}{1+m} - 2^{-\frac{1}{2}-\frac{m}{2}}e^{2a}\left(-bx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-2bx^2\right) - 2^{-\frac{1}{2}-\frac{m}{2}}e^{-2a}\left(bx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},2bx^2\right)\right)$$

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2]^2,x]
```

```
[Out] (x*(e*x)^m*(-4/(1 + m) - 2^(-1/2 - m/2)*E^(2*a)*(-(b*x^2))^(-1/2 - m/2)*Gam
ma[(1 + m)/2, -2*b*x^2] - (2^(-1/2 - m/2)*(b*x^2)^(-1/2 - m/2)*Gamma[(1 + m
)/2, 2*b*x^2])/E^(2*a)))/8
```

Maple [F]

$$\int (ex)^m \sinh\left(x^2 b + a\right)^2 dx$$

```
[In] int((e*x)^m*sinh(b*x^2+a)^2,x)
```

```
[Out] int((e*x)^m*sinh(b*x^2+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\int (ex)^m \sinh^2\left(a + bx^2\right)\, dx = -\frac{8\,bx\cosh\left(m\log\left(ex\right)\right) + \left(em+e\right)\cosh\left(\frac{1}{2}\left(m-1\right)\log\left(\frac{2\,b}{e^2}\right) + 2\,a\right)\Gamma\left(\frac{1}{2}\,m + \frac{1}{2}, 2\,bx^2\right) - \left(em+e\right)\cosh\left(\frac{1}{2}\,(\ldots}{\ldots}$$

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*b*x*cosh(m*log(e*x)) + (e*m + e)*cosh(1/2*(m - 1)*log(2*b/e^2) + 2
*a)*gamma(1/2*m + 1/2, 2*b*x^2) - (e*m + e)*cosh(1/2*(m - 1)*log(-2*b/e^2)
- 2*a)*gamma(1/2*m + 1/2, -2*b*x^2) + 8*b*x*sinh(m*log(e*x)) - (e*m + e)*ga
mma(1/2*m + 1/2, 2*b*x^2)*sinh(1/2*(m - 1)*log(2*b/e^2) + 2*a) + (e*m + e)*
gamma(1/2*m + 1/2, -2*b*x^2)*sinh(1/2*(m - 1)*log(-2*b/e^2) - 2*a))/(b*m +
b)
```

Sympy [F]

$$\int (ex)^m \sinh^2\left(a + bx^2\right)\, dx = \int (ex)^m \sinh^2\left(a + bx^2\right) dx$$

```
[In] integrate((e*x)**m*sinh(b*x**2+a)**2,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**2)**2, x)
```

Maxima [F]

$$\int (ex)^m \sinh^2(a+bx^2)\ dx = \int (ex)^m \sinh\left(bx^2+a\right)^2\ dx$$

[In] integrate((e*x)^m*sinh(b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/4*e^m*integrate(e^(2*b*x^2 + m*log(x) + 2*a), x) + 1/4*e^m*integrate(e^(-2*b*x^2 + m*log(x) - 2*a), x) - 1/2*(e*x)^(m + 1)/(e*(m + 1))

Giac [F]

$$\int (ex)^m \sinh^2(a+bx^2)\ dx = \int (ex)^m \sinh\left(bx^2+a\right)^2\ dx$$

[In] integrate((e*x)^m*sinh(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate((e*x)^m*sinh(b*x^2 + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh^2(a+bx^2)\ dx = \int \sinh\left(b\,x^2+a\right)^2 (e\,x)^m\,dx$$

[In] int(sinh(a + b*x^2)^2*(e*x)^m,x)

[Out] int(sinh(a + b*x^2)^2*(e*x)^m, x)

3.26 $\int (ex)^m \sinh\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 95

$$\int (ex)^m \sinh\left(a+bx^2\right)\,dx = -\frac{e^a (ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-bx^2\right)}{4e} + \frac{e^{-a}(ex)^{1+m}\left(bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},bx^2\right)}{4e}$$

```
[Out] -1/4*exp(a)*(e*x)^(1+m)*(-b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-b*x^2)/e+1/4
*(e*x)^(1+m)*(b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,b*x^2)/e/exp(a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5436, 2250}

$$\int (ex)^m \sinh\left(a+bx^2\right)\,dx = \frac{e^{-a}\left(bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},bx^2\right)}{4e} - \frac{e^a\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-bx^2\right)}{4e}$$

```
[In] Int[(e*x)^m*Sinh[a + b*x^2],x]
```

```
[Out] -1/4*(E^a*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -(b*x^2)])
/e + ((e*x)^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, b*x^2])/(4*e*E^a)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5436

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int e^{-a-bx^2}(ex)^m\,dx\right)+\frac{1}{2}\int e^{a+bx^2}(ex)^m\,dx\\ &= -\frac{e^a(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-bx^2\right)}{4e}+\frac{e^{-a}(ex)^{1+m}\left(bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},bx^2\right)}{4e}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int (ex)^m\sinh\left(a+bx^2\right)\,dx=\frac{1}{4}e^{-a}x(ex)^m\left(-e^{2a}\left(-bx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-bx^2\right)+\left(bx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},bx^2\right)\right)$$

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2],x]
[Out] (x*(e*x)^m*(-(E^(2*a)*(-(b*x^2))^(-1/2 - m/2)*Gamma[(1 + m)/2, -(b*x^2)]) +
 (b*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, b*x^2]))/(4*E^a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.70 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

method	result	size
meijerg	$\frac{(ex)^m x\,\text{hypergeom}\left(\left[\frac{1}{4}+\frac{m}{4}\right],\left[\frac{1}{2},\frac{m}{4}+\frac{5}{4}\right],\frac{x^4b^2}{4}\right)\sinh(a)}{1+m}+\frac{(ex)^m b\,x^3\,\text{hypergeom}\left(\left[\frac{3}{4}+\frac{m}{4}\right],\left[\frac{3}{2},\frac{7}{4}+\frac{m}{4}\right],\frac{x^4b^2}{4}\right)\cosh(a)}{3+m}$	77

```
[In] int((e*x)^m*sinh(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (e*x)^m/(1+m)*x*hypergeom([1/4+1/4*m],[1/2,1/4*m+5/4],1/4*x^4*b^2)*sinh(a)+
(e*x)^m*b/(3+m)*x^3*hypergeom([3/4+1/4*m],[3/2,7/4+1/4*m],1/4*x^4*b^2)*cosh
(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.31

$$\int (ex)^m \sinh\left(a+bx^2\right)\,dx = \frac{e\cosh\left(\frac{1}{2}(m-1)\log\left(\frac{b}{e^2}\right)+a\right)\Gamma\left(\frac{1}{2}m+\frac{1}{2},bx^2\right)+e\cosh\left(\frac{1}{2}(m-1)\log\left(-\frac{b}{e^2}\right)-a\right)\Gamma\left(\frac{1}{2}m+\frac{1}{2},-bx^2\right)-e\Gamma}{4b}$$

```
[In] integrate((e*x)^m*sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(e*cosh(1/2*(m - 1)*log(b/e^2) + a)*gamma(1/2*m + 1/2, b*x^2) + e*cosh(
1/2*(m - 1)*log(-b/e^2) - a)*gamma(1/2*m + 1/2, -b*x^2) - e*gamma(1/2*m + 1
/2, b*x^2)*sinh(1/2*(m - 1)*log(b/e^2) + a) - e*gamma(1/2*m + 1/2, -b*x^2)*
sinh(1/2*(m - 1)*log(-b/e^2) - a))/b
```

Sympy [F]

$$\int (ex)^m \sinh\left(a+bx^2\right)\,dx = \int (ex)^m \sinh\left(a+bx^2\right)dx$$

```
[In] integrate((e*x)**m*sinh(b*x**2+a),x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**2), x)
```

Maxima [F]

$$\int (ex)^m \sinh\left(a+bx^2\right)\,dx = \int (ex)^m \sinh\left(bx^2+a\right)\,dx$$

```
[In] integrate((e*x)^m*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a), x)
```

Giac [F]

$$\int (ex)^m \sinh \left(a+bx^2\right) \, dx = \int (ex)^m \sinh \left(bx^2+a\right) \, dx$$

```
[In] integrate((e*x)^m*sinh(b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh \left(a+bx^2\right) \, dx = \int \sinh\left(b\,x^2+a\right)\,{\left(e\,x\right)}^m \,dx$$

```
[In] int(sinh(a + b*x^2)*(e*x)^m,x)
[Out] int(sinh(a + b*x^2)*(e*x)^m, x)
```

3.27 $\int (ex)^m \operatorname{csch}(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (ex)^m \operatorname{csch}\left(a+bx^2\right)\,dx = x^{-m}(ex)^m \mathrm{Int}\left(x^m \operatorname{csch}\left(a+bx^2\right),x\right)$$

```
[Out] (e*x)^m*Unintegrable(x^m*csch(b*x^2+a),x)/(x^m)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \operatorname{csch}\left(a+bx^2\right)\,dx = \int (ex)^m \operatorname{csch}\left(a+bx^2\right)\,dx$$

```
[In] Int[(e*x)^m*Csch[a + b*x^2],x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b*x^2], x])/x^m
```

Rubi steps

$$\text{integral} = \left(x^{-m}(ex)^m\right)\int x^m \operatorname{csch}\left(a+bx^2\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (ex)^m \operatorname{csch}\left(a+bx^2\right) \, dx = \int (ex)^m \operatorname{csch}\left(a+bx^2\right) \, dx$$

```
[In] Integrate[(e*x)^m*Csch[a + b*x^2],x]
[Out] Integrate[(e*x)^m*Csch[a + b*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{(ex)^m}{\sinh\left(x^2 b + a\right)} dx$$

```
[In] int((e*x)^m/sinh(b*x^2+a),x)
[Out] int((e*x)^m/sinh(b*x^2+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (ex)^m \operatorname{csch}\left(a+bx^2\right) \, dx = \int \frac{(ex)^m}{\sinh\left(bx^2+a\right)} \, dx$$

```
[In] integrate((e*x)^m/sinh(b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x)^m/sinh(b*x^2 + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (ex)^m \operatorname{csch}\left(a+bx^2\right) \, dx = \int \frac{(ex)^m}{\sinh\left(a+bx^2\right)} \, dx$$

```
[In] integrate((e*x)**m/sinh(b*x**2+a),x)
[Out] Integral((e*x)**m/sinh(a + b*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (ex)^m \mathrm{csch}\left(a+bx^2\right) \, dx = \int \frac{(ex)^m}{\sinh\left(bx^2+a\right)} \, dx$$

```
[In] integrate((e*x)^m/sinh(b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x)^m/sinh(b*x^2 + a), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (ex)^m \mathrm{csch}\left(a+bx^2\right) \, dx = \int \frac{(ex)^m}{\sinh\left(bx^2+a\right)} \, dx$$

```
[In] integrate((e*x)^m/sinh(b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x)^m/sinh(b*x^2 + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (ex)^m \mathrm{csch}\left(a+bx^2\right) \, dx = \int \frac{(e\,x)^m}{\sinh\left(b\,x^2+a\right)} \, dx$$

```
[In] int((e*x)^m/sinh(a + b*x^2),x)
[Out] int((e*x)^m/sinh(a + b*x^2), x)
```

3.28 $\int x^3 \sinh(a + bx^4) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int x^3 \sinh\left(a + bx^4\right) \, dx = \frac{\cosh\left(a + bx^4\right)}{4b}$$

```
[Out] 1/4*cosh(b*x^4+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5428, 2718}

$$\int x^3 \sinh\left(a + bx^4\right) \, dx = \frac{\cosh\left(a + bx^4\right)}{4b}$$

```
[In] Int[x^3*Sinh[a + b*x^4],x]
```

```
[Out] Cosh[a + b*x^4]/(4*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
```

```
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{4}\text{Subst}\left(\int \sinh(a+bx)\,dx, x, x^4\right) \\ &= \frac{\cosh\left(a+bx^4\right)}{4b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x^3 \sinh\left(a+bx^4\right)\,dx = \frac{\cosh\left(a+bx^4\right)}{4b}$$

```
[In] Integrate[x^3*Sinh[a + b*x^4],x]
[Out] Cosh[a + b*x^4]/(4*b)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh\left(b\,x^{4}+a\right)}{4b}$	14
default	$\frac{\cosh\left(b\,x^{4}+a\right)}{4b}$	14
parallelrisch	$\frac{1+\cosh\left(b\,x^{4}+a\right)}{4b}$	16
risch	$\frac{{\mathrm e}^{b\,x^{4}+a}}{8b}+\frac{{\mathrm e}^{-b\,x^{4}-a}}{8b}$	31
meijerg	$\frac{\sinh\left(a\right)\sinh\left(b\,x^{4}\right)}{4b}-\frac{\cosh\left(a\right)\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(b\,x^{4}\right)}{\sqrt{\pi}}\right)}{4b}$	40

```
[In] int(x^3*sinh(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4*cosh(b*x^4+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x^3 \sinh\left(a+bx^4\right)\,dx = \frac{\cosh\left(bx^4+a\right)}{4\,b}$$

```
[In] integrate(x^3*sinh(b*x^4+a),x, algorithm="fricas")
[Out] 1/4*cosh(b*x^4 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x^3 \sinh\left(a+bx^4\right)\,dx = \begin{cases} \frac{\cosh\left(a+bx^4\right)}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \sinh\left(a\right)}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*sinh(b*x**4+a),x)
[Out] Piecewise((cosh(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*sinh(a)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x^3 \sinh\left(a+bx^4\right)\,dx = \frac{\cosh\left(bx^4+a\right)}{4\,b}$$

```
[In] integrate(x^3*sinh(b*x^4+a),x, algorithm="maxima")
[Out] 1/4*cosh(b*x^4 + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int x^3 \sinh\left(a+bx^4\right)\,dx = \frac{e^{\left(bx^4+a\right)} + e^{\left(-bx^4-a\right)}}{8\,b}$$

```
[In] integrate(x^3*sinh(b*x^4+a),x, algorithm="giac")
[Out] 1/8*(e^(b*x^4 + a) + e^(-b*x^4 - a))/b
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x^3 \sinh \left(a+bx^4\right) \, dx = \frac{\cosh(b\,x^4+a)}{4\,b}$$

```
[In] int(x^3*sinh(a + b*x^4),x)
[Out] cosh(a + b*x^4)/(4*b)
```

3.29 $\int x^2 \sinh\left(a+\frac{b}{x}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int x^2 \sinh\left(a+\frac{b}{x}\right) dx = \frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right)-\frac{1}{6}b^3\cosh(a)\text{Chi}\left(\frac{b}{x}\right)+\frac{1}{6}b^2x\sinh\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)-\frac{1}{6}b^3\sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[Out] -1/6*b^3*Chi(b/x)*cosh(a)+1/6*b*x^2*cosh(a+b/x)-1/6*b^3*Shi(b/x)*sinh(a)+1/
6*b^2*x*sinh(a+b/x)+1/3*x^3*sinh(a+b/x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5428, 3378, 3384, 3379, 3382}

$$\int x^2 \sinh\left(a+\frac{b}{x}\right) dx = -\frac{1}{6}b^3\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-\frac{1}{6}b^3\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+\frac{1}{6}b^2x\sinh\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)+\frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right)$$

```
[In] Int[x^2*Sinh[a + b/x],x]
```

```
[Out] (b*x^2*Cosh[a + b/x])/6 - (b^3*Cosh[a]*CoshIntegral[b/x])/6 + (b^2*x*Sinh[a
 + b/x])/6 + (x^3*Sinh[a + b/x])/3 - (b^3*Sinh[a]*SinhIntegral[b/x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right) - \frac{1}{3}b\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right) - \frac{1}{6}b^2\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right) + \frac{1}{6}b^2x\sinh\left(a+\frac{b}{x}\right) \\
&\quad + \frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right) - \frac{1}{6}b^3\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{6} b x^2 \cosh\left(a+\frac{b}{x}\right) + \frac{1}{6} b^2 x \sinh\left(a+\frac{b}{x}\right) + \frac{1}{3} x^3 \sinh\left(a+\frac{b}{x}\right) \\
&\quad - \frac{1}{6}\left(b^3 \cosh(a)\right) \text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, \frac{1}{x}\right) \\
&\quad - \frac{1}{6}\left(b^3 \sinh(a)\right) \text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{6} b x^2 \cosh\left(a+\frac{b}{x}\right) - \frac{1}{6} b^3 \cosh(a) \text{Chi}\left(\frac{b}{x}\right) + \frac{1}{6} b^2 x \sinh\left(a+\frac{b}{x}\right) \\
&\quad + \frac{1}{3} x^3 \sinh\left(a+\frac{b}{x}\right) - \frac{1}{6} b^3 \sinh(a) \text{Shi}\left(\frac{b}{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$
\begin{aligned}
\int x^2 \sinh\left(a+\frac{b}{x}\right)\,dx = \frac{1}{6}\Bigg(&-b^3 \cosh(a) \text{Chi}\left(\frac{b}{x}\right) \\
&+ x\left(bx \cosh\left(a+\frac{b}{x}\right) + b^2 \sinh\left(a+\frac{b}{x}\right) + 2x^2 \sinh\left(a+\frac{b}{x}\right)\right) \\
&- b^3 \sinh(a) \text{Shi}\left(\frac{b}{x}\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[x^2*Sinh[a + b/x],x]
```

```
[Out] (-(b^3*Cosh[a]*CoshIntegral[b/x]) + x*(b*x*Cosh[a + b/x] + b^2*Sinh[a + b/x
] + 2*x^2*Sinh[a + b/x]) - b^3*Sinh[a]*SinhIntegral[b/x])/6
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.67

method	result
risch	$\frac{e^{-a} \operatorname{Ei}_1\left(\frac{b}{x}\right) b^3}{12} - \frac{e^{-\frac{ax+b}{x}} b^2 x}{12} + \frac{e^{-\frac{ax+b}{x}} b x^2}{12} - \frac{e^{-\frac{ax+b}{x}} x^3}{6} + \frac{e^{a} \operatorname{Ei}_1\left(-\frac{b}{x}\right) b^3}{12} + \frac{e^{\frac{ax+b}{x}} x b^2}{12} + \frac{e^{\frac{ax+b}{x}} x^2 b}{12} + \frac{e^{\frac{ax+b}{x}} x^3}{6}$
meijerg	$\frac{b^3 \sqrt{\pi} \cosh(a) \left(-\frac{8x^2\left(\frac{55b^2}{2x^2}+45\right)}{45\sqrt{\pi}\, b^2} + \frac{8x^2 \cosh\left(\frac{b}{x}\right)}{3\sqrt{\pi}\, b^2} + \frac{16x^3\left(\frac{5b^2}{2x^2}+5\right)\sinh\left(\frac{b}{x}\right)}{15\sqrt{\pi}\, b^3} - \frac{8\left(\text{Chi}\left(\frac{b}{x}\right)-\ln\left(\frac{b}{x}\right)-\gamma\right)}{3\sqrt{\pi}} - \frac{4\left(2\gamma-\frac{11}{3}-2\ln(x)+2\ln(ib)\right)}{3\sqrt{\pi}} + \frac{8x^2}{\sqrt{\pi}\, b^2}\right)}{16}$

```
[In] int(x^2*sinh(a+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*exp(-a)*Ei(1,b/x)*b^3-1/12*exp(-(a*x+b)/x)*b^2*x+1/12*exp(-(a*x+b)/x)*
b*x^2-1/6*exp(-(a*x+b)/x)*x^3+1/12*exp(a)*Ei(1,-b/x)*b^3+1/12*exp((a*x+b)/x
)*x*b^2+1/12*exp((a*x+b)/x)*x^2*b+1/6*exp((a*x+b)/x)*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int x^2 \sinh\left(a+\frac{b}{x}\right)\,dx = \frac{1}{6}bx^2\cosh\left(\frac{ax+b}{x}\right) - \frac{1}{12}\left(b^3\mathrm{Ei}\left(\frac{b}{x}\right)+b^3\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\cosh(a) - \frac{1}{12}\left(b^3\mathrm{Ei}\left(\frac{b}{x}\right)-b^3\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\sinh(a) + \frac{1}{6}\left(b^2x+2\,x^3\right)\sinh\left(\frac{ax+b}{x}\right)$$

```
[In] integrate(x^2*sinh(a+b/x),x, algorithm="fricas")
[Out] 1/6*b*x^2*cosh((a*x + b)/x) - 1/12*(b^3*Ei(b/x) + b^3*Ei(-b/x))*cosh(a) - 1
/12*(b^3*Ei(b/x) - b^3*Ei(-b/x))*sinh(a) + 1/6*(b^2*x + 2*x^3)*sinh((a*x +
b)/x)
```

Sympy [F]

$$\int x^2 \sinh\left(a+\frac{b}{x}\right)\,dx = \int x^2 \sinh\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate(x**2*sinh(a+b/x),x)
[Out] Integral(x**2*sinh(a + b/x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int x^2 \sinh\left(a+\frac{b}{x}\right)\,dx = \frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right) + \frac{1}{6}\left(b^2e^{(-a)}\Gamma\left(-2,\frac{b}{x}\right)+b^2e^a\Gamma\left(-2,-\frac{b}{x}\right)\right)b$$

```
[In] integrate(x^2*sinh(a+b/x),x, algorithm="maxima")
[Out] 1/3*x^3*sinh(a + b/x) + 1/6*(b^2*e^(-a)*gamma(-2, b/x) + b^2*e^a*gamma(-2,
-b/x))*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(68) = 136$.

Time = 0.26 (sec) , antiderivative size = 534, normalized size of antiderivative = 6.85

$$\int x^2 \sinh\left(a + \frac{b}{x}\right) dx =$$

$$-\frac{a^3b^4\text{Ei}\left(a - \frac{ax+b}{x}\right)e^{(-a)} + a^3b^4\text{Ei}\left(-a + \frac{ax+b}{x}\right)e^a - \frac{3(ax+b)a^2b^4\text{Ei}\left(a - \frac{ax+b}{x}\right)e^{(-a)}}{x} - \frac{3(ax+b)a^2b^4\text{Ei}\left(-a + \frac{ax+b}{x}\right)e^a}{x} + \cdots}{\cdots}$$

```
[In] integrate(x^2*sinh(a+b/x),x, algorithm="giac")
```

```
[Out] -1/12*(a^3*b^4*Ei(a - (a*x + b)/x)*e^(-a) + a^3*b^4*Ei(-a + (a*x + b)/x)*e^
a - 3*(a*x + b)*a^2*b^4*Ei(a - (a*x + b)/x)*e^(-a)/x - 3*(a*x + b)*a^2*b^4*
Ei(-a + (a*x + b)/x)*e^a/x + 3*(a*x + b)^2*a*b^4*Ei(a - (a*x + b)/x)*e^(-a)
/x^2 + 3*(a*x + b)^2*a*b^4*Ei(-a + (a*x + b)/x)*e^a/x^2 + a^2*b^4*e^((a*x +
 b)/x) - a^2*b^4*e^(-(a*x + b)/x) - (a*x + b)^3*b^4*Ei(a - (a*x + b)/x)*e^(
-a)/x^3 - (a*x + b)^3*b^4*Ei(-a + (a*x + b)/x)*e^a/x^3 - a*b^4*e^((a*x + b)
/x) - 2*(a*x + b)*a*b^4*e^((a*x + b)/x)/x - a*b^4*e^(-(a*x + b)/x) + 2*(a*x
 + b)*a*b^4*e^(-(a*x + b)/x)/x + 2*b^4*e^((a*x + b)/x) + (a*x + b)^2*b^4*e^
((a*x + b)/x)/x^2 + (a*x + b)*b^4*e^((a*x + b)/x)/x - 2*b^4*e^(-(a*x + b)/x
) - (a*x + b)^2*b^4*e^(-(a*x + b)/x)/x^2 + (a*x + b)*b^4*e^(-(a*x + b)/x)/x
)/((a^3 - 3*(a*x + b)*a^2/x + 3*(a*x + b)^2*a/x^2 - (a*x + b)^3/x^3)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh\left(a + \frac{b}{x}\right) dx = \int x^2 \sinh\left(a + \frac{b}{x}\right) dx$$

```
[In] int(x^2*sinh(a + b/x),x)
```

```
[Out] int(x^2*sinh(a + b/x), x)
```

3.30 $\int x \sinh\left(a+\frac{b}{x}\right)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int x \sinh\left(a+\frac{b}{x}\right) dx = \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right) - \frac{1}{2}b^2\text{Chi}\left(\frac{b}{x}\right)\sinh(a) + \frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right) - \frac{1}{2}b^2\cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[Out] 1/2*b*x*cosh(a+b/x)-1/2*b^2*cosh(a)*Shi(b/x)-1/2*b^2*Chi(b/x)*sinh(a)+1/2*x
^2*sinh(a+b/x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5428, 3378, 3384, 3379, 3382}

$$\int x \sinh\left(a+\frac{b}{x}\right) dx = -\frac{1}{2}b^2\sinh(a)\text{Chi}\left(\frac{b}{x}\right) - \frac{1}{2}b^2\cosh(a)\text{Shi}\left(\frac{b}{x}\right) + \frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right) + \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)$$

```
[In] Int[x*Sinh[a + b/x],x]
```

```
[Out] (b*x*Cosh[a + b/x])/2 - (b^2*CoshIntegral[b/x]*Sinh[a])/2 + (x^2*Sinh[a + b
/x])/2 - (b^2*Cosh[a]*SinhIntegral[b/x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right) - \frac{1}{2}b\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right) + \frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right) - \frac{1}{2}b^2\text{Subst}\left(\int \frac{\sinh(a+bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right) + \frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right) - \frac{1}{2}\left(b^2\cosh(a)\right)\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad - \frac{1}{2}\left(b^2\sinh(a)\right)\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right) - \frac{1}{2}b^2\text{Chi}\left(\frac{b}{x}\right)\sinh(a) + \frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right) - \frac{1}{2}b^2\cosh(a)\text{Shi}\left(\frac{b}{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int x\sinh\left(a+\frac{b}{x}\right)\,dx=\frac{1}{2}\left(-b^2\mathrm{Chi}\left(\frac{b}{x}\right)\sinh(a)+x\left(b\cosh\left(a+\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)\right)-b^2\cosh(a)\mathrm{Shi}\left(\frac{b}{x}\right)\right)$$

```
[In] Integrate[x*Sinh[a + b/x],x]
[Out] (-(b^2*CoshIntegral[b/x]*Sinh[a]) + x*(b*Cosh[a + b/x] + x*Sinh[a + b/x]) -
 b^2*Cosh[a]*SinhIntegral[b/x])/2
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{\mathrm{e}^{-a}\,\mathrm{Ei}_1\left(\frac{b}{x}\right)b^2}{4}+\frac{\mathrm{e}^{-\frac{ax+b}{x}}bx}{4}-\frac{\mathrm{e}^{-\frac{ax+b}{x}}x^2}{4}+\frac{\mathrm{e}^{a}\,\mathrm{Ei}_1\left(-\frac{b}{x}\right)b^2}{4}+\frac{\mathrm{e}^{\frac{ax+b}{x}}xb}{4}+\frac{\mathrm{e}^{\frac{ax+b}{x}}x^2}{4}$
meijerg	$-\frac{ib^2\sqrt{\pi}\,\cosh(a)\left(\frac{4ix\cosh\left(\frac{b}{x}\right)}{b\sqrt{\pi}}+\frac{4ix^2\sinh\left(\frac{b}{x}\right)}{b^2\sqrt{\pi}}-\frac{4i\,\mathrm{Shi}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{8}+\frac{b^2\sqrt{\pi}\,\sinh(a)\left(-\frac{4x^2\left(\frac{9b^2}{2x^2}+3\right)}{3\sqrt{\pi}\,b^2}+\frac{4x^2\cosh\left(\frac{b}{x}\right)}{\sqrt{\pi}\,b^2}+\frac{4x\sinh\left(\frac{b}{x}\right)}{\sqrt{\pi}\,b}-\frac{4\left(\mathrm{Chi}\left(\frac{b}{x}\right)\right.}{\ldots}\ldots\right.}{8}$

```
[In] int(x*sinh(a+b/x),x,method=_RETURNVERBOSE)
[Out] -1/4*exp(-a)*Ei(1,b/x)*b^2+1/4*exp(-(a*x+b)/x)*b*x-1/4*exp(-(a*x+b)/x)*x^2+
1/4*exp(a)*Ei(1,-b/x)*b^2+1/4*exp((a*x+b)/x)*x*b+1/4*exp((a*x+b)/x)*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int x\sinh\left(a+\frac{b}{x}\right)\,dx=\frac{1}{2}\,bx\cosh\left(\frac{ax+b}{x}\right)+\frac{1}{2}\,x^2\sinh\left(\frac{ax+b}{x}\right)-\frac{1}{4}\left(b^2\mathrm{Ei}\left(\frac{b}{x}\right)-b^2\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\cosh\left(a\right)-\frac{1}{4}\left(b^2\mathrm{Ei}\left(\frac{b}{x}\right)+b^2\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\sinh\left(a\right)$$

```
[In] integrate(x*sinh(a+b/x),x, algorithm="fricas")
[Out] 1/2*b*x*cosh((a*x + b)/x) + 1/2*x^2*sinh((a*x + b)/x) - 1/4*(b^2*Ei(b/x) -
b^2*Ei(-b/x))*cosh(a) - 1/4*(b^2*Ei(b/x) + b^2*Ei(-b/x))*sinh(a)
```

Sympy [F]

$$\int x \sinh\left(a + \frac{b}{x}\right) dx = \int x \sinh\left(a + \frac{b}{x}\right) dx$$

```
[In] integrate(x*sinh(a+b/x),x)
[Out] Integral(x*sinh(a + b/x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int x \sinh\left(a + \frac{b}{x}\right) dx = \frac{1}{2} x^2 \sinh\left(a + \frac{b}{x}\right) + \frac{1}{4}\left(be^{(-a)}\Gamma\left(-1, \frac{b}{x}\right) - be^{a}\Gamma\left(-1, -\frac{b}{x}\right)\right)b$$

```
[In] integrate(x*sinh(a+b/x),x, algorithm="maxima")
[Out] 1/2*x^2*sinh(a + b/x) + 1/4*(b*e^(-a)*gamma(-1, b/x) - b*e^a*gamma(-1, -b/x
))*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(52) = 104.

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 5.22

$$\int x \sinh\left(a + \frac{b}{x}\right) dx$$

$$= \frac{a^2b^3\text{Ei}\left(a - \frac{ax+b}{x}\right)e^{(-a)} - a^2b^3\text{Ei}\left(-a + \frac{ax+b}{x}\right)e^{a} - \frac{2\,(ax+b)ab^3\text{Ei}\left(a - \frac{ax+b}{x}\right)e^{(-a)}}{x} + \frac{2\,(ax+b)ab^3\text{Ei}\left(-a + \frac{ax+b}{x}\right)e^{a}}{x} + \frac{(ax+}{}}{4}$$

```
[In] integrate(x*sinh(a+b/x),x, algorithm="giac")
[Out] 1/4*(a^2*b^3*Ei(a - (a*x + b)/x)*e^(-a) - a^2*b^3*Ei(-a + (a*x + b)/x)*e^a
- 2*(a*x + b)*a*b^3*Ei(a - (a*x + b)/x)*e^(-a)/x + 2*(a*x + b)*a*b^3*Ei(-a
+ (a*x + b)/x)*e^a/x + (a*x + b)^2*b^3*Ei(a - (a*x + b)/x)*e^(-a)/x^2 - (a*
x + b)^2*b^3*Ei(-a + (a*x + b)/x)*e^a/x^2 - a*b^3*e^((a*x + b)/x) - a*b^3*e
^(-(a*x + b)/x) + b^3*e^((a*x + b)/x) + (a*x + b)*b^3*e^((a*x + b)/x)/x - b
^3*e^(-(a*x + b)/x) + (a*x + b)*b^3*e^(-(a*x + b)/x)/x)/((a^2 - 2*(a*x + b)
*a/x + (a*x + b)^2/x^2)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x \sinh\left(a + \frac{b}{x}\right) dx = \int x \sinh\left(a + \frac{b}{x}\right) dx$$

```
[In] int(x*sinh(a + b/x),x)
[Out] int(x*sinh(a + b/x), x)
```

3.31 $\int \sinh\left(a+\frac{b}{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 33

$$\int \sinh\left(a+\frac{b}{x}\right)\,dx=-b\cosh(a)\text{Chi}\left(\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[Out] -b*Chi(b/x)*cosh(a)-b*Shi(b/x)*sinh(a)+x*sinh(a+b/x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5410, 3378, 3384, 3379, 3382}

$$\int \sinh\left(a+\frac{b}{x}\right)\,dx=-b\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)$$

```
[In] Int[Sinh[a + b/x],x]
```

```
[Out] -(b*Cosh[a]*CoshIntegral[b/x]) + x*Sinh[a + b/x] - b*Sinh[a]*SinhIntegral[b
/x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5410

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= x\sinh\left(a+\frac{b}{x}\right) - b\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= x\sinh\left(a+\frac{b}{x}\right) - (b\cosh(a))\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad - (b\sinh(a))\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= -b\cosh(a)\text{Chi}\left(\frac{b}{x}\right) + x\sinh\left(a+\frac{b}{x}\right) - b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \sinh\left(a+\frac{b}{x}\right)\,dx = -b\cosh(a)\text{Chi}\left(\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[In] Integrate[Sinh[a + b/x],x]
```

```
[Out] -(b*Cosh[a]*CoshIntegral[b/x]) + x*Sinh[a + b/x] - b*Sinh[a]*SinhIntegral[b
/x]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

method	result	s
risch	$\frac{e^{-a}\,\text{Ei}_1\left(\frac{b}{x}\right)b}{2}-\frac{e^{-\frac{ax+b}{x}}x}{2}+\frac{e^{a}\,\text{Ei}_1\left(-\frac{b}{x}\right)b}{2}+\frac{e^{\frac{ax+b}{x}}x}{2}$	5
meijerg	$-\frac{\sqrt{\pi}\,\cosh(a)b\left(\frac{4}{\sqrt{\pi}}-\frac{4x\sinh\left(\frac{b}{x}\right)}{\sqrt{\pi}\,b}+\frac{4\,\text{Chi}\left(\frac{b}{x}\right)-4\ln\left(\frac{b}{x}\right)-4\gamma}{\sqrt{\pi}}+\frac{4\gamma-4-4\ln(x)+4\ln(ib)}{\sqrt{\pi}}\right)}{4}-\frac{i\sqrt{\pi}\,\sinh(a)b\left(\frac{4ix\cosh\left(\frac{b}{x}\right)}{b\sqrt{\pi}}-\frac{4i\,\text{Shi}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{4}$	1

```
[In] int(sinh(a+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(-a)*Ei(1,b/x)*b-1/2*exp(-(a*x+b)/x)*x+1/2*exp(a)*Ei(1,-b/x)*b+1/2*e
xp((a*x+b)/x)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\begin{aligned}\int \sinh\left(a+\frac{b}{x}\right)\,dx = &-\frac{1}{2}\left(b\text{Ei}\left(\frac{b}{x}\right)+b\text{Ei}\left(-\frac{b}{x}\right)\right)\cosh\left(a\right)\\&-\frac{1}{2}\left(b\text{Ei}\left(\frac{b}{x}\right)-b\text{Ei}\left(-\frac{b}{x}\right)\right)\sinh\left(a\right)+x\sinh\left(\frac{ax+b}{x}\right)\end{aligned}$$

```
[In] integrate(sinh(a+b/x),x, algorithm="fricas")
```

```
[Out] -1/2*(b*Ei(b/x) + b*Ei(-b/x))*cosh(a) - 1/2*(b*Ei(b/x) - b*Ei(-b/x))*sinh(a
) + x*sinh((a*x + b)/x)
```

Sympy [F]

$$\int \sinh\left(a+\frac{b}{x}\right)\,dx=\int \sinh\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate(sinh(a+b/x),x)
[Out] Integral(sinh(a + b/x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \sinh\left(a+\frac{b}{x}\right)\,dx=-\frac{1}{2}\left(\mathrm{Ei}\left(-\frac{b}{x}\right)e^{(-a)}+\mathrm{Ei}\left(\frac{b}{x}\right)e^{a}\right)b+x\sinh\left(a+\frac{b}{x}\right)$$

```
[In] integrate(sinh(a+b/x),x, algorithm="maxima")
[Out] -1/2*(Ei(-b/x)*e^(-a) + Ei(b/x)*e^a)*b + x*sinh(a + b/x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(33) = 66.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 5.24

$$\int \sinh\left(a+\frac{b}{x}\right)\,dx=-\frac{ab^2\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}-\frac{(ax+b)b^2\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}}{x}-b^2e^{\left(-\frac{ax+b}{x}\right)}}{2\left(a-\frac{ax+b}{x}\right)b}-\frac{ab^2\mathrm{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}-\frac{(ax+b)b^2\mathrm{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}}{x}+b^2e^{\left(\frac{ax+b}{x}\right)}}{2\left(a-\frac{ax+b}{x}\right)b}$$

```
[In] integrate(sinh(a+b/x),x, algorithm="giac")
[Out] -1/2*(a*b^2*Ei(a - (a*x + b)/x)*e^(-a) - (a*x + b)*b^2*Ei(a - (a*x + b)/x)*
e^(-a)/x - b^2*e^(-(a*x + b)/x))/((a - (a*x + b)/x)*b) - 1/2*(a*b^2*Ei(-a +
 (a*x + b)/x)*e^a - (a*x + b)*b^2*Ei(-a + (a*x + b)/x)*e^a/x + b^2*e^((a*x
+ b)/x))/((a - (a*x + b)/x)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh\left(a+\frac{b}{x}\right)\,dx=\int \sinh\left(a+\frac{b}{x}\right)\,dx$$

```
[In] int(sinh(a + b/x),x)
[Out] int(sinh(a + b/x), x)
```

3.32 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx = -\text{Chi}\left(\frac{b}{x}\right)\sinh(a) - \cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[Out] -cosh(a)*Shi(b/x)-Chi(b/x)*sinh(a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5426, 5425, 5424}

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx = \sinh(a)\left(-\text{Chi}\left(\frac{b}{x}\right)\right) - \cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[In] Int[Sinh[a + b/x]/x,x]
```

```
[Out] -(CoshIntegral[b/x]*Sinh[a]) - Cosh[a]*SinhIntegral[b/x]
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5426

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \cosh(a)\int \frac{\sinh\left(\frac{b}{x}\right)}{x}\,dx + \sinh(a)\int \frac{\cosh\left(\frac{b}{x}\right)}{x}\,dx \\ &= -\text{Chi}\left(\frac{b}{x}\right)\sinh(a) - \cosh(a)\text{Shi}\left(\frac{b}{x}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx = -\text{Chi}\left(\frac{b}{x}\right)\sinh(a) - \cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[In] Integrate[Sinh[a + b/x]/x,x]
[Out] -(CoshIntegral[b/x]*Sinh[a]) - Cosh[a]*SinhIntegral[b/x]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
risch	$-\frac{e^{-a}\,\text{Ei}_1\left(\frac{b}{x}\right)}{2}+\frac{e^{a}\,\text{Ei}_1\left(-\frac{b}{x}\right)}{2}$	27
meijerg	$-\cosh\left(a\right)\text{Shi}\left(\frac{b}{x}\right)-\frac{\sqrt{\pi}\,\sinh\left(a\right)\left(\frac{2\,\text{Chi}\left(\frac{b}{x}\right)-2\ln\left(\frac{b}{x}\right)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma-2\ln\left(x\right)+2\ln\left(ib\right)}{\sqrt{\pi}}\right)}{2}$	62

```
[In] int(sinh(a+b/x)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*exp(-a)*Ei(1,b/x)+1/2*exp(a)*Ei(1,-b/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx = -\frac{1}{2}\left(\operatorname{Ei}\left(\frac{b}{x}\right)-\operatorname{Ei}\left(-\frac{b}{x}\right)\right)\cosh(a) - \frac{1}{2}\left(\operatorname{Ei}\left(\frac{b}{x}\right)+\operatorname{Ei}\left(-\frac{b}{x}\right)\right)\sinh(a)$$

```
[In] integrate(sinh(a+b/x)/x,x, algorithm="fricas")
[Out] -1/2*(Ei(b/x) - Ei(-b/x))*cosh(a) - 1/2*(Ei(b/x) + Ei(-b/x))*sinh(a)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx = -\sinh(a)\operatorname{Chi}\left(\frac{b}{x}\right) - \cosh(a)\operatorname{Shi}\left(\frac{b}{x}\right)$$

```
[In] integrate(sinh(a+b/x)/x,x)
[Out] -sinh(a)*Chi(b/x) - cosh(a)*Shi(b/x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx = \frac{1}{2}\operatorname{Ei}\left(-\frac{b}{x}\right)e^{(-a)} - \frac{1}{2}\operatorname{Ei}\left(\frac{b}{x}\right)e^{a}$$

```
[In] integrate(sinh(a+b/x)/x,x, algorithm="maxima")
[Out] 1/2*Ei(-b/x)*e^(-a) - 1/2*Ei(b/x)*e^a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.10

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx = \frac{b\operatorname{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)} - b\operatorname{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}}{2\,b}$$

```
[In] integrate(sinh(a+b/x)/x,x, algorithm="giac")
[Out] 1/2*(b*Ei(a - (a*x + b)/x)*e^(-a) - b*Ei(-a + (a*x + b)/x)*e^a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx = -\sinh(a)\ \mathrm{coshint}\left(\frac{b}{x}\right) - \cosh(a)\ \mathrm{sinhint}\left(\frac{b}{x}\right)$$

```
[In] int(sinh(a + b/x)/x,x)
[Out] - sinh(a)*coshint(b/x) - cosh(a)*sinhint(b/x)
```

3.33 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2} dx = -\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

```
[Out] -cosh(a+b/x)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5428, 2718}

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2} dx = -\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

```
[In] Int[Sinh[a + b/x]/x^2,x]
```

```
[Out] -(Cosh[a + b/x]/b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
```

```
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x}\right) \\ &= -\frac{\cosh\left(a+\frac{b}{x}\right)}{b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

```
[In] Integrate[Sinh[a + b/x]/x^2,x]
[Out] -(Cosh[a + b/x]/b)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$	14
default	$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$	14
parallelrisch	$\frac{-\cosh\left(\frac{ax+b}{x}\right)-1}{b}$	19
risch	$-\frac{e^{\frac{ax+b}{x}}}{2b}-\frac{e^{-\frac{ax+b}{x}}}{2b}$	33
meijerg	$\frac{\sqrt{\pi}\,\cosh(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{b}-\frac{\sinh(a)\sinh\left(\frac{b}{x}\right)}{b}$	39

```
[In] int(sinh(a+b/x)/x^2,x,method=_RETURNVERBOSE)
[Out] -cosh(a+b/x)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\cosh\left(\frac{ax+b}{x}\right)}{b}$$

```
[In] integrate(sinh(a+b/x)/x^2,x, algorithm="fricas")
[Out] -cosh((a*x + b)/x)/b
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{\cosh\left(a+\frac{b}{x}\right)}{b} & \text{for}\ b \neq 0 \\ -\frac{\sinh(a)}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(a+b/x)/x**2,x)
[Out] Piecewise((-cosh(a + b/x)/b, Ne(b, 0)), (-sinh(a)/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

```
[In] integrate(sinh(a+b/x)/x^2,x, algorithm="maxima")
[Out] -cosh(a + b/x)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{e^{\left(\frac{ax+b}{x}\right)} + e^{\left(-\frac{ax+b}{x}\right)}}{2\,b}$$

```
[In] integrate(sinh(a+b/x)/x^2,x, algorithm="giac")
[Out] -1/2*(e^((a*x + b)/x) + e^(-(a*x + b)/x))/b
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

```
[In] int(sinh(a + b/x)/x^2,x)
[Out] -cosh(a + b/x)/b
```

3.34 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx}+\frac{\sinh\left(a+\frac{b}{x}\right)}{b^2}$$

```
[Out] -cosh(a+b/x)/b/x+sinh(a+b/x)/b^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5428, 3377, 2717}

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{\sinh\left(a+\frac{b}{x}\right)}{b^2}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx}$$

```
[In] Int[Sinh[a + b/x]/x^3,x]
```

```
[Out] -(Cosh[a + b/x]/(b*x)) + Sinh[a + b/x]/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx}+\frac{\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\ &= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx}+\frac{\sinh\left(a+\frac{b}{x}\right)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3}\,dx=\frac{-b\cosh\left(a+\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)}{b^2x}$$

```
[In] Integrate[Sinh[a + b/x]/x^3,x]
[Out] (-(b*Cosh[a + b/x]) + x*Sinh[a + b/x])/(b^2*x)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

method	result	size
parallelrisch	$\frac{-b\cosh\left(\frac{ax+b}{x}\right)+\sinh\left(\frac{ax+b}{x}\right)x}{x\,b^2}$	34
derivativedivides	$-\frac{\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)-\sinh\left(a+\frac{b}{x}\right)-a\cosh\left(a+\frac{b}{x}\right)}{b^2}$	44
default	$-\frac{\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)-\sinh\left(a+\frac{b}{x}\right)-a\cosh\left(a+\frac{b}{x}\right)}{b^2}$	44
risch	$-\frac{(-x+b)\,e^{\frac{ax+b}{x}}}{2b^2x}-\frac{(x+b)\,e^{-\frac{ax+b}{x}}}{2b^2x}$	47
meijerg	$-\frac{\cosh(a)\left(\frac{\cosh\left(\frac{b}{x}\right)b}{x}-\sinh\left(\frac{b}{x}\right)\right)}{b^2}+\frac{2\sqrt{\pi}\,\sinh(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}}-\frac{b\sinh\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^2}$	71

```
[In] int(sinh(a+b/x)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/x/b^2*(-b*cosh((a*x+b)/x)+sinh((a*x+b)/x)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\sinh \left(a+\frac{b}{x}\right)}{x^3}\,dx=-\frac{b\cosh \left(\frac{ax+b}{x}\right)-x\sinh \left(\frac{ax+b}{x}\right)}{b^2x}$$

```
[In] integrate(sinh(a+b/x)/x^3,x, algorithm="fricas")
[Out] -(b*cosh((a*x + b)/x) - x*sinh((a*x + b)/x))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sinh \left(a+\frac{b}{x}\right)}{x^3}\,dx=\begin{cases}-\frac{\cosh \left(a+\frac{b}{x}\right)}{bx}+\frac{\sinh \left(a+\frac{b}{x}\right)}{b^2} & \text{for } b\neq 0\\ -\frac{\sinh (a)}{2x^2} & \text{otherwise}\end{cases}$$

```
[In] integrate(sinh(a+b/x)/x**3,x)
[Out] Piecewise((-cosh(a + b/x)/(b*x) + sinh(a + b/x)/b**2, Ne(b, 0)), (-sinh(a)/
(2*x**2), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \frac{\sinh \left(a+\frac{b}{x}\right)}{x^3}\,dx=-\frac{1}{4}\,b\left(\frac{e^{(-a)}\Gamma\left(3,\frac{b}{x}\right)}{b^3}-\frac{e^a\Gamma\left(3,-\frac{b}{x}\right)}{b^3}\right)-\frac{\sinh \left(a+\frac{b}{x}\right)}{2\,x^2}$$

```
[In] integrate(sinh(a+b/x)/x^3,x, algorithm="maxima")
[Out] -1/4*b*(e^(-a)*gamma(3, b/x)/b^3 - e^a*gamma(3, -b/x)/b^3) - 1/2*sinh(a + b
/x)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{ae^{\left(\frac{ax+b}{x}\right)} + ae^{\left(-\frac{ax+b}{x}\right)} - \frac{(ax+b)e^{\left(\frac{ax+b}{x}\right)}}{x} - \frac{(ax+b)e^{\left(-\frac{ax+b}{x}\right)}}{x} + e^{\left(\frac{ax+b}{x}\right)} - e^{\left(-\frac{ax+b}{x}\right)}}{2\,b^2}$$

```
[In] integrate(sinh(a+b/x)/x^3,x, algorithm="giac")
[Out] 1/2*(a*e^((a*x + b)/x) + a*e^(-(a*x + b)/x) - (a*x + b)*e^((a*x + b)/x)/x -
 (a*x + b)*e^(-(a*x + b)/x)/x + e^((a*x + b)/x) - e^(-(a*x + b)/x))/b^2
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{\sinh\left(a+\frac{b}{x}\right)}{b^2} - \frac{\cosh\left(a+\frac{b}{x}\right)}{b\,x}$$

```
[In] int(sinh(a + b/x)/x^3,x)
[Out] sinh(a + b/x)/b^2 - cosh(a + b/x)/(b*x)
```

3.35 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3} - \frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}$$

```
[Out] -2*cosh(a+b/x)/b^3-cosh(a+b/x)/b/x^2+2*sinh(a+b/x)/b^2/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5428, 3377, 2718}

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3} + \frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x} - \frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2}$$

```
[In] Int[Sinh[a + b/x]/x^4,x]
```

```
[Out] (-2*Cosh[a + b/x])/b^3 - Cosh[a + b/x]/(b*x^2) + (2*Sinh[a + b/x])/(b^2*x)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^2 \sinh(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x} - \frac{2\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}\\
&= -\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3} - \frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{-\left((b^2+2x^2)\cosh\left(a+\frac{b}{x}\right)\right)+2bx\sinh\left(a+\frac{b}{x}\right)}{b^3x^2}$$

```
[In] Integrate[Sinh[a + b/x]/x^4,x]
[Out] (-((b^2 + 2*x^2)*Cosh[a + b/x]) + 2*b*x*Sinh[a + b/x])/(b^3*x^2)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{(b^2-2bx+2x^2)\mathrm{e}^{\frac{ax+b}{x}}}{2b^3x^2}-\frac{(b^2+2bx+2x^2)\mathrm{e}^{-\frac{ax+b}{x}}}{2b^3x^2}$
parallelrisch	$\frac{\tanh\left(\frac{ax+b}{2x}\right)^2 b^2-4\tanh\left(\frac{ax+b}{2x}\right)xb+4x^2+b^2}{x^2b^3\left(\tanh\left(\frac{ax+b}{2x}\right)^2-1\right)}$
derivativedivides	$-\frac{a^2\cosh\left(a+\frac{b}{x}\right)-2a\left(\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)-\sinh\left(a+\frac{b}{x}\right)\right)+\left(a+\frac{b}{x}\right)^2\cosh\left(a+\frac{b}{x}\right)-2\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)+2\cosh\left(a+\frac{b}{x}\right)}{b^3}$
default	$-\frac{a^2\cosh\left(a+\frac{b}{x}\right)-2a\left(\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)-\sinh\left(a+\frac{b}{x}\right)\right)+\left(a+\frac{b}{x}\right)^2\cosh\left(a+\frac{b}{x}\right)-2\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)+2\cosh\left(a+\frac{b}{x}\right)}{b^3}$
meijerg	$-\frac{4\sqrt{\pi}\,\cosh(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{b^2}{2x^2}+1\right)\cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}}-\frac{b\sinh\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^3}-\frac{4i\sqrt{\pi}\,\sinh(a)\left(\frac{ib\cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}-\frac{i\left(\frac{3b^2}{2x^2}+3\right)\sinh\left(\frac{b}{x}\right)}{6\sqrt{\pi}}\right)}{b^3}$

```
[In] int(sinh(a+b/x)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/2*(b^2-2*b*x+2*x^2)/b^3/x^2*exp((a*x+b)/x)-1/2*(b^2+2*b*x+2*x^2)/b^3/x^2
*exp(-(a*x+b)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx=\frac{2\,bx\sinh\left(\frac{ax+b}{x}\right)-(b^2+2\,x^2)\cosh\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

```
[In] integrate(sinh(a+b/x)/x^4,x, algorithm="fricas")
[Out] (2*b*x*sinh((a*x + b)/x) - (b^2 + 2*x^2)*cosh((a*x + b)/x))/(b^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx=\begin{cases}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3} & \text{for } b\neq 0\\ -\frac{\sinh(a)}{3x^3} & \text{otherwise}\end{cases}$$

```
[In] integrate(sinh(a+b/x)/x**4,x)
[Out] Piecewise((-cosh(a + b/x)/(b*x**2) + 2*sinh(a + b/x)/(b**2*x) - 2*cosh(a +
b/x)/b**3, Ne(b, 0)), (-sinh(a)/(3*x**3), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{1}{6}\,b\left(\frac{e^{(-a)}\Gamma\left(4,\frac{b}{x}\right)}{b^4}+\frac{e^{a}\Gamma\left(4,-\frac{b}{x}\right)}{b^4}\right)-\frac{\sinh\left(a+\frac{b}{x}\right)}{3\,x^3}$$

```
[In] integrate(sinh(a+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*b*(e^(-a)*gamma(4, b/x)/b^4 + e^a*gamma(4, -b/x)/b^4) - 1/3*sinh(a + b
/x)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(46) = 92.

Time = 0.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 4.65

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx =$$
$$-\frac{a^2e^{\left(\frac{ax+b}{x}\right)}+a^2e^{\left(-\frac{ax+b}{x}\right)}+2\,ae^{\left(\frac{ax+b}{x}\right)}-\frac{2\,(ax+b)ae^{\left(\frac{ax+b}{x}\right)}}{x}-2\,ae^{\left(-\frac{ax+b}{x}\right)}-\frac{2\,(ax+b)ae^{\left(-\frac{ax+b}{x}\right)}}{x}+\frac{(ax+b)^2e^{\left(\frac{ax+b}{x}\right)}}{x^2}}{2\,b^3}$$

```
[In] integrate(sinh(a+b/x)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*e^((a*x + b)/x) + a^2*e^(-(a*x + b)/x) + 2*a*e^((a*x + b)/x) - 2*
(a*x + b)*a*e^((a*x + b)/x)/x - 2*a*e^(-(a*x + b)/x) - 2*(a*x + b)*a*e^(-(a
*x + b)/x)/x + (a*x + b)^2*e^((a*x + b)/x)/x^2 - 2*(a*x + b)*e^((a*x + b)/x
)/x + (a*x + b)^2*e^(-(a*x + b)/x)/x^2 + 2*(a*x + b)*e^(-(a*x + b)/x)/x + 2
*e^((a*x + b)/x) + 2*e^(-(a*x + b)/x))/b^3
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{\mathrm{e}^{a+\frac{b}{x}}\left(\frac{1}{2\,b}-\frac{x}{b^2}+\frac{x^2}{b^3}\right)}{x^2}-\frac{\mathrm{e}^{-a-\frac{b}{x}}\left(\frac{x}{b^2}+\frac{1}{2\,b}+\frac{x^2}{b^3}\right)}{x^2}$$

```
[In] int(sinh(a + b/x)/x^4,x)
```

```
[Out] - (exp(a + b/x)*(1/(2*b) - x/b^2 + x^2/b^3))/x^2 - (exp(- a - b/x)*(x/b^2 +
 1/(2*b) + x^2/b^3))/x^2
```

3.36 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 62

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4} + \frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}$$

```
[Out] -cosh(a+b/x)/b/x^3-6*cosh(a+b/x)/b^3/x+6*sinh(a+b/x)/b^4+3*sinh(a+b/x)/b^2/
x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5428, 3377, 2717}

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx = \frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4} - \frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x} + \frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2} - \frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}$$

```
[In] Int[Sinh[a + b/x]/x^5,x]
```

```
[Out] -(Cosh[a + b/x]/(b*x^3)) - (6*Cosh[a + b/x])/(b^3*x) + (6*Sinh[a + b/x])/b^
4 + (3*Sinh[a + b/x])/(b^2*x^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^3 \sinh(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3} + \frac{3\text{Subst}\left(\int x^2\cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3} + \frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2} - \frac{6\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}\\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x} + \frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2} + \frac{6\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b^3}\\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4} + \frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx = \frac{-b(b^2+6x^2)\cosh\left(a+\frac{b}{x}\right)+3x(b^2+2x^2)\sinh\left(a+\frac{b}{x}\right)}{b^4x^3}$$

```
[In] Integrate[Sinh[a + b/x]/x^5,x]
[Out] (-(b*(b^2 + 6*x^2)*Cosh[a + b/x]) + 3*x*(b^2 + 2*x^2)*Sinh[a + b/x])/(b^4*x
^3)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{\left(b^{3}-3 b^{2} x+6 x^{2} b-6 x^{3}\right) {\mathrm e}^{\frac{a x+b}{x}}}{2 x^{3} b^{4}}-\frac{\left(b^{3}+3 b^{2} x+6 x^{2} b+6 x^{3}\right) {\mathrm e}^{-\frac{a x+b}{x}}}{2 x^{3} b^{4}}$
parallelrisch	$\frac{\left(b^{3}+6 x^{2} b\right) \tanh \left(\frac{a x+b}{2 x}\right)^{2}+\left(-6 b^{2} x-12 x^{3}\right) \tanh \left(\frac{a x+b}{2 x}\right)+b^{3}+6 x^{2} b}{x^{3} b^{4} \left(\tanh \left(\frac{a x+b}{2 x}\right)^{2}-1\right)}$
meijerg	$\frac{8 i \sqrt{\pi}\, \cosh \left(a \right) \left(\frac{i b \left(\frac{5 b^{2}}{2 x^{2}}+15\right) \cosh \left(\frac{b}{x}\right)}{20 \sqrt{\pi}\, x}-\frac{i \left(\frac{15 b^{2}}{2 x^{2}}+15\right) \sinh \left(\frac{b}{x}\right)}{20 \sqrt{\pi}}\right)}{b^{4}}-\frac{8 \sqrt{\pi}\, \sinh \left(a \right) \left(\frac{3}{4 \sqrt{\pi}}-\frac{\left(\frac{3 b^{2}}{2 x^{2}}+3\right) \cosh \left(\frac{b}{x}\right)}{4 \sqrt{\pi}}+\frac{b \left(\frac{b^{2}}{2 x^{2}}+3\right) \sin}{4 \sqrt{\pi}\, x}\right.}{b^{4}}$
derivativedivides	$-\frac{-a^{3} \cosh \left(a +\frac{b}{x}\right)+3 a^{2} \left(\left(a +\frac{b}{x}\right) \cosh \left(a +\frac{b}{x}\right)-\sinh \left(a +\frac{b}{x}\right)\right)-3 a \left(\left(a +\frac{b}{x}\right)^{2} \cosh \left(a +\frac{b}{x}\right)-2 \left(a +\frac{b}{x}\right) \sinh \left(a +\frac{b}{x}\right)+2 \cosh (\right.}{b^{4}}$
default	$-\frac{-a^{3} \cosh \left(a +\frac{b}{x}\right)+3 a^{2} \left(\left(a +\frac{b}{x}\right) \cosh \left(a +\frac{b}{x}\right)-\sinh \left(a +\frac{b}{x}\right)\right)-3 a \left(\left(a +\frac{b}{x}\right)^{2} \cosh \left(a +\frac{b}{x}\right)-2 \left(a +\frac{b}{x}\right) \sinh \left(a +\frac{b}{x}\right)+2 \cosh (\right.}{b^{4}}$

```
[In] int(sinh(a+b/x)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/2*(b^3-3*b^2*x+6*b*x^2-6*x^3)/x^3/b^4*exp((a*x+b)/x)-1/2*(b^3+3*b^2*x+6*
b*x^2+6*x^3)/x^3/b^4*exp(-(a*x+b)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{\sinh \left(a+\frac{b}{x}\right)}{x^5} \, dx=-\frac{(b^3+6 \, b x^2) \cosh \left(\frac{a x+b}{x}\right)-3 \, (b^2 x+2 \, x^3) \sinh \left(\frac{a x+b}{x}\right)}{b^4 x^3}$$

```
[In] integrate(sinh(a+b/x)/x^5,x, algorithm="fricas")
[Out] -((b^3 + 6*b*x^2)*cosh((a*x + b)/x) - 3*(b^2*x + 2*x^3)*sinh((a*x + b)/x))/
(b^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{\sinh \left(a+\frac{b}{x}\right)}{x^5} \, dx=\begin{cases} -\frac{\cosh \left(a+\frac{b}{x}\right)}{b x^3}+\frac{3 \sinh \left(a+\frac{b}{x}\right)}{b^2 x^2}-\frac{6 \cosh \left(a+\frac{b}{x}\right)}{b^3 x}+\frac{6 \sinh \left(a+\frac{b}{x}\right)}{b^4} & \text{for}\: b \neq 0 \\ -\frac{\sinh (a)}{4 x^4} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(a+b/x)/x**5,x)
```

```
[Out] Piecewise((-cosh(a + b/x)/(b*x**3) + 3*sinh(a + b/x)/(b**2*x**2) - 6*cosh(a
 + b/x)/(b**3*x) + 6*sinh(a + b/x)/b**4, Ne(b, 0)), (-sinh(a)/(4*x**4), Tru
e))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\frac{1}{8}\,b\left(\frac{e^{(-a)}\Gamma\left(5,\frac{b}{x}\right)}{b^5}-\frac{e^{a}\Gamma\left(5,-\frac{b}{x}\right)}{b^5}\right)-\frac{\sinh\left(a+\frac{b}{x}\right)}{4\,x^4}$$

```
[In] integrate(sinh(a+b/x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/8*b*(e^(-a)*gamma(5, b/x)/b^5 - e^a*gamma(5, -b/x)/b^5) - 1/4*sinh(a + b
/x)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(62) = 124$.

Time = 0.27 (sec) , antiderivative size = 386, normalized size of antiderivative = 6.23

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx = \frac{a^3e^{\left(\frac{ax+b}{x}\right)}+a^3e^{\left(-\frac{ax+b}{x}\right)}+3\,a^2e^{\left(\frac{ax+b}{x}\right)}-\frac{3\,(ax+b)a^2e^{\left(\frac{ax+b}{x}\right)}}{x}-3\,a^2e^{\left(-\frac{ax+b}{x}\right)}-\frac{3\,(ax+b)a^2e^{\left(-\frac{ax+b}{x}\right)}}{x}+6\,ae^{\left(\frac{ax+b}{x}\right)}+}{}$$

```
[In] integrate(sinh(a+b/x)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*e^((a*x + b)/x) + a^3*e^(-(a*x + b)/x) + 3*a^2*e^((a*x + b)/x) - 3
*(a*x + b)*a^2*e^((a*x + b)/x)/x - 3*a^2*e^(-(a*x + b)/x) - 3*(a*x + b)*a^2
*e^(-(a*x + b)/x)/x + 6*a*e^((a*x + b)/x) + 3*(a*x + b)^2*a*e^((a*x + b)/x)
/x^2 - 6*(a*x + b)*a*e^((a*x + b)/x)/x + 6*a*e^(-(a*x + b)/x) + 3*(a*x + b)
^2*a*e^(-(a*x + b)/x)/x^2 + 6*(a*x + b)*a*e^(-(a*x + b)/x)/x - (a*x + b)^3*
e^((a*x + b)/x)/x^3 + 3*(a*x + b)^2*e^((a*x + b)/x)/x^2 - 6*(a*x + b)*e^((a
*x + b)/x)/x - (a*x + b)^3*e^(-(a*x + b)/x)/x^3 - 3*(a*x + b)^2*e^(-(a*x +
b)/x)/x^2 - 6*(a*x + b)*e^(-(a*x + b)/x)/x + 6*e^((a*x + b)/x) - 6*e^(-(a*x
 + b)/x))/b^4
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx = \frac{e^{a+\frac{b}{x}}\left(\frac{3\,x}{2\,b^2}-\frac{1}{2\,b}-\frac{3\,x^2}{b^3}+\frac{3\,x^3}{b^4}\right)}{x^3}-\frac{e^{-a-\frac{b}{x}}\left(\frac{3\,x}{2\,b^2}+\frac{1}{2\,b}+\frac{3\,x^2}{b^3}+\frac{3\,x^3}{b^4}\right)}{x^3}$$

```
[In] int(sinh(a + b/x)/x^5,x)
```

```
[Out] (exp(a + b/x)*((3*x)/(2*b^2) - 1/(2*b) - (3*x^2)/b^3 + (3*x^3)/b^4))/x^3 -
(exp(- a - b/x)*((3*x)/(2*b^2) + 1/(2*b) + (3*x^2)/b^3 + (3*x^3)/b^4))/x^3
```

3.37 $\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 146

$$\begin{aligned}\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx = &-\frac{1}{8}3^{1+m}be^{3a}\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,-\frac{3b}{x}\right)\\&+\frac{3}{8}be^a\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,-\frac{b}{x}\right)\\&+\frac{3}{8}be^{-a}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,\frac{b}{x}\right)\\&-\frac{1}{8}3^{1+m}be^{-3a}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,\frac{3b}{x}\right)\end{aligned}$$

```
[Out] -1/8*3^(1+m)*b*exp(3*a)*(-b/x)^m*(e*x)^m*GAMMA(-1-m,-3*b/x)+3/8*b*exp(a)*(-
b/x)^m*(e*x)^m*GAMMA(-1-m,-b/x)+3/8*b*(b/x)^m*(e*x)^m*GAMMA(-1-m,b/x)/exp(a
)-1/8*3^(1+m)*b*(b/x)^m*(e*x)^m*GAMMA(-1-m,3*b/x)/exp(3*a)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5458, 3393, 3389, 2212}

$$\begin{aligned}\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx = &-\frac{1}{8}e^{3a}b3^{m+1}\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-m-1,-\frac{3b}{x}\right)\\&+\frac{3}{8}e^ab\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-m-1,-\frac{b}{x}\right)\\&+\frac{3}{8}e^{-a}b\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-m-1,\frac{b}{x}\right)\\&-\frac{1}{8}e^{-3a}b3^{m+1}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-m-1,\frac{3b}{x}\right)\end{aligned}$$

```
[In] Int[(e*x)^m*Sinh[a + b/x]^3,x]
```

```
[Out] -1/8*(3^(1 + m)*b*E^(3*a)*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, (-3*b)/x]) + (3*
b*E^a*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, -(b/x)])/8 + (3*b*(b/x)^m*(e*x)^m*Ga
mma[-1 - m, b/x])/(8*E^a) - (3^(1 + m)*b*(b/x)^m*(e*x)^m*Gamma[-1 - m, (3*b
)/x])/(8*E^(3*a))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5458

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^
p/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p]
 && ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\sinh^3(a+bx)\,dx, x, \frac{1}{x}\right)\right)\right) \\
&= \\
&\quad -\left(\left(\left(i\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int \left(\frac{3}{4}ix^{-2-m}\sinh(a+bx) - \frac{1}{4}ix^{-2-m}\sinh(3a+3bx)\right)\,dx, x, \frac{1}{x}\right)\right)\right) \\
&= -\left(\frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\sinh(3a+3bx)\,dx, x, \frac{1}{x}\right)\right) \\
&\quad + \frac{1}{4}\left(3\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\sinh(a+bx)\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$= -\left(\frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-i(3ia+3ibx)} x^{-2-m}\, dx, x, \frac{1}{x}\right)\right)$$
$$+ \frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{i(3ia+3ibx)} x^{-2-m}\, dx, x, \frac{1}{x}\right)$$
$$+ \frac{1}{8}\left(3\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-i(ia+ibx)} x^{-2-m}\, dx, x, \frac{1}{x}\right)$$
$$- \frac{1}{8}\left(3\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{i(ia+ibx)} x^{-2-m}\, dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{8}3^{1+m}be^{3a}\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m, -\frac{3b}{x}\right) + \frac{3}{8}be^a\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m, -\frac{b}{x}\right)$$
$$+ \frac{3}{8}be^{-a}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m, \frac{b}{x}\right) - \frac{1}{8}3^{1+m}be^{-3a}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m, \frac{3b}{x}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.79

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx = -\frac{3}{8}be^{-3a}(ex)^m \left(3^m e^{6a}\left(-\frac{b}{x}\right)^m \Gamma\left(-1-m, -\frac{3b}{x}\right) - e^{4a}\left(-\frac{b}{x}\right)^m \Gamma\left(-1-m, -\frac{b}{x}\right) + \left(\frac{b}{x}\right)^m \left(-e^{2a}\Gamma\left(-1-m, \frac{b}{x}\right) + 3^m\Gamma\left(-1-m, \frac{3b}{x}\right)\right)\right)$$

```
[In] Integrate[(e*x)^m*Sinh[a + b/x]^3,x]
```

```
[Out] (-3*b*(e*x)^m*(3^m*E^(6*a)*(-(b/x))^m*Gamma[-1 - m, (-3*b)/x] - E^(4*a)*(-(
b/x))^m*Gamma[-1 - m, -(b/x)] + (b/x)^m*(-(E^(2*a)*Gamma[-1 - m, b/x]) + 3^
m*Gamma[-1 - m, (3*b)/x])))/(8*E^(3*a))
```

Maple [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)^3 dx$$

```
[In] int((e*x)^m*sinh(a+b/x)^3,x)
```

```
[Out] int((e*x)^m*sinh(a+b/x)^3,x)
```

Fricas [F]

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx = \int (ex)^m \sinh\left(a+\frac{b}{x}\right)^3 dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x)^3,x, algorithm="fricas")
[Out] integral((e*x)^m*sinh((a*x + b)/x)^3, x)
```

Sympy [F]

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx = \int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx$$

```
[In] integrate((e*x)**m*sinh(a+b/x)**3,x)
[Out] Integral((e*x)**m*sinh(a + b/x)**3, x)
```

Maxima [F]

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx = \int (ex)^m \sinh\left(a+\frac{b}{x}\right)^3 dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x)^3,x, algorithm="maxima")
[Out] integrate((e*x)^m*sinh(a + b/x)^3, x)
```

Giac [F]

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx = \int (ex)^m \sinh\left(a+\frac{b}{x}\right)^3 dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x)^3,x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(a + b/x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right)\,dx=\int \sinh\left(a+\frac{b}{x}\right)^3\,(e\,x)^m\,dx$$

```
[In] int(sinh(a + b/x)^3*(e*x)^m,x)
[Out] int(sinh(a + b/x)^3*(e*x)^m, x)
```

3.38 $\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx = -\frac{x(ex)^m}{2(1+m)} - 2^{-1+m} b e^{2a} \left(-\frac{b}{x}\right)^m (ex)^m \Gamma\left(-1-m,-\frac{2b}{x}\right) + 2^{-1+m} b e^{-2a} \left(\frac{b}{x}\right)^m (ex)^m \Gamma\left(-1-m,\frac{2b}{x}\right)$$

```
[Out] -1/2*x*(e*x)^m/(1+m)-2^(-1+m)*b*exp(2*a)*(-b/x)^m*(e*x)^m*GAMMA(-1-m,-2*b/x
)+2^(-1+m)*b*(b/x)^m*(e*x)^m*GAMMA(-1-m,2*b/x)/exp(2*a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5458, 3393, 3388, 2212}

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx = -e^{2a} b 2^{m-1} \left(-\frac{b}{x}\right)^m (ex)^m \Gamma\left(-m-1,-\frac{2b}{x}\right) + e^{-2a} b 2^{m-1} \left(\frac{b}{x}\right)^m (ex)^m \Gamma\left(-m-1,\frac{2b}{x}\right) - \frac{x(ex)^m}{2(m+1)}$$

```
[In] Int[(e*x)^m*Sinh[a + b/x]^2,x]
```

```
[Out] -1/2*(x*(e*x)^m)/(1 + m) - 2^(-1 + m)*b*E^(2*a)*(-(b/x))^m*(e*x)^m*Gamma[-1
 - m, (-2*b)/x] + (2^(-1 + m)*b*(b/x)^m*(e*x)^m*Gamma[-1 - m, (2*b)/x])/E^(
2*a)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5458

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^
p/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p]
 && ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \sinh^2(a+bx)\, dx, x, \frac{1}{x}\right)\right)\\
&= \left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int \left(\frac{x^{-2-m}}{2} - \frac{1}{2}x^{-2-m}\cosh(2a+2bx)\right) dx, x, \frac{1}{x}\right)\\
&= -\frac{x(ex)^m}{2(1+m)} - \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\cosh(2a+2bx)\, dx, x, \frac{1}{x}\right)\\
&= -\frac{x(ex)^m}{2(1+m)} - \frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-i(2ia+2ibx)}x^{-2-m}\, dx, x, \frac{1}{x}\right)\\
&\quad - \frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{i(2ia+2ibx)}x^{-2-m}\, dx, x, \frac{1}{x}\right)\\
&= -\frac{x(ex)^m}{2(1+m)} - 2^{-1+m}be^{2a}\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m, -\frac{2b}{x}\right)\\
&\quad + 2^{-1+m}be^{-2a}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m, \frac{2b}{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx = \frac{1}{2}(ex)^m\left(-\frac{x}{1+m}-2^m b e^{2a}\left(-\frac{b}{x}\right)^m \Gamma\left(-1-m,-\frac{2b}{x}\right) + 2^m b e^{-2a}\left(\frac{b}{x}\right)^m \Gamma\left(-1-m,\frac{2b}{x}\right)\right)$$

```
[In] Integrate[(e*x)^m*Sinh[a + b/x]^2,x]
[Out] ((e*x)^m*(-(x/(1 + m)) - 2^m*b*E^(2*a)*(-(b/x))^m*Gamma[-1 - m, (-2*b)/x] +
 (2^m*b*(b/x)^m*Gamma[-1 - m, (2*b)/x])/E^(2*a)))/2
```

Maple [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)^2 dx$$

```
[In] int((e*x)^m*sinh(a+b/x)^2,x)
[Out] int((e*x)^m*sinh(a+b/x)^2,x)
```

Fricas [F]

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx = \int (ex)^m \sinh\left(a+\frac{b}{x}\right)^2 dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x)^2,x, algorithm="fricas")
[Out] integral((e*x)^m*sinh((a*x + b)/x)^2, x)
```

Sympy [F]

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx = \int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx$$

```
[In] integrate((e*x)**m*sinh(a+b/x)**2,x)
[Out] Integral((e*x)**m*sinh(a + b/x)**2, x)
```

Maxima [F]

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx = \int (ex)^m \sinh\left(a+\frac{b}{x}\right)^2 dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*e^m*integrate(e^(m*log(x) + 2*a + 2*b/x), x) + 1/4*e^m*integrate(e^(m*l
og(x) - 2*a - 2*b/x), x) - 1/2*(e*x)^(m + 1)/(e*(m + 1))
```

Giac [F]

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx = \int (ex)^m \sinh\left(a+\frac{b}{x}\right)^2 dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx = \int \sinh\left(a+\frac{b}{x}\right)^2 (e\,x)^m\, dx$$

```
[In] int(sinh(a + b/x)^2*(e*x)^m,x)
```

```
[Out] int(sinh(a + b/x)^2*(e*x)^m, x)
```

3.39 $\int (ex)^m \sinh\left(a+\frac{b}{x}\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right) dx = -\frac{1}{2}be^a\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,-\frac{b}{x}\right) - \frac{1}{2}be^{-a}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,\frac{b}{x}\right)$$

```
[Out] -1/2*b*exp(a)*(-b/x)^m*(e*x)^m*GAMMA(-1-m,-b/x)-1/2*b*(b/x)^m*(e*x)^m*GAMMA
(-1-m,b/x)/exp(a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5458, 3389, 2212}

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right) dx = -\frac{1}{2}e^ab\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-m-1,-\frac{b}{x}\right) - \frac{1}{2}e^{-a}b\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-m-1,\frac{b}{x}\right)$$

```
[In] Int[(e*x)^m*Sinh[a + b/x],x]
```

```
[Out] -1/2*(b*E^a*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, -(b/x)]) - (b*(b/x)^m*(e*x)^m*
Gamma[-1 - m, b/x])/(2*E^a)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5458

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^
p/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p]
 && ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \sinh(a+bx)\,dx, x, \frac{1}{x}\right)\right) \\
&= -\left(\frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-i(ia+ibx)} x^{-2-m}\,dx, x, \frac{1}{x}\right)\right) \\
&\quad + \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{i(ia+ibx)} x^{-2-m}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{1}{2} b e^a \left(-\frac{b}{x}\right)^m (ex)^m \Gamma\left(-1-m, -\frac{b}{x}\right) - \frac{1}{2} b e^{-a} \left(\frac{b}{x}\right)^m (ex)^m \Gamma\left(-1-m, \frac{b}{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$
\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx = -\frac{1}{2} b e^{-a} (ex)^m \left(e^{2a}\left(-\frac{b}{x}\right)^m \Gamma\left(-1-m, -\frac{b}{x}\right) + \left(\frac{b}{x}\right)^m \Gamma\left(-1-m, \frac{b}{x}\right)\right)
$$

```
[In] Integrate[(e*x)^m*Sinh[a + b/x],x]
[Out] -1/2*(b*(e*x)^m*(E^(2*a)*(-(b/x))^m*Gamma[-1 - m, -(b/x)] + (b/x)^m*Gamma[-
1 - m, b/x]))/E^a
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

method	result	size
meijerg	$\frac{(ex)^m b\,\mathrm{hypergeom}\left(\left[-\frac{m}{2}\right],\left[\frac{3}{2},1-\frac{m}{2}\right],\frac{b^2}{4x^2}\right)\cosh(a)}{m}+\frac{(ex)^m x\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}-\frac{m}{2}\right],\left[\frac{1}{2},\frac{1}{2}-\frac{m}{2}\right],\frac{b^2}{4x^2}\right)\sinh(a)}{1+m}$	70

```
[In] int((e*x)^m*sinh(a+b/x),x,method=_RETURNVERBOSE)
[Out] (e*x)^m*b/m*hypergeom([-1/2*m],[3/2,1-1/2*m],1/4*b^2/x^2)*cosh(a)+(e*x)^m/(
1+m)*x*hypergeom([-1/2-1/2*m],[1/2,1/2-1/2*m],1/4*b^2/x^2)*sinh(a)
```

Fricas [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx=\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x),x, algorithm="fricas")
[Out] integral((e*x)^m*sinh((a*x + b)/x), x)
```

Sympy [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx=\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate((e*x)**m*sinh(a+b/x),x)
[Out] Integral((e*x)**m*sinh(a + b/x), x)
```

Maxima [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx=\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x),x, algorithm="maxima")
[Out] integrate((e*x)^m*sinh(a + b/x), x)
```

Giac [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx = \int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x),x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(a + b/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx = \int \sinh\left(a+\frac{b}{x}\right)\,(e\,x)^m\,dx$$

```
[In] int(sinh(a + b/x)*(e*x)^m,x)
[Out] int(sinh(a + b/x)*(e*x)^m, x)
```

3.40 $\int (ex)^m \mathbf{csch}\left(a+\frac{b}{x}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) \, dx = x^{-m}(ex)^m \operatorname{Int}\left(x^m \operatorname{csch}\left(a+\frac{b}{x}\right),x\right)$$

```
[Out] (e*x)^m*Unintegrable(x^m*csch(a+b/x),x)/(x^m)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) \, dx = \int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) \, dx$$

```
[In] Int[(e*x)^m*Csch[a + b/x],x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b/x], x])/x^m
```

Rubi steps

$$\text{integral} = \left(x^{-m}(ex)^m\right) \int x^m \operatorname{csch}\left(a+\frac{b}{x}\right) \, dx$$

Mathematica [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx = \int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx$$

```
[In] Integrate[(e*x)^m*Csch[a + b/x],x]
[Out] Integrate[(e*x)^m*Csch[a + b/x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x}\right)}dx$$

```
[In] int((e*x)^m/sinh(a+b/x),x)
[Out] int((e*x)^m/sinh(a+b/x),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx = \int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x}\right)} dx$$

```
[In] integrate((e*x)^m/sinh(a+b/x),x, algorithm="fricas")
[Out] integral((e*x)^m/sinh((a*x + b)/x), x)
```

Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx = \int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x}\right)} dx$$

```
[In] integrate((e*x)**m/sinh(a+b/x),x)
[Out] Integral((e*x)**m/sinh(a + b/x), x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx = \int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x}\right)} dx$$

```
[In] integrate((e*x)^m/sinh(a+b/x),x, algorithm="maxima")
[Out] integrate((e*x)^m/sinh(a + b/x), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx = \int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x}\right)} dx$$

```
[In] integrate((e*x)^m/sinh(a+b/x),x, algorithm="giac")
[Out] integrate((e*x)^m/sinh(a + b/x), x)
```

Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx = \int \frac{(e\,x)^m}{\sinh\left(a+\frac{b}{x}\right)} dx$$

```
[In] int((e*x)^m/sinh(a + b/x),x)
[Out] int((e*x)^m/sinh(a + b/x), x)
```

3.41 $\int x^4 \sinh\left(a+\frac{b}{x^2}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 104

$$\int x^4 \sinh\left(a+\frac{b}{x^2}\right)\,dx = \frac{2}{15}bx^3\cosh\left(a+\frac{b}{x^2}\right) - \frac{2}{15}b^{5/2}e^{-a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{2}{15}b^{5/2}e^{a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right) + \frac{4}{15}b^2x\sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right)$$

```
[Out] 2/15*b*x^3*cosh(a+b/x^2)+4/15*b^2*x*sinh(a+b/x^2)+1/5*x^5*sinh(a+b/x^2)-2/1
5*b^(5/2)*erf(b^(1/2)/x)*Pi^(1/2)/exp(a)-2/15*b^(5/2)*exp(a)*erfi(b^(1/2)/x
)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5454, 5434, 5435, 5407, 2235, 2236}

$$\int x^4 \sinh\left(a+\frac{b}{x^2}\right)\,dx = -\frac{2}{15}\sqrt{\pi}e^{-a}b^{5/2}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{2}{15}\sqrt{\pi}e^{a}b^{5/2}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right) + \frac{4}{15}b^2x\sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right) + \frac{2}{15}bx^3\cosh\left(a+\frac{b}{x^2}\right)$$

```
[In] Int[x^4*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x^3*Cosh[a + b/x^2])/15 - (2*b^(5/2)*Sqrt[Pi]*Erf[Sqrt[b]/x])/(15*E^a)
 - (2*b^(5/2)*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/15 + (4*b^2*x*Sinh[a + b/x^2])/
15 + (x^5*Sinh[a + b/x^2])/5
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5434

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x
)^(m + 1)*(Sinh[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5435

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x
)^(m + 1)*(Cosh[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int
[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5454

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sinh(a+bx^2)}{x^6}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{5}(2b)\text{Subst}\left(\int \frac{\cosh(a+bx^2)}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{15}bx^3\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{15}(4b^2)\,\text{Subst}\left(\int \frac{\sinh(a+bx^2)}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{15}bx^3\cosh\left(a+\frac{b}{x^2}\right) + \frac{4}{15}b^2x\sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{15}(8b^3)\,\text{Subst}\left(\int \cosh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{15}bx^3\cosh\left(a+\frac{b}{x^2}\right) + \frac{4}{15}b^2x\sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{15}(4b^3)\,\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right) - \frac{1}{15}(4b^3)\,\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{15}bx^3\cosh\left(a+\frac{b}{x^2}\right) - \frac{2}{15}b^{5/2}e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{2}{15}b^{5/2}e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right) + \frac{4}{15}b^2x\sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int x^4\sinh\left(a+\frac{b}{x^2}\right)\,dx$$

$$= \frac{1}{15}\left(2bx^3\cosh\left(a+\frac{b}{x^2}\right) + 2b^{5/2}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)(-\cosh(a)+\sinh(a)) - 2b^{5/2}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a)) + 4b^2x\sinh\left(a+\frac{b}{x^2}\right) + 3x^5\sinh\left(a+\frac{b}{x^2}\right)\right)$$

```
[In] Integrate[x^4*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x^3*Cosh[a + b/x^2] + 2*b^(5/2)*Sqrt[Pi]*Erf[Sqrt[b]/x]*(-Cosh[a] + Si
nh[a]) - 2*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]) + 4*b^2*x*S
inh[a + b/x^2] + 3*x^5*Sinh[a + b/x^2])/15
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{e^{-a} x^{5} e^{-\frac{b}{x^{2}}}}{10}+\frac{e^{-a} b\, x^{3} e^{-\frac{b}{x^{2}}}}{15}-\frac{2 b^{\frac{5}{2}} \operatorname{erf}\left(\frac{\sqrt{b}}{x}\right) \sqrt{\pi}\, e^{-a}}{15}-\frac{2 e^{-a} e^{-\frac{b}{x^{2}}} b^{2} x}{15}+\frac{e^{a} x^{5} e^{\frac{b}{x^{2}}}}{10}+\frac{e^{a} b\, x^{3} e^{\frac{b}{x^{2}}}}{15}+\frac{2 e^{a} b^{2} x\, e^{\frac{b}{x^{2}}}}{15}-\frac{2 e^{a} b^{3} \ldots}{}$
meijerg	$-\frac{i b^{2} \sqrt{\pi}\, \cosh(a) \sqrt{2}\, \sqrt{i b}\left(\frac{8 x^{5} \sqrt{2}\left(\frac{4 b^{2}}{x^{4}}-\frac{2 b}{x^{2}}+3\right) e^{-\frac{b}{x^{2}}}}{15 \sqrt{\pi}\, (i b)^{\frac{3}{2}} b}-\frac{8 x^{5} \sqrt{2}\left(\frac{4 b^{2}}{x^{4}}+\frac{2 b}{x^{2}}+3\right) e^{\frac{b}{x^{2}}}}{15 \sqrt{\pi}\, (i b)^{\frac{3}{2}} b}+\frac{32 \sqrt{2}\, b^{\frac{3}{2}} \operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{15 (i b)^{\frac{3}{2}}}+\frac{32 \sqrt{2}\, b^{\frac{3}{2}} \operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{15 (i b)^{\frac{3}{2}}}\right)}{32}+\frac{b^{2} \sqrt{\pi}\, \sin \ldots}{}$

```
[In] int(x^4*sinh(a+b/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/exp(a)*x^5*exp(-b/x^2)+1/15/exp(a)*b*x^3*exp(-b/x^2)-2/15*b^(5/2)*erf
(b^(1/2)/x)*Pi^(1/2)/exp(a)-2/15/exp(a)*exp(-b/x^2)*b^2*x+1/10*exp(a)*x^5*e
xp(b/x^2)+1/15*exp(a)*b*x^3*exp(b/x^2)+2/15*exp(a)*b^2*x*exp(b/x^2)-2/15*ex
p(a)*b^3*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(80) = 160.

Time = 0.25 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.11

$$\int x^{4} \sinh\left(a+\frac{b}{x^{2}}\right) dx =$$
$$-\frac{3 x^{5}-2 b x^{3}+4 b^{2} x-\left(3 x^{5}+2 b x^{3}+4 b^{2} x\right) \cosh\left(\frac{a x^{2}+b}{x^{2}}\right)^{2}-4 \sqrt{\pi}\left(b^{2} \cosh(a) \cosh\left(\frac{a x^{2}+b}{x^{2}}\right)+b^{2} \cosh\left(\ldots\right.\right.}{}$$

```
[In] integrate(x^4*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/30*(3*x^5 - 2*b*x^3 + 4*b^2*x - (3*x^5 + 2*b*x^3 + 4*b^2*x)*cosh((a*x^2
 + b)/x^2)^2 - 4*sqrt(pi)*(b^2*cosh(a)*cosh((a*x^2 + b)/x^2) + b^2*cosh((a*x
^2 + b)/x^2)*sinh(a) + (b^2*cosh(a) + b^2*sinh(a))*sinh((a*x^2 + b)/x^2))*s
qrt(-b)*erf(sqrt(-b)/x) + 4*sqrt(pi)*(b^2*cosh(a)*cosh((a*x^2 + b)/x^2) - b
^2*cosh((a*x^2 + b)/x^2)*sinh(a) + (b^2*cosh(a) - b^2*sinh(a))*sinh((a*x^2
 + b)/x^2))*sqrt(b)*erf(sqrt(b)/x) - 2*(3*x^5 + 2*b*x^3 + 4*b^2*x)*cosh((a*x
^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) - (3*x^5 + 2*b*x^3 + 4*b^2*x)*sinh((a*x^
2 + b)/x^2)^2)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

Sympy [F]

$$\int x^4 \sinh\left(a+\frac{b}{x^2}\right) dx = \int x^4 \sinh\left(a+\frac{b}{x^2}\right) dx$$

```
[In] integrate(x**4*sinh(a+b/x**2),x)
[Out] Integral(x**4*sinh(a + b/x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

$$\int x^4 \sinh\left(a+\frac{b}{x^2}\right) dx = \frac{1}{5}x^5 \sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{10}\left(x^3\left(\frac{b}{x^2}\right)^{\frac{3}{2}} e^{(-a)}\Gamma\left(-\frac{3}{2},\frac{b}{x^2}\right) + x^3\left(-\frac{b}{x^2}\right)^{\frac{3}{2}} e^{a}\Gamma\left(-\frac{3}{2},-\frac{b}{x^2}\right)\right)b$$

```
[In] integrate(x^4*sinh(a+b/x^2),x, algorithm="maxima")
[Out] 1/5*x^5*sinh(a + b/x^2) + 1/10*(x^3*(b/x^2)^(3/2)*e^(-a)*gamma(-3/2, b/x^2)
 + x^3*(-b/x^2)^(3/2)*e^a*gamma(-3/2, -b/x^2))*b
```

Giac [F]

$$\int x^4 \sinh\left(a+\frac{b}{x^2}\right) dx = \int x^4 \sinh\left(a+\frac{b}{x^2}\right) dx$$

```
[In] integrate(x^4*sinh(a+b/x^2),x, algorithm="giac")
[Out] integrate(x^4*sinh(a + b/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sinh\left(a+\frac{b}{x^2}\right) dx = \int x^4 \sinh\left(a+\frac{b}{x^2}\right) dx$$

```
[In] int(x^4*sinh(a + b/x^2),x)
[Out] int(x^4*sinh(a + b/x^2), x)
```

3.42 $\int x^3 \sinh\left(a+\frac{b}{x^2}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 62

$$\int x^3 \sinh\left(a+\frac{b}{x^2}\right)\,dx=\frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right)-\frac{1}{4}b^2\text{Chi}\left(\frac{b}{x^2}\right)\sinh(a)+\frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{4}b^2\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

```
[Out] 1/4*b*x^2*cosh(a+b/x^2)-1/4*b^2*cosh(a)*Shi(b/x^2)-1/4*b^2*Chi(b/x^2)*sinh(
a)+1/4*x^4*sinh(a+b/x^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5428, 3378, 3384, 3379, 3382}

$$\int x^3 \sinh\left(a+\frac{b}{x^2}\right)\,dx=-\frac{1}{4}b^2\sinh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{4}b^2\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)+\frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right)+\frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right)$$

```
[In] Int[x^3*Sinh[a + b/x^2],x]
```

```
[Out] (b*x^2*Cosh[a + b/x^2])/4 - (b^2*CoshIntegral[b/x^2]*Sinh[a])/4 + (x^4*Sinh
[a + b/x^2])/4 - (b^2*Cosh[a]*SinhIntegral[b/x^2])/4
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}b\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}b^2\text{Subst}\left(\int \frac{\sinh(a+bx)}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right)\\
&\quad - \frac{1}{4}\left(b^2\cosh(a)\right)\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, \frac{1}{x^2}\right)\\
&\quad - \frac{1}{4}\left(b^2\sinh(a)\right)\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, \frac{1}{x^2}\right)
\end{aligned}$$

$$= \frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right)-\frac{1}{4}b^2\text{Chi}\left(\frac{b}{x^2}\right)\sinh(a)+\frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{4}b^2\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int x^3\sinh\left(a+\frac{b}{x^2}\right)\,dx = \frac{1}{4}\left(bx^2\cosh\left(a+\frac{b}{x^2}\right)-b^2\text{Chi}\left(\frac{b}{x^2}\right)\sinh(a)+x^4\sinh\left(a+\frac{b}{x^2}\right)-b^2\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)\right)$$

```
[In] Integrate[x^3*Sinh[a + b/x^2],x]
```

```
[Out] (b*x^2*Cosh[a + b/x^2] - b^2*CoshIntegral[b/x^2]*Sinh[a] + x^4*Sinh[a + b/x
^2] - b^2*Cosh[a]*SinhIntegral[b/x^2])/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(54) = 108$.

Time = 0.65 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.89

method	result
risch	$\frac{e^{\frac{2a x^2+b}{x^2}} e^{-a} x^4}{8}-\frac{e^{-a} x^4 e^{-\frac{b}{x^2}}}{8}+\frac{e^{\frac{2a x^2+b}{x^2}} e^{-a} b x^2}{8}+\frac{e^{2a} e^{-a}\,\text{Ei}_1\left(-\frac{b}{x^2}\right) b^2}{8}+\frac{e^{-a} b x^2 e^{-\frac{b}{x^2}}}{8}-\frac{e^{-a} b^2\,\text{Ei}_1\left(\frac{b}{x^2}\right)}{8}$
meijerg	$-\frac{i b^2\sqrt{\pi}\cosh(a)\left(\frac{4i x^2\cosh\left(\frac{b}{x^2}\right)}{b\sqrt{\pi}}+\frac{4i x^4\sinh\left(\frac{b}{x^2}\right)}{b^2\sqrt{\pi}}-\frac{4i\,\text{Shi}\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{16}+\frac{b^2\sqrt{\pi}\sinh(a)\left(-\frac{4x^4\left(\frac{9b^2}{2x^4}+3\right)}{3\sqrt{\pi}\,b^2}+\frac{4x^4\cosh\left(\frac{b}{x^2}\right)}{\sqrt{\pi}\,b^2}+\frac{4x^2\sinh\left(\frac{b}{x^2}\right)}{\sqrt{\pi}\,b}-\right.}{16}$

```
[In] int(x^3*sinh(a+b/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*exp((2*a*x^2+b)/x^2)*exp(-a)*x^4-1/8*exp(-a)*x^4*exp(-b/x^2)+1/8*exp((2
*a*x^2+b)/x^2)*exp(-a)*b*x^2+1/8*exp(2*a)*exp(-a)*Ei(1,-b/x^2)*b^2+1/8*exp(
-a)*b*x^2*exp(-b/x^2)-1/8*exp(-a)*b^2*Ei(1,b/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.44

$$\int x^3 \sinh\left(a+\frac{b}{x^2}\right)\,dx = \frac{1}{4}\,x^4\sinh\left(\frac{ax^2+b}{x^2}\right)+\frac{1}{4}\,bx^2\cosh\left(\frac{ax^2+b}{x^2}\right) - \frac{1}{8}\left(b^2\mathrm{Ei}\left(\frac{b}{x^2}\right)-b^2\mathrm{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh\left(a\right) - \frac{1}{8}\left(b^2\mathrm{Ei}\left(\frac{b}{x^2}\right)+b^2\mathrm{Ei}\left(-\frac{b}{x^2}\right)\right)\sinh\left(a\right)$$

```
[In] integrate(x^3*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*sinh((a*x^2 + b)/x^2) + 1/4*b*x^2*cosh((a*x^2 + b)/x^2) - 1/8*(b^2*
Ei(b/x^2) - b^2*Ei(-b/x^2))*cosh(a) - 1/8*(b^2*Ei(b/x^2) + b^2*Ei(-b/x^2))*
sinh(a)
```

Sympy [F]

$$\int x^3 \sinh\left(a+\frac{b}{x^2}\right)\,dx = \int x^3 \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate(x**3*sinh(a+b/x**2),x)
```

```
[Out] Integral(x**3*sinh(a + b/x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int x^3 \sinh\left(a+\frac{b}{x^2}\right)\,dx = \frac{1}{4}\,x^4\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{8}\left(be^{(-a)}\Gamma\left(-1,\frac{b}{x^2}\right)-be^a\Gamma\left(-1,-\frac{b}{x^2}\right)\right)b$$

```
[In] integrate(x^3*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*sinh(a + b/x^2) + 1/8*(b*e^(-a)*gamma(-1, b/x^2) - b*e^a*gamma(-1,
-b/x^2))*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 353, normalized size of antiderivative = 5.69

$$\int x^3 \sinh\left(a + \frac{b}{x^2}\right) dx$$

$$= \frac{a^2b^3\text{Ei}\left(a - \frac{ax^2+b}{x^2}\right)e^{(-a)} - a^2b^3\text{Ei}\left(-a + \frac{ax^2+b}{x^2}\right)e^a - \frac{2\,(ax^2+b)ab^3\text{Ei}\left(a-\frac{ax^2+b}{x^2}\right)e^{(-a)}}{x^2} + \frac{2\,(ax^2+b)ab^3\text{Ei}\left(-a+\frac{ax^2+b}{x^2}\right)e^a}{x^2} -}{}$$

```
[In] integrate(x^3*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2*b^3*Ei(a - (a*x^2 + b)/x^2)*e^(-a) - a^2*b^3*Ei(-a + (a*x^2 + b)/x
^2)*e^a - 2*(a*x^2 + b)*a*b^3*Ei(a - (a*x^2 + b)/x^2)*e^(-a)/x^2 + 2*(a*x^2
 + b)*a*b^3*Ei(-a + (a*x^2 + b)/x^2)*e^a/x^2 - a*b^3*e^((a*x^2 + b)/x^2) -
a*b^3*e^(-(a*x^2 + b)/x^2) + b^3*e^((a*x^2 + b)/x^2) - b^3*e^(-(a*x^2 + b)/
x^2) + (a*x^2 + b)^2*b^3*Ei(a - (a*x^2 + b)/x^2)*e^(-a)/x^4 - (a*x^2 + b)^2
*b^3*Ei(-a + (a*x^2 + b)/x^2)*e^a/x^4 + (a*x^2 + b)*b^3*e^((a*x^2 + b)/x^2)
/x^2 + (a*x^2 + b)*b^3*e^(-(a*x^2 + b)/x^2)/x^2)/((a^2 - 2*(a*x^2 + b)*a/x^
2 + (a*x^2 + b)^2/x^4)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sinh\left(a + \frac{b}{x^2}\right) dx = \int x^3 \sinh\left(a + \frac{b}{x^2}\right) dx$$

```
[In] int(x^3*sinh(a + b/x^2),x)
```

```
[Out] int(x^3*sinh(a + b/x^2), x)
```

3.43 $\int x^2 \sinh\left(a+\frac{b}{x^2}\right)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 86

$$\int x^2 \sinh\left(a+\frac{b}{x^2}\right)\, dx = \frac{2}{3} b x \cosh\left(a+\frac{b}{x^2}\right)+\frac{1}{3} b^{3/2} e^{-a} \sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right) -\frac{1}{3} b^{3/2} e^{a} \sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{1}{3} x^3 \sinh\left(a+\frac{b}{x^2}\right)$$

```
[Out] 2/3*b*x*cosh(a+b/x^2)+1/3*x^3*sinh(a+b/x^2)+1/3*b^(3/2)*erf(b^(1/2)/x)*Pi^(
1/2)/exp(a)-1/3*b^(3/2)*exp(a)*erfi(b^(1/2)/x)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5454, 5434, 5435, 5406, 2235, 2236}

$$\int x^2 \sinh\left(a+\frac{b}{x^2}\right)\, dx = \frac{1}{3}\sqrt{\pi} e^{-a} b^{3/2}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right) -\frac{1}{3}\sqrt{\pi} e^{a} b^{3/2}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{2}{3} b x \cosh\left(a+\frac{b}{x^2}\right)+\frac{1}{3} x^3 \sinh\left(a+\frac{b}{x^2}\right)$$

```
[In] Int[x^2*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x*Cosh[a + b/x^2])/3 + (b^(3/2)*Sqrt[Pi]*Erf[Sqrt[b]/x])/(3*E^a) - (b^
(3/2)*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/3 + (x^3*Sinh[a + b/x^2])/3
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5434

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x
)^(m + 1)*(Sinh[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5435

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x
)^(m + 1)*(Cosh[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int
[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5454

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sinh\left(a+bx^2\right)}{x^4}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{3}(2b)\text{Subst}\left(\int \frac{\cosh\left(a+bx^2\right)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{3}\left(4b^2\right)\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2}{3} b x \cosh \left(a+\frac{b}{x^2}\right)+\frac{1}{3} x^3 \sinh \left(a+\frac{b}{x^2}\right) \\
&\quad +\frac{1}{3}\left(2 b^2\right) \text{Subst}\left(\int e^{-a-b x^2} \, dx, x, \frac{1}{x}\right)-\frac{1}{3}\left(2 b^2\right) \text{Subst}\left(\int e^{a+b x^2} \, dx, x, \frac{1}{x}\right) \\
&= \frac{2}{3} b x \cosh \left(a+\frac{b}{x^2}\right)+\frac{1}{3} b^{3/2} e^{-a} \sqrt{\pi} \text{erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{3} b^{3/2} e^a \sqrt{\pi} \text{erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{1}{3} x^3 \sinh \left(a+\frac{b}{x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int x^2 \sinh \left(a+\frac{b}{x^2}\right) \, dx = \frac{1}{3}\Bigg(& 2 b x \cosh \left(a+\frac{b}{x^2}\right)+b^{3/2} \sqrt{\pi} \text{erf}\left(\frac{\sqrt{b}}{x}\right)(\cosh (a)-\sinh (a)) \\
& -b^{3/2} \sqrt{\pi} \text{erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh (a)+\sinh (a))+x^3 \sinh \left(a+\frac{b}{x^2}\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[x^2*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x*Cosh[a + b/x^2] + b^(3/2)*Sqrt[Pi]*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]
) - b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]) + x^3*Sinh[a + b/x
^2])/3
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{e^{-a} x^3 e^{-\frac{b}{x^2}}}{6}+\frac{b^{\frac{3}{2}} \text{erf}\left(\frac{\sqrt{b}}{x}\right) \sqrt{\pi}\, e^{-a}}{3}+\frac{e^{-a} e^{-\frac{b}{x^2}} b x}{3}+\frac{e^{a} x^3 e^{\frac{b}{x^2}}}{6}+\frac{e^{a} b x\, e^{\frac{b}{x^2}}}{3}-\frac{e^{a} b^2 \sqrt{\pi}\, \text{erf}\left(\frac{\sqrt{-b}}{x}\right)}{3 \sqrt{-b}}$
meijerg	$-\frac{b \sqrt{\pi}\, \cosh (a) \sqrt{2}\, \sqrt{i b}\left(-\frac{4 x^3 \sqrt{2}\left(\frac{2 b}{x^2}+1\right) e^{\frac{b}{x^2}}}{3 \sqrt{\pi}\, \sqrt{i b}\, b}+\frac{4 x^3 \sqrt{2}\left(-\frac{2 b}{x^2}+1\right) e^{-\frac{b}{x^2}}}{3 \sqrt{\pi}\, \sqrt{i b}\, b}-\frac{8 \sqrt{2}\, \sqrt{b}\, \text{erf}\left(\frac{\sqrt{b}}{x}\right)}{3 \sqrt{i b}}+\frac{8 \sqrt{2}\, \sqrt{b}\, \text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{3 \sqrt{i b}}\right)}{16}-\frac{i b \sqrt{\pi}\, \sinh (a) \sqrt{2}}{}$

```
[In] int(x^2*sinh(a+b/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/exp(a)*x^3*exp(-b/x^2)+1/3*b^(3/2)*erf(b^(1/2)/x)*Pi^(1/2)/exp(a)+1/3/
exp(a)*exp(-b/x^2)*b*x+1/6*exp(a)*x^3*exp(b/x^2)+1/3*exp(a)*b*x*exp(b/x^2)-
1/3*exp(a)*b^2*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.10

$$\int x^2 \sinh\left(a+\frac{b}{x^2}\right)\,dx =$$
$$-\frac{x^3 - (x^3 + 2\,bx)\cosh\left(\frac{ax^2+b}{x^2}\right)^2 - 2\,\sqrt{\pi}\Big(b\cosh\left(a\right)\cosh\left(\frac{ax^2+b}{x^2}\right) + b\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh\left(a\right) + (b\cosh\left(a\right) + \cdots}{\cdots}$$

```
[In] integrate(x^2*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/6*(x^3 - (x^3 + 2*b*x)*cosh((a*x^2 + b)/x^2)^2 - 2*sqrt(pi)*(b*cosh(a)*c
osh((a*x^2 + b)/x^2) + b*cosh((a*x^2 + b)/x^2)*sinh(a) + (b*cosh(a) + b*sin
h(a))*sinh((a*x^2 + b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) - 2*sqrt(pi)*(b*cosh(
a)*cosh((a*x^2 + b)/x^2) - b*cosh((a*x^2 + b)/x^2)*sinh(a) + (b*cosh(a) - b
*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b)*erf(sqrt(b)/x) - 2*(x^3 + 2*b*x)*c
osh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) - (x^3 + 2*b*x)*sinh((a*x^2 + b)
/x^2)^2 - 2*b*x)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

Sympy [F]

$$\int x^2 \sinh\left(a+\frac{b}{x^2}\right)\,dx = \int x^2 \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate(x**2*sinh(a+b/x**2),x)
```

```
[Out] Integral(x**2*sinh(a + b/x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int x^2 \sinh\left(a+\frac{b}{x^2}\right)\,dx = \frac{1}{3}\,x^3 \sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{6}\left(x\sqrt{\frac{b}{x^2}}e^{(-a)}\Gamma\left(-\frac{1}{2},\frac{b}{x^2}\right) + x\sqrt{-\frac{b}{x^2}}e^{a}\Gamma\left(-\frac{1}{2},-\frac{b}{x^2}\right)\right)b$$

```
[In] integrate(x^2*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sinh(a + b/x^2) + 1/6*(x*sqrt(b/x^2)*e^(-a)*gamma(-1/2, b/x^2) + x*
sqrt(-b/x^2)*e^a*gamma(-1/2, -b/x^2))*b
```

Giac [F]

$$\int x^2 \sinh\left(a+\frac{b}{x^2}\right)\,dx = \int x^2 \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate(x^2*sinh(a+b/x^2),x, algorithm="giac")
[Out] integrate(x^2*sinh(a + b/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh\left(a+\frac{b}{x^2}\right)\,dx = \int x^2 \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] int(x^2*sinh(a + b/x^2),x)
[Out] int(x^2*sinh(a + b/x^2), x)
```

3.44 $\int x \sinh \left(a+\frac{b}{x^2}\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int x \sinh \left(a+\frac{b}{x^2}\right) \, dx=-\frac{1}{2} b \cosh (a) \text{Chi}\left(\frac{b}{x^2}\right)+\frac{1}{2} x^2 \sinh \left(a+\frac{b}{x^2}\right)-\frac{1}{2} b \sinh (a) \text{Shi}\left(\frac{b}{x^2}\right)$$

```
[Out] -1/2*b*Chi(b/x^2)*cosh(a)-1/2*b*Shi(b/x^2)*sinh(a)+1/2*x^2*sinh(a+b/x^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5428, 3378, 3384, 3379, 3382}

$$\int x \sinh \left(a+\frac{b}{x^2}\right) \, dx=-\frac{1}{2} b \cosh (a) \text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2} b \sinh (a) \text{Shi}\left(\frac{b}{x^2}\right)+\frac{1}{2} x^2 \sinh \left(a+\frac{b}{x^2}\right)$$

```
[In] Int[x*Sinh[a + b/x^2],x]
```

```
[Out] -1/2*(b*Cosh[a]*CoshIntegral[b/x^2]) + (x^2*Sinh[a + b/x^2])/2 - (b*Sinh[a]
*SinhIntegral[b/x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{2}b\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{2}(b\cosh(a))\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, \frac{1}{x^2}\right)\\
&\quad - \frac{1}{2}(b\sinh(a))\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}b\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right) + \frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{2}b\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int x \sinh\left(a+\frac{b}{x^2}\right) dx = \frac{1}{2}\left(-b\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right)+x^2\sinh\left(a+\frac{b}{x^2}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)\right)$$

```
[In] Integrate[x*Sinh[a + b/x^2],x]
```

```
[Out] (-(b*Cosh[a]*CoshIntegral[b/x^2]) + x^2*Sinh[a + b/x^2] - b*Sinh[a]*SinhInt
egral[b/x^2])/2
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

method	result
risch	$\frac{e^{\frac{2a x^2+b}{x^2}} e^{-a} x^2}{4}+\frac{e^{2a} e^{-a} \text{Ei}_1\left(-\frac{b}{x^2}\right) b}{4}-\frac{e^{-a} x^2 e^{-\frac{b}{x^2}}}{4}+\frac{e^{-a} b \text{ Ei}_1\left(\frac{b}{x^2}\right)}{4}$
meijerg	$-\frac{b\sqrt{\pi}\cosh(a)\left(\frac{4}{\sqrt{\pi}}-\frac{4x^2\sinh\left(\frac{b}{x^2}\right)}{\sqrt{\pi}\,b}+\frac{4\text{ Chi}\left(\frac{b}{x^2}\right)-4\ln\left(\frac{b}{x^2}\right)-4\gamma}{\sqrt{\pi}}+\frac{4\gamma-4-8\ln(x)+4\ln(ib)}{\sqrt{\pi}}\right)}{8}-\frac{ib\sqrt{\pi}\sinh(a)\left(\frac{4ix^2\cosh\left(\frac{b}{x^2}\right)}{b\sqrt{\pi}}-\frac{4i\text{ Shi}\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right.}{8}$

```
[In] int(x*sinh(a+b/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp((2*a*x^2+b)/x^2)*exp(-a)*x^2+1/4*exp(2*a)*exp(-a)*Ei(1,-b/x^2)*b-1/
4*exp(-a)*x^2*exp(-b/x^2)+1/4*exp(-a)*b*Ei(1,b/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

$$\begin{aligned}\int x\sinh\left(a+\frac{b}{x^2}\right) dx = & \frac{1}{2}x^2\sinh\left(\frac{ax^2+b}{x^2}\right)-\frac{1}{4}\left(b\text{Ei}\left(\frac{b}{x^2}\right)+b\text{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh(a) \\ & -\frac{1}{4}\left(b\text{Ei}\left(\frac{b}{x^2}\right)-b\text{Ei}\left(-\frac{b}{x^2}\right)\right)\sinh(a)\end{aligned}$$

```
[In] integrate(x*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*sinh((a*x^2 + b)/x^2) - 1/4*(b*Ei(b/x^2) + b*Ei(-b/x^2))*cosh(a) -
1/4*(b*Ei(b/x^2) - b*Ei(-b/x^2))*sinh(a)
```

Sympy [F]

$$\int x \sinh\left(a+\frac{b}{x^2}\right)\,dx=\int x\sinh\left(a+\frac{b}{x^2}\right)dx$$

```
[In] integrate(x*sinh(a+b/x**2),x)
[Out] Integral(x*sinh(a + b/x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int x \sinh\left(a+\frac{b}{x^2}\right)dx=\frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{4}\left(\mathrm{Ei}\left(-\frac{b}{x^2}\right)e^{(-a)}+\mathrm{Ei}\left(\frac{b}{x^2}\right)e^a\right)b$$

```
[In] integrate(x*sinh(a+b/x^2),x, algorithm="maxima")
[Out] 1/2*x^2*sinh(a + b/x^2) - 1/4*(Ei(-b/x^2)*e^(-a) + Ei(b/x^2)*e^a)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 4.60

$$\int x\sinh\left(a+\frac{b}{x^2}\right)dx=-\frac{ab^2\mathrm{Ei}\left(a-\frac{ax^2+b}{x^2}\right)e^{(-a)}-\frac{(ax^2+b)b^2\mathrm{Ei}\left(a-\frac{ax^2+b}{x^2}\right)e^{(-a)}}{x^2}-b^2e^{\left(-\frac{ax^2+b}{x^2}\right)}}{4\left(a-\frac{ax^2+b}{x^2}\right)b}-\frac{ab^2\mathrm{Ei}\left(-a+\frac{ax^2+b}{x^2}\right)e^a-\frac{(ax^2+b)b^2\mathrm{Ei}\left(-a+\frac{ax^2+b}{x^2}\right)e^a}{x^2}+b^2e^{\left(\frac{ax^2+b}{x^2}\right)}}{4\left(a-\frac{ax^2+b}{x^2}\right)b}$$

```
[In] integrate(x*sinh(a+b/x^2),x, algorithm="giac")
[Out] -1/4*(a*b^2*Ei(a - (a*x^2 + b)/x^2)*e^(-a) - (a*x^2 + b)*b^2*Ei(a - (a*x^2
+ b)/x^2)*e^(-a)/x^2 - b^2*e^(-(a*x^2 + b)/x^2))/((a - (a*x^2 + b)/x^2)*b)
- 1/4*(a*b^2*Ei(-a + (a*x^2 + b)/x^2)*e^a - (a*x^2 + b)*b^2*Ei(-a + (a*x^2
+ b)/x^2)*e^a/x^2 + b^2*e^((a*x^2 + b)/x^2))/((a - (a*x^2 + b)/x^2)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x \sinh \left(a+\frac{b}{x^2}\right) dx = \int x \sinh\left(a+\frac{b}{x^2}\right) dx$$

```
[In] int(x*sinh(a + b/x^2),x)
[Out] int(x*sinh(a + b/x^2), x)
```

3.45 $\int \sinh\left(a+\frac{b}{x^2}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \sinh\left(a+\frac{b}{x^2}\right)\,dx = -\frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{b}e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)+x\sinh\left(a+\frac{b}{x^2}\right)$$

```
[Out] x*sinh(a+b/x^2)-1/2*erf(b^(1/2)/x)*b^(1/2)*Pi^(1/2)/exp(a)-1/2*exp(a)*erfi(
b^(1/2)/x)*b^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5410, 5434, 5407, 2235, 2236}

$$\int \sinh\left(a+\frac{b}{x^2}\right)\,dx = -\frac{1}{2}\sqrt{\pi}e^{-a}\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{\pi}e^{a}\sqrt{b}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)+x\sinh\left(a+\frac{b}{x^2}\right)$$

```
[In] Int[Sinh[a + b/x^2],x]
```

```
[Out] -1/2*(Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]/x])/E^a - (Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqr
t[b]/x])/2 + x*Sinh[a + b/x^2]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5410

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5434

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x
)^(m + 1)*(Sinh[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sinh\left(a+bx^2\right)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= x\sinh\left(a+\frac{b}{x^2}\right) - (2b)\text{Subst}\left(\int \cosh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= x\sinh\left(a+\frac{b}{x^2}\right) - b\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right) - b\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{1}{2}\sqrt{b}e^{a}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}}{x}\right) + x\sinh\left(a+\frac{b}{x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \sinh\left(a+\frac{b}{x^2}\right)\,dx = x\cosh\left(\frac{b}{x^2}\right)\sinh(a) - \frac{1}{2}\sqrt{b}\sqrt{\pi}\left(\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)-\sinh(a)) + \operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a))\right) + x\cosh(a)\sinh\left(\frac{b}{x^2}\right)$$

```
[In] Integrate[Sinh[a + b/x^2],x]
```

```
[Out] x*Cosh[b/x^2]*Sinh[a] - (Sqrt[b]*Sqrt[Pi]*(Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a
]) + Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a])))/2 + x*Cosh[a]*Sinh[b/x^2]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{b}\,\sqrt{\pi}\,\mathrm{e}^{-a}}{2}-\frac{\mathrm{e}^{-a}x\,\mathrm{e}^{-\frac{b}{x^2}}}{2}+\frac{\mathrm{e}^{a}x\,\mathrm{e}^{\frac{b}{x^2}}}{2}-\frac{\mathrm{e}^{a}b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{2\sqrt{-b}}$
meijerg	$\frac{i\sqrt{\pi}\,\cosh(a)\sqrt{2}\,\sqrt{ib}\left(\frac{2x\sqrt{2}\,\sqrt{ib}\,\mathrm{e}^{-\frac{b}{x^2}}}{\sqrt{\pi}\,b}-\frac{2x\sqrt{2}\,\sqrt{ib}\,\mathrm{e}^{\frac{b}{x^2}}}{\sqrt{\pi}\,b}+\frac{2\sqrt{ib}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{\sqrt{b}}+\frac{2\sqrt{ib}\,\sqrt{2}\,\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{\sqrt{b}}\right)}{8}-\frac{\sqrt{\pi}\,\sinh(a)\sqrt{2}\,\sqrt{ib}\left(-\frac{2x\sqrt{2}\,\mathrm{e}^{\frac{b}{x^2}}}{\sqrt{\pi}\,\sqrt{ib}}\cdots\right.}{\cdots}$

```
[In] int(sinh(a+b/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*erf(b^(1/2)/x)*b^(1/2)*Pi^(1/2)/exp(a)-1/2/exp(a)*x*exp(-b/x^2)+1/2*ex
p(a)*x*exp(b/x^2)-1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(49) = 98.

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.40

$$\int \sinh\left(a+\frac{b}{x^2}\right)\,dx = \frac{x\cosh\left(\frac{ax^2+b}{x^2}\right)^2+\sqrt{\pi}\left(\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right)+\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a)+(\cosh(a)+\sinh(a))\sinh\left(\frac{ax^2+b}{x^2}\right)\cdots\right.}{\cdots}$$

```
[In] integrate(sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) +
cosh((a*x^2 + b)/x^2)*sinh(a) + (cosh(a) + sinh(a))*sinh((a*x^2 + b)/x^2))*
sqrt(-b)*erf(sqrt(-b)/x) - sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) - cosh((
a*x^2 + b)/x^2)*sinh(a) + (cosh(a) - sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b
)*erf(sqrt(b)/x) + 2*x*cosh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) + x*sinh
((a*x^2 + b)/x^2)^2 - x)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

Sympy [F]

$$\int \sinh\left(a+\frac{b}{x^2}\right)\,dx=\int \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate(sinh(a+b/x**2),x)
```

```
[Out] Integral(sinh(a + b/x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \sinh\left(a+\frac{b}{x^2}\right)\,dx=-\frac{1}{2}\,b\left(\frac{\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{b}{x^2}}\right)-1\right)e^{(-a)}}{x\sqrt{\frac{b}{x^2}}}+\frac{\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{b}{x^2}}\right)-1\right)e^{a}}{x\sqrt{-\frac{b}{x^2}}}\right)+x\sinh\left(a+\frac{b}{x^2}\right)$$

```
[In] integrate(sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] -1/2*b*(sqrt(pi)*(erf(sqrt(b/x^2)) - 1)*e^(-a)/(x*sqrt(b/x^2)) + sqrt(pi)*(
erf(sqrt(-b/x^2)) - 1)*e^a/(x*sqrt(-b/x^2))) + x*sinh(a + b/x^2)
```

Giac [F]

$$\int \sinh\left(a+\frac{b}{x^2}\right)\,dx=\int \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate(sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh\left(a+\frac{b}{x^2}\right)\,dx = \int \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] int(sinh(a + b/x^2),x)
[Out] int(sinh(a + b/x^2), x)
```

3.46 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx=-\frac{1}{2}\text{Chi}\left(\frac{b}{x^2}\right)\sinh(a)-\frac{1}{2}\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

```
[Out] -1/2*cosh(a)*Shi(b/x^2)-1/2*Chi(b/x^2)*sinh(a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5426, 5425, 5424}

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx=-\frac{1}{2}\sinh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

```
[In] Int[Sinh[a + b/x^2]/x,x]
```

```
[Out] -1/2*(CoshIntegral[b/x^2]*Sinh[a]) - (Cosh[a]*SinhIntegral[b/x^2])/2
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5426

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(a)\int \frac{\sinh\left(\frac{b}{x^2}\right)}{x}\,dx + \sinh(a)\int \frac{\cosh\left(\frac{b}{x^2}\right)}{x}\,dx\\ &= -\frac{1}{2}\text{Chi}\left(\frac{b}{x^2}\right)\sinh(a) - \frac{1}{2}\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx = \frac{1}{2}\left(-\text{Chi}\left(\frac{b}{x^2}\right)\sinh(a) - \cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)\right)$$

```
[In] Integrate[Sinh[a + b/x^2]/x,x]
[Out] (-(CoshIntegral[b/x^2]*Sinh[a]) - Cosh[a]*SinhIntegral[b/x^2])/2
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{e^{2a}e^{-a}\,\text{Ei}_1\left(-\frac{b}{x^2}\right)}{4} - \frac{e^{-a}\,\text{Ei}_1\left(\frac{b}{x^2}\right)}{4}$	33
meijerg	$-\frac{\cosh(a)\,\text{Shi}\left(\frac{b}{x^2}\right)}{2} - \frac{\sqrt{\pi}\,\sinh(a)\left(\frac{2\,\text{Chi}\left(\frac{b}{x^2}\right)-2\ln\left(\frac{b}{x^2}\right)-2\gamma}{\sqrt{\pi}} + \frac{2\gamma-4\ln(x)+2\ln(ib)}{\sqrt{\pi}}\right)}{4}$	62

```
[In] int(sinh(a+b/x^2)/x,x,method=_RETURNVERBOSE)
[Out] 1/4*exp(2*a)*exp(-a)*Ei(1,-b/x^2)-1/4*exp(-a)*Ei(1,b/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx = -\frac{1}{4}\left(\mathrm{Ei}\left(\frac{b}{x^2}\right) - \mathrm{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh(a) - \frac{1}{4}\left(\mathrm{Ei}\left(\frac{b}{x^2}\right) + \mathrm{Ei}\left(-\frac{b}{x^2}\right)\right)\sinh(a)$$

```
[In] integrate(sinh(a+b/x^2)/x,x, algorithm="fricas")
[Out] -1/4*(Ei(b/x^2) - Ei(-b/x^2))*cosh(a) - 1/4*(Ei(b/x^2) + Ei(-b/x^2))*sinh(a
)
```

Sympy [F]

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

```
[In] integrate(sinh(a+b/x**2)/x,x)
[Out] Integral(sinh(a + b/x**2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx = \frac{1}{4}\,\mathrm{Ei}\left(-\frac{b}{x^2}\right)e^{(-a)} - \frac{1}{4}\,\mathrm{Ei}\left(\frac{b}{x^2}\right)e^{a}$$

```
[In] integrate(sinh(a+b/x^2)/x,x, algorithm="maxima")
[Out] 1/4*Ei(-b/x^2)*e^(-a) - 1/4*Ei(b/x^2)*e^a
```

Giac [F]

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

```
[In] integrate(sinh(a+b/x^2)/x,x, algorithm="giac")
[Out] integrate(sinh(a + b/x^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx = -\frac{\sinh(a)\ \mathrm{coshint}\left(\frac{b}{x^2}\right)}{2} - \frac{\cosh(a)\ \mathrm{sinhint}\left(\frac{b}{x^2}\right)}{2}$$

```
[In] int(sinh(a + b/x^2)/x,x)
[Out] - (sinh(a)*coshint(b/x^2))/2 - (cosh(a)*sinhint(b/x^2))/2
```

3.47 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2} dx = \frac{e^{-a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}} - \frac{e^{a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

```
[Out] 1/4*erf(b^(1/2)/x)*Pi^(1/2)/exp(a)/b^(1/2)-1/4*exp(a)*erfi(b^(1/2)/x)*Pi^(1
/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5454, 5406, 2235, 2236}

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2} dx = \frac{\sqrt{\pi}e^{-a}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}} - \frac{\sqrt{\pi}e^{a}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

```
[In] Int[Sinh[a + b/x^2]/x^2,x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[b]/x])/(4*Sqrt[b]*E^a) - (E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/
(4*Sqrt[b])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5454

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right) - \frac{1}{2}\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}} - \frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \frac{\sqrt{\pi}\left(\text{erf}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)-\sinh(a)) - \text{erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{b}}$$

```
[In] Integrate[Sinh[a + b/x^2]/x^2,x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]) - Erfi[Sqrt[b]/x]*(Cosh[a] +
Sinh[a])))/(4*Sqrt[b])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	siz
risch	$\frac{\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{\pi}\,\mathrm{e}^{-a}}{4\sqrt{b}}-\frac{\mathrm{e}^{a}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{4\sqrt{-b}}$	44
meijerg	$\frac{\sqrt{\pi}\,\cosh(a)\sqrt{2}\,\sqrt{ib}\left(-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{2b^{\frac{3}{2}}}+\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{2b^{\frac{3}{2}}}\right)}{4b}+\frac{i\sqrt{\pi}\,\sinh(a)\sqrt{2}\,\sqrt{ib}\left(\frac{\sqrt{ib}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{2\sqrt{b}}+\frac{\sqrt{ib}\,\sqrt{2}\,\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{2\sqrt{b}}\right)}{4b}$	131

```
[In] int(sinh(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/4*erf(b^(1/2)/x)*Pi^(1/2)/exp(a)/b^(1/2)-1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)*e
rf((-b)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \frac{\sqrt{\pi}\sqrt{-b}(\cosh(a)+\sinh(a))\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)+\sqrt{\pi}\sqrt{b}(\cosh(a)-\sinh(a))\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\,b}$$

```
[In] integrate(sinh(a+b/x^2)/x^2,x, algorithm="fricas")
[Out] 1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)/x) + sqrt(pi)*sqrt(
b)*(cosh(a) - sinh(a))*erf(sqrt(b)/x))/b
```

Sympy [F]

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

```
[In] integrate(sinh(a+b/x**2)/x**2,x)
[Out] Integral(sinh(a + b/x**2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{1}{2}\,b\left(\frac{e^{(-a)}\Gamma\left(\frac{3}{2},\frac{b}{x^2}\right)}{x^3\left(\frac{b}{x^2}\right)^{\frac{3}{2}}}+\frac{e^{a}\Gamma\left(\frac{3}{2},-\frac{b}{x^2}\right)}{x^3\left(-\frac{b}{x^2}\right)^{\frac{3}{2}}}\right)-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}$$

```
[In] integrate(sinh(a+b/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(-a)*gamma(3/2, b/x^2)/(x^3*(b/x^2)^(3/2)) + e^a*gamma(3/2, -b/x^
2)/(x^3*(-b/x^2)^(3/2))) - sinh(a + b/x^2)/x
```

Giac [F]

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

```
[In] integrate(sinh(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

```
[In] int(sinh(a + b/x^2)/x^2,x)
```

```
[Out] int(sinh(a + b/x^2)/x^2, x)
```

3.48 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^3} dx = -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[Out] -1/2*cosh(a+b/x^2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5428, 2718}

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^3} dx = -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[In] Int[Sinh[a + b/x^2]/x^3,x]
```

```
[Out] -1/2*Cosh[a + b/x^2]/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
```

```
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)\right) \\ &= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[In] Integrate[Sinh[a + b/x^2]/x^3,x]
[Out] -1/2*Cosh[a + b/x^2]/b
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$	14
default	$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$	14
parallelrisch	$\frac{-\cosh\left(\frac{a\,x^2+b}{x^2}\right)-1}{2b}$	22
risch	$-\frac{e^{\frac{a\,x^2+b}{x^2}}}{4b}-\frac{e^{-\frac{a\,x^2+b}{x^2}}}{4b}$	37
meijerg	$\frac{\sqrt{\pi}\,\cosh(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{2b}-\frac{\sinh(a)\sinh\left(\frac{b}{x^2}\right)}{2b}$	40

```
[In] int(sinh(a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*cosh(a+b/x^2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\sinh \left(a+\frac{b}{x^2}\right)}{x^3} \, dx = -\frac{\cosh \left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

```
[In] integrate(sinh(a+b/x^2)/x^3,x, algorithm="fricas")
[Out] -1/2*cosh((a*x^2 + b)/x^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\sinh \left(a+\frac{b}{x^2}\right)}{x^3} \, dx = \begin{cases} -\frac{\cosh \left(a+\frac{b}{x^2}\right)}{2b} & \text{for}\ b \neq 0 \\ -\frac{\sinh (a)}{2x^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(a+b/x**2)/x**3,x)
[Out] Piecewise((-cosh(a + b/x**2)/(2*b), Ne(b, 0)), (-sinh(a)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sinh \left(a+\frac{b}{x^2}\right)}{x^3} \, dx = -\frac{\cosh \left(a+\frac{b}{x^2}\right)}{2\,b}$$

```
[In] integrate(sinh(a+b/x^2)/x^3,x, algorithm="maxima")
[Out] -1/2*cosh(a + b/x^2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{e^{\left(\frac{ax^2+b}{x^2}\right)} + e^{\left(-\frac{ax^2+b}{x^2}\right)}}{4\,b}$$

```
[In] integrate(sinh(a+b/x^2)/x^3,x, algorithm="giac")
[Out] -1/4*(e^((a*x^2 + b)/x^2) + e^(-(a*x^2 + b)/x^2))/b
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2\,b}$$

```
[In] int(sinh(a + b/x^2)/x^3,x)
[Out] -cosh(a + b/x^2)/(2*b)
```

3.49 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} + \frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}$$

```
[Out] -1/2*cosh(a+b/x^2)/b/x+1/8*erf(b^(1/2)/x)*Pi^(1/2)/b^(3/2)/exp(a)+1/8*exp(a
)*erfi(b^(1/2)/x)*Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5454, 5432, 5407, 2235, 2236}

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \frac{\sqrt{\pi}e^{-a}\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} + \frac{\sqrt{\pi}e^{a}\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} - \frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx}$$

```
[In] Int[Sinh[a + b/x^2]/x^4,x]
```

```
[Out] -1/2*Cosh[a + b/x^2]/(b*x) + (Sqrt[Pi]*Erf[Sqrt[b]/x])/(8*b^(3/2)*E^a) + (E
^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/(8*b^(3/2))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5454

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^2 \sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{\text{Subst}\left(\int \cosh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right)}{4b} + \frac{\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)}{4b}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} + \frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$
$$=\frac{-4\sqrt{b}\cosh\left(a+\frac{b}{x^2}\right)+\sqrt{\pi}x\text{erf}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)-\sinh(a))+\sqrt{\pi}x\text{erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a))}{8b^{3/2}x}$$

```
[In] Integrate[Sinh[a + b/x^2]/x^4,x]
[Out] (-4*Sqrt[b]*Cosh[a + b/x^2] + Sqrt[Pi]*x*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a])
 + Sqrt[Pi]*x*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]))/(8*b^(3/2)*x)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{e^{-a} e^{-\frac{b}{x^2}}}{4bx}+\frac{\text{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{\pi}\, e^{-a}}{8b^{\frac{3}{2}}}-\frac{e^{a} e^{\frac{b}{x^2}}}{4xb}+\frac{e^{a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{-b}}{x}\right)}{8b\sqrt{-b}}$
meijerg	$-\frac{i\sqrt{\pi}\,\cosh(a)\sqrt{2}\,\sqrt{ib}\left(\frac{\sqrt{2}\,(ib)^{\frac{5}{2}} e^{-\frac{b}{x^2}}}{4\sqrt{\pi}\,x\,b^2}+\frac{\sqrt{2}\,(ib)^{\frac{5}{2}} e^{\frac{b}{x^2}}}{4\sqrt{\pi}\,x\,b^2}-\frac{(ib)^{\frac{5}{2}}\sqrt{2}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{\frac{5}{2}}}-\frac{(ib)^{\frac{5}{2}}\sqrt{2}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{\frac{5}{2}}}\right)}{2b^2}+\frac{\sqrt{\pi}\,\sinh(a)\sqrt{2}\,\sqrt{ib}\left(\frac{\sqrt{2}\,(ib)^{\frac{3}{2}} e^{\frac{b}{x^2}}}{4\sqrt{\pi}\,xb}\right.}{}$

```
[In] int(sinh(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/4/exp(a)/b/x*exp(-b/x^2)+1/8*erf(b^(1/2)/x)*Pi^(1/2)/b^(3/2)/exp(a)-1/4*
exp(a)*exp(b/x^2)/x/b+1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 251, normalized size of antiderivative = 3.35

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx =$$
$$-\frac{2\,b\cosh\left(\frac{ax^2+b}{x^2}\right)^2+\sqrt{\pi}\left(x\cosh\left(a\right)\cosh\left(\frac{ax^2+b}{x^2}\right)+x\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh\left(a\right)+\left(x\cosh\left(a\right)+x\sinh\left(a\right)\right)\sinh\right.}{}$$

```
[In] integrate(sinh(a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/x^
2) + x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) + x*sinh(a))*sinh((a*x^2
+ b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) - sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/
x^2) - x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) - x*sinh(a))*sinh((a*x^
2 + b)/x^2))*sqrt(b)*erf(sqrt(b)/x) + 4*b*cosh((a*x^2 + b)/x^2)*sinh((a*x^2
 + b)/x^2) + 2*b*sinh((a*x^2 + b)/x^2)^2 + 2*b)/(b^2*x*cosh((a*x^2 + b)/x^2
) + b^2*x*sinh((a*x^2 + b)/x^2))
```

Sympy [F]

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

```
[In] integrate(sinh(a+b/x**2)/x**4,x)
[Out] Integral(sinh(a + b/x**2)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = -\frac{1}{6}\,b\left(\frac{e^{(-a)}\Gamma\left(\frac{5}{2},\frac{b}{x^2}\right)}{x^5\left(\frac{b}{x^2}\right)^{\frac{5}{2}}}+\frac{e^{a}\Gamma\left(\frac{5}{2},-\frac{b}{x^2}\right)}{x^5\left(-\frac{b}{x^2}\right)^{\frac{5}{2}}}\right)-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{3\,x^3}$$

```
[In] integrate(sinh(a+b/x^2)/x^4,x, algorithm="maxima")
[Out] -1/6*b*(e^(-a)*gamma(5/2, b/x^2)/(x^5*(b/x^2)^(5/2)) + e^a*gamma(5/2, -b/x^
2)/(x^5*(-b/x^2)^(5/2))) - 1/3*sinh(a + b/x^2)/x^3
```

Giac [F]

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

```
[In] integrate(sinh(a+b/x^2)/x^4,x, algorithm="giac")
[Out] integrate(sinh(a + b/x^2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

```
[In] int(sinh(a + b/x^2)/x^4,x)
[Out] int(sinh(a + b/x^2)/x^4, x)
```

3.50 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5}\,dx=-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2}+\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b^2}$$

```
[Out] -1/2*cosh(a+b/x^2)/b/x^2+1/2*sinh(a+b/x^2)/b^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5428, 3377, 2717}

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5}\,dx=\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2}$$

```
[In] Int[Sinh[a + b/x^2]/x^5,x]
```

```
[Out] -1/2*Cosh[a + b/x^2]/(b*x^2) + Sinh[a + b/x^2]/(2*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2} + \frac{\text{Subst}\left(\int\cosh(a+bx)\,dx, x, \frac{1}{x^2}\right)}{2b}\\ &= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int\frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5}\,dx = \frac{-b\cosh\left(a+\frac{b}{x^2}\right)+x^2\sinh\left(a+\frac{b}{x^2}\right)}{2b^2x^2}$$

```
[In] Integrate[Sinh[a + b/x^2]/x^5,x]
[Out] (-(b*Cosh[a + b/x^2]) + x^2*Sinh[a + b/x^2])/(2*b^2*x^2)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

method	result	size
parallelrisch	$\frac{\sinh\left(\frac{a\,x^2+b}{x^2}\right)x^2-b\cosh\left(\frac{a\,x^2+b}{x^2}\right)}{2b^2x^2}$	41
risch	$-\frac{(-x^2+b)\,\mathrm{e}^{\frac{a\,x^2+b}{x^2}}}{4b^2x^2}-\frac{(x^2+b)\,\mathrm{e}^{-\frac{a\,x^2+b}{x^2}}}{4b^2x^2}$	55
meijerg	$-\frac{\cosh(a)\left(\frac{\cosh\left(\frac{b}{x^2}\right)b}{x^2}-\sinh\left(\frac{b}{x^2}\right)\right)}{2b^2}+\frac{\sqrt{\pi}\,\sinh(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}}-\frac{b\sinh\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}\,x^2}\right)}{b^2}$	70

```
[In] int(sinh(a+b/x^2)/x^5,x,method=_RETURNVERBOSE)
[Out] 1/2*(sinh((a*x^2+b)/x^2)*x^2-b*cosh((a*x^2+b)/x^2))/b^2/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \frac{\sinh \left(a+\frac{b}{x^2}\right)}{x^5}\,dx=\frac{x^2\sinh \left(\frac{ax^2+b}{x^2}\right)-b\cosh \left(\frac{ax^2+b}{x^2}\right)}{2\,b^2x^2}$$

```
[In] integrate(sinh(a+b/x^2)/x^5,x, algorithm="fricas")
[Out] 1/2*(x^2*sinh((a*x^2 + b)/x^2) - b*cosh((a*x^2 + b)/x^2))/(b^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\sinh \left(a+\frac{b}{x^2}\right)}{x^5}\,dx=\begin{cases}-\frac{\cosh \left(a+\frac{b}{x^2}\right)}{2bx^2}+\frac{\sinh \left(a+\frac{b}{x^2}\right)}{2b^2} & \text{for } b\neq 0\\ -\frac{\sinh (a)}{4x^4} & \text{otherwise}\end{cases}$$

```
[In] integrate(sinh(a+b/x**2)/x**5,x)
[Out] Piecewise((-cosh(a + b/x**2)/(2*b*x**2) + sinh(a + b/x**2)/(2*b**2), Ne(b,
0)), (-sinh(a)/(4*x**4), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{\sinh \left(a+\frac{b}{x^2}\right)}{x^5}\,dx=-\frac{1}{8}\,b\left(\frac{e^{(-a)}\Gamma \left(3,\frac{b}{x^2}\right)}{b^3}-\frac{e^a\Gamma \left(3,-\frac{b}{x^2}\right)}{b^3}\right)-\frac{\sinh \left(a+\frac{b}{x^2}\right)}{4\,x^4}$$

```
[In] integrate(sinh(a+b/x^2)/x^5,x, algorithm="maxima")
[Out] -1/8*b*(e^(-a)*gamma(3, b/x^2)/b^3 - e^a*gamma(3, -b/x^2)/b^3) - 1/4*sinh(a
 + b/x^2)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5}\,dx = -\frac{\left(\left(\frac{b}{x^2}-1\right)e^{\left(2\,a+\frac{b}{x^2}\right)}+\left(\frac{b}{x^2}+1\right)e^{\left(-\frac{b}{x^2}\right)}\right)e^{(-a)}}{4\,b^2}$$

```
[In] integrate(sinh(a+b/x^2)/x^5,x, algorithm="giac")
[Out] -1/4*((b/x^2 - 1)*e^(2*a + b/x^2) + (b/x^2 + 1)*e^(-b/x^2))*e^(-a)/b^2
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5}\,dx = -\frac{\mathrm{e}^{a+\frac{b}{x^2}}\left(\frac{1}{4\,b}-\frac{x^2}{4\,b^2}\right)}{x^2}-\frac{\mathrm{e}^{-a-\frac{b}{x^2}}\left(\frac{1}{4\,b}+\frac{x^2}{4\,b^2}\right)}{x^2}$$

```
[In] int(sinh(a + b/x^2)/x^5,x)
[Out] - (exp(a + b/x^2)*(1/(4*b) - x^2/(4*b^2)))/x^2 - (exp(- a - b/x^2)*(1/(4*b)
 + x^2/(4*b^2)))/x^2
```

3.51 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 93

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx=-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}+\frac{3e^{-a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}-\frac{3e^{a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}+\frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}$$

```
[Out] -1/2*cosh(a+b/x^2)/b/x^3+3/4*sinh(a+b/x^2)/b^2/x+3/16*erf(b^(1/2)/x)*Pi^(1/
2)/b^(5/2)/exp(a)-3/16*exp(a)*erfi(b^(1/2)/x)*Pi^(1/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5454, 5432, 5433, 5406, 2235, 2236}

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx=\frac{3\sqrt{\pi}e^{-a}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}-\frac{3\sqrt{\pi}e^{a}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}+\frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}$$

```
[In] Int[Sinh[a + b/x^2]/x^6,x]
```

```
[Out] -1/2*Cosh[a + b/x^2]/(b*x^3) + (3*Sqrt[Pi]*Erf[Sqrt[b]/x])/(16*b^(5/2)*E^a)
 - (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/(16*b^(5/2)) + (3*Sinh[a + b/x^2])/(4*b
^2*x)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5454

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^4 \sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3} + \frac{3\text{Subst}\left(\int x^2\cosh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3} + \frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x} - \frac{3\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)}{4b^2}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3} + \frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x} + \frac{3\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right)}{8b^2} - \frac{3\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)}{8b^2}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3} + \frac{3e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}} - \frac{3e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}} + \frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx$$
$$= \frac{3\sqrt{\pi}x^3\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)-\sinh(a))-3\sqrt{\pi}x^3\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a))+4\sqrt{b}\left(-2b\cosh\left(a+\frac{b}{x^2}\right)+3x\right.}{16b^{5/2}x^3}$$

```
[In] Integrate[Sinh[a + b/x^2]/x^6,x]
```

```
[Out] (3*Sqrt[Pi]*x^3*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]) - 3*Sqrt[Pi]*x^3*Erfi[Sq
rt[b]/x]*(Cosh[a] + Sinh[a]) + 4*Sqrt[b]*(-2*b*Cosh[a + b/x^2] + 3*x^2*Sinh
[a + b/x^2]))/(16*b^(5/2)*x^3)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{e^{-a} e^{-\frac{b}{x^2}}}{4b\,x^3}-\frac{3\,e^{-a} e^{-\frac{b}{x^2}}}{8b^2 x}+\frac{3\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{\pi}\,e^{-a}}{16b^{\frac{5}{2}}}-\frac{e^{a} e^{\frac{b}{x^2}}}{4x^3 b}+\frac{3\,e^{a} e^{\frac{b}{x^2}}}{8b^2 x}-\frac{3\,e^{a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{16b^2\sqrt{-b}}$
meijerg	$-\frac{\sqrt{\pi}\,\cosh(a)\sqrt{2}\sqrt{ib}\left(-\frac{\sqrt{2}\,(ib)^{\frac{7}{2}}\left(-\frac{14b}{x^2}+21\right)e^{\frac{b}{x^2}}}{112\sqrt{\pi}\,x\,b^3}+\frac{\sqrt{2}\,(ib)^{\frac{7}{2}}\left(\frac{14b}{x^2}+21\right)e^{-\frac{b}{x^2}}}{112\sqrt{\pi}\,x\,b^3}-\frac{3(ib)^{\frac{7}{2}}\sqrt{2}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{32b^{\frac{7}{2}}}+\frac{3(ib)^{\frac{7}{2}}\sqrt{2}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{32b^{\frac{7}{2}}}\right)}{b^3}-\frac{i\sqrt{\pi}\,\sin}{}$

```
[In] int(sinh(a+b/x^2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/exp(a)/b/x^3*exp(-b/x^2)-3/8/exp(a)/b^2/x*exp(-b/x^2)+3/16*erf(b^(1/2)
/x)*Pi^(1/2)/b^(5/2)/exp(a)-1/4*exp(a)*exp(b/x^2)/x^3/b+3/8*exp(a)/b^2*exp(
b/x^2)/x-3/16*exp(a)/b^2*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(71) = 142.

Time = 0.26 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.37

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx =$$
$$-\frac{6\,bx^2-2\left(3\,bx^2-2\,b^2\right)\cosh\left(\frac{ax^2+b}{x^2}\right)^2-3\sqrt{\pi}\left(x^3\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right)+x^3\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a)+(x\right.}{}$$

```
[In] integrate(sinh(a+b/x^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/16*(6*b*x^2 - 2*(3*b*x^2 - 2*b^2)*cosh((a*x^2 + b)/x^2)^2 - 3*sqrt(pi)*(
x^3*cosh(a)*cosh((a*x^2 + b)/x^2) + x^3*cosh((a*x^2 + b)/x^2)*sinh(a) + (x^
3*cosh(a) + x^3*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) -
3*sqrt(pi)*(x^3*cosh(a)*cosh((a*x^2 + b)/x^2) - x^3*cosh((a*x^2 + b)/x^2)*s
inh(a) + (x^3*cosh(a) - x^3*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b)*erf(sqr
t(b)/x) - 4*(3*b*x^2 - 2*b^2)*cosh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) -
 2*(3*b*x^2 - 2*b^2)*sinh((a*x^2 + b)/x^2)^2 + 4*b^2)/(b^3*x^3*cosh((a*x^2
+ b)/x^2) + b^3*x^3*sinh((a*x^2 + b)/x^2))
```

Sympy [F]

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx$$

```
[In] integrate(sinh(a+b/x**2)/x**6,x)
```

```
[Out] Integral(sinh(a + b/x**2)/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx = -\frac{1}{10}\,b\left(\frac{e^{(-a)}\Gamma\left(\frac{7}{2},\frac{b}{x^2}\right)}{x^7\left(\frac{b}{x^2}\right)^{\frac{7}{2}}}+\frac{e^{a}\Gamma\left(\frac{7}{2},-\frac{b}{x^2}\right)}{x^7\left(-\frac{b}{x^2}\right)^{\frac{7}{2}}}\right)-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{5\,x^5}$$

```
[In] integrate(sinh(a+b/x^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/10*b*(e^(-a)*gamma(7/2, b/x^2)/(x^7*(b/x^2)^(7/2)) + e^a*gamma(7/2, -b/x
^2)/(x^7*(-b/x^2)^(7/2))) - 1/5*sinh(a + b/x^2)/x^5
```

Giac [F]

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx$$

```
[In] integrate(sinh(a+b/x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx$$

```
[In] int(sinh(a + b/x^2)/x^6,x)
[Out] int(sinh(a + b/x^2)/x^6, x)
```

3.52 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 47

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx = -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3} - \frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}$$

```
[Out] -cosh(a+b/x^2)/b^3-1/2*cosh(a+b/x^2)/b/x^4+sinh(a+b/x^2)/b^2/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5428, 3377, 2718}

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx = -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2} - \frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4}$$

```
[In] Int[Sinh[a + b/x^2]/x^7,x]
```

```
[Out] -(Cosh[a + b/x^2]/b^3) - Cosh[a + b/x^2]/(2*b*x^4) + Sinh[a + b/x^2]/(b^2*x
^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int x^2\sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4} + \frac{\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \frac{1}{x^2}\right)}{b}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2} - \frac{\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)}{b^2}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3} - \frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx = \frac{-\left(\left(b^2+2x^4\right)\cosh\left(a+\frac{b}{x^2}\right)\right)+2bx^2\sinh\left(a+\frac{b}{x^2}\right)}{2b^3x^4}$$

```
[In] Integrate[Sinh[a + b/x^2]/x^7,x]
[Out] (-((b^2 + 2*x^4)*Cosh[a + b/x^2]) + 2*b*x^2*Sinh[a + b/x^2])/(2*b^3*x^4)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

method	result	size
risch	$-\frac{(2x^4-2x^2b+b^2)\,\mathrm{e}^{\frac{a x^2+b}{x^2}}}{4b^3x^4}-\frac{(2x^4+2x^2b+b^2)\,\mathrm{e}^{-\frac{a x^2+b}{x^2}}}{4b^3x^4}$	73
parallelrisch	$\frac{4x^4-4\tanh\left(\frac{a x^2+b}{2x^2}\right)x^2b+\tanh\left(\frac{a x^2+b}{2x^2}\right)^2b^2+b^2}{2x^4b^3\left(\tanh\left(\frac{a x^2+b}{2x^2}\right)^2-1\right)}$	75
meijerg	$-\frac{2\sqrt{\pi}\,\cosh(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{b^2}{2x^4}+1\right)\cosh\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}}-\frac{b\sinh\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}\,x^2}\right)}{b^3}-\frac{2i\sqrt{\pi}\,\sinh(a)\left(\frac{ib\cosh\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}\,x^2}-\frac{i\left(\frac{3b^2}{2x^4}+3\right)\sinh\left(\frac{b}{x^2}\right)}{6\sqrt{\pi}}\right)}{b^3}$	104

```
[In] int(sinh(a+b/x^2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*x^4-2*b*x^2+b^2)/b^3/x^4*exp((a*x^2+b)/x^2)-1/4*(2*x^4+2*b*x^2+b^2)
/b^3/x^4*exp(-(a*x^2+b)/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx=\frac{2\,bx^2\sinh\left(\frac{ax^2+b}{x^2}\right)-\left(2\,x^4+b^2\right)\cosh\left(\frac{ax^2+b}{x^2}\right)}{2\,b^3x^4}$$

```
[In] integrate(sinh(a+b/x^2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*sinh((a*x^2 + b)/x^2) - (2*x^4 + b^2)*cosh((a*x^2 + b)/x^2))/(
b^3*x^4)
```

Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx=\begin{cases}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4}+\frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3} & \text{for } b\neq 0\\ -\frac{\sinh(a)}{6x^6} & \text{otherwise}\end{cases}$$

```
[In] integrate(sinh(a+b/x**2)/x**7,x)
```

```
[Out] Piecewise((-cosh(a + b/x**2)/(2*b*x**4) + sinh(a + b/x**2)/(b**2*x**2) - co
sh(a + b/x**2)/b**3, Ne(b, 0)), (-sinh(a)/(6*x**6), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx = -\frac{1}{12}\,b\left(\frac{e^{(-a)}\Gamma\left(4,\frac{b}{x^2}\right)}{b^4}+\frac{e^{a}\Gamma\left(4,-\frac{b}{x^2}\right)}{b^4}\right)-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{6\,x^6}$$

```
[In] integrate(sinh(a+b/x^2)/x^7,x, algorithm="maxima")
[Out] -1/12*b*(e^(-a)*gamma(4, b/x^2)/b^4 + e^a*gamma(4, -b/x^2)/b^4) - 1/6*sinh(
a + b/x^2)/x^6
```

Giac [F]

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx = \int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx$$

```
[In] integrate(sinh(a+b/x^2)/x^7,x, algorithm="giac")
[Out] integrate(sinh(a + b/x^2)/x^7, x)
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx = -\frac{\mathrm{e}^{a+\frac{b}{x^2}}\left(\frac{1}{4\,b}-\frac{x^2}{2\,b^2}+\frac{x^4}{2\,b^3}\right)}{x^4}-\frac{\mathrm{e}^{-a-\frac{b}{x^2}}\left(\frac{1}{4\,b}+\frac{x^2}{2\,b^2}+\frac{x^4}{2\,b^3}\right)}{x^4}$$

```
[In] int(sinh(a + b/x^2)/x^7,x)
[Out] - (exp(a + b/x^2)*(1/(4*b) - x^2/(2*b^2) + x^4/(2*b^3)))/x^4 - (exp(- a - b
/x^2)*(1/(4*b) + x^2/(2*b^2) + x^4/(2*b^3)))/x^4
```

3.53 $\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 194

$$\begin{aligned}\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right)\,dx &= \frac{1}{16}3^{\frac{1+m}{2}}e^{3a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),-\frac{3b}{x^2}\right)\\&\quad-\frac{3}{16}e^a\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),-\frac{b}{x^2}\right)\\&\quad+\frac{3}{16}e^{-a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),\frac{b}{x^2}\right)\\&\quad-\frac{1}{16}3^{\frac{1+m}{2}}e^{-3a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),\frac{3b}{x^2}\right)\end{aligned}$$

```
[Out] 1/16*3^(1/2+1/2*m)*exp(3*a)*(-b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m
,-3*b/x^2)-3/16*exp(a)*(-b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m,-b/x
^2)+3/16*(b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m,b/x^2)/exp(a)-1/16*
3^(1/2+1/2*m)*(b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m,3*b/x^2)/exp(3
*a)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{5458, 5448, 5436, 2250\}$

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right)\,dx = \frac{1}{16}e^{3a}3^{\frac{m+1}{2}}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),-\frac{3b}{x^2}\right) - \frac{3}{16}e^{a}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right) + \frac{3}{16}e^{-a}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),\frac{b}{x^2}\right) - \frac{1}{16}e^{-3a}3^{\frac{m+1}{2}}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),\frac{3b}{x^2}\right)$$

```
[In] Int[(e*x)^m*Sinh[a + b/x^2]^3,x]
```

```
[Out] (3^((1 + m)/2)*E^(3*a)*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, (
-3*b)/x^2])/16 - (3*E^a*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2,
-(b/x^2)])/16 + (3*(b/x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, b/x^2])/
(16*E^a) - (3^((1 + m)/2)*(b/x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, (
3*b)/x^2])/(16*E^(3*a))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5436

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 5448

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5458

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^
p/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p]
 && ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \sinh^3\left(a+bx^2\right)\, dx, x, \frac{1}{x}\right)\right) \\
&= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int \left(-\frac{3}{4}x^{-2-m}\sinh\left(a+bx^2\right)+\frac{1}{4}x^{-2-m}\sinh\left(3a+3bx^2\right)\right)\, dx, x, \frac{1}{x}\right)\right) \\
&= -\left(\frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\sinh\left(3a+3bx^2\right)\, dx, x, \frac{1}{x}\right)\right) \\
&\quad + \frac{1}{4}\left(3\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\sinh\left(a+bx^2\right)\, dx, x, \frac{1}{x}\right) \\
&= \frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-3a-3bx^2}x^{-2-m}\, dx, x, \frac{1}{x}\right) \\
&\quad - \frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{3a+3bx^2}x^{-2-m}\, dx, x, \frac{1}{x}\right) \\
&\quad - \frac{1}{8}\left(3\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-a-bx^2}x^{-2-m}\, dx, x, \frac{1}{x}\right) \\
&\quad + \frac{1}{8}\left(3\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{a+bx^2}x^{-2-m}\, dx, x, \frac{1}{x}\right) \\
&= \frac{1}{16}3^{\frac{1+m}{2}}e^{3a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m), -\frac{3b}{x^2}\right) \\
&\quad - \frac{3}{16}e^{a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m), -\frac{b}{x^2}\right) \\
&\quad + \frac{3}{16}e^{-a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m), \frac{b}{x^2}\right) \\
&\quad - \frac{1}{16}3^{\frac{1+m}{2}}e^{-3a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m), \frac{3b}{x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.84

$$\begin{aligned}
\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right)\, dx &= \frac{1}{16}e^{-3a}x(ex)^m \left(3^{\frac{1+m}{2}}e^{6a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1}{2}(-1-m), -\frac{3b}{x^2}\right)\right. \\
&\quad - 3e^{4a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1}{2}(-1-m), -\frac{b}{x^2}\right) \\
&\quad + \left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}\left(3e^{2a}\Gamma\left(\frac{1}{2}(-1-m), \frac{b}{x^2}\right)\right. \\
&\quad \left.\left. - 3^{\frac{1+m}{2}}\Gamma\left(\frac{1}{2}(-1-m), \frac{3b}{x^2}\right)\right)\right)
\end{aligned}$$

```
[In] Integrate[(e*x)^m*Sinh[a + b/x^2]^3,x]
```

```
[Out] (x*(e*x)^m*(3^((1 + m)/2)*E^(6*a)*(-(b/x^2))^((1 + m)/2)*Gamma[(-1 - m)/2,
(-3*b)/x^2] - 3*E^(4*a)*(-(b/x^2))^((1 + m)/2)*Gamma[(-1 - m)/2, -(b/x^2)]
+ (b/x^2)^((1 + m)/2)*(3*E^(2*a)*Gamma[(-1 - m)/2, b/x^2] - 3^((1 + m)/2)*G
amma[(-1 - m)/2, (3*b)/x^2])))/(16*E^(3*a))
```

Maple [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^3 dx$$

```
[In] int((e*x)^m*sinh(a+b/x^2)^3,x)
```

```
[Out] int((e*x)^m*sinh(a+b/x^2)^3,x)
```

Fricas [F]

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right) dx = \int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^3 dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x^2 + b)/x^2)^3, x)
```

Sympy [F]

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right) dx = \int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right) dx$$

```
[In] integrate((e*x)**m*sinh(a+b/x**2)**3,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b/x**2)**3, x)
```

Maxima [F]

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right) dx = \int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^3 dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2)^3, x)
```

Giac [F]

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right)\,dx=\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^3\,dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^3,x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(a + b/x^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right)\,dx=\int \sinh\left(a+\frac{b}{x^2}\right)^3 (e\,x)^m\,dx$$

```
[In] int(sinh(a + b/x^2)^3*(e*x)^m,x)
[Out] int(sinh(a + b/x^2)^3*(e*x)^m, x)
```

3.54 $\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx = -\frac{x(ex)^m}{2(1+m)} + 2^{\frac{1}{2}(-5+m)}e^{2a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),-\frac{2b}{x^2}\right) + 2^{\frac{1}{2}(-5+m)}e^{-2a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),\frac{2b}{x^2}\right)$$

```
[Out] -1/2*x*(e*x)^m/(1+m)+2^(-5/2+1/2*m)*exp(2*a)*(-b/x^2)^(1/2+1/2*m)*x*(e*x)^m
*GAMMA(-1/2-1/2*m,-2*b/x^2)+2^(-5/2+1/2*m)*(b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GA
MMA(-1/2-1/2*m,2*b/x^2)/exp(2*a)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5458, 5448, 5437, 2250}

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx = e^{2a}2^{\frac{m-5}{2}}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),-\frac{2b}{x^2}\right) + e^{-2a}2^{\frac{m-5}{2}}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),\frac{2b}{x^2}\right) - \frac{x(ex)^m}{2(m+1)}$$

```
[In] Int[(e*x)^m*Sinh[a + b/x^2]^2,x]
```

```
[Out] -1/2*(x*(e*x)^m)/(1 + m) + 2^((-5 + m)/2)*E^(2*a)*(-(b/x^2))^((1 + m)/2)*x*
(e*x)^m*Gamma[(-1 - m)/2, (-2*b)/x^2] + (2^((-5 + m)/2)*(b/x^2)^((1 + m)/2)
*x*(e*x)^m*Gamma[(-1 - m)/2, (2*b)/x^2])/E^(2*a)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5437

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 5448

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5458

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^
p/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p]
 && ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \operatorname{Subst}\left(\int x^{-2-m} \sinh^2\left(a+bx^2\right)\, dx, x, \frac{1}{x}\right)\right) \\
&= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \operatorname{Subst}\left(\int \left(-\frac{1}{2}x^{-2-m}+\frac{1}{2}x^{-2-m}\cosh\left(2a+2bx^2\right)\right) dx, x, \frac{1}{x}\right)\right) \\
&= -\frac{x(ex)^m}{2(1+m)} - \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \operatorname{Subst}\left(\int x^{-2-m}\cosh\left(2a+2bx^2\right)\, dx, x, \frac{1}{x}\right) \\
&= -\frac{x(ex)^m}{2(1+m)} - \frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \operatorname{Subst}\left(\int e^{-2a-2bx^2} x^{-2-m}\, dx, x, \frac{1}{x}\right) \\
&\quad - \frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \operatorname{Subst}\left(\int e^{2a+2bx^2} x^{-2-m}\, dx, x, \frac{1}{x}\right) \\
&= -\frac{x(ex)^m}{2(1+m)} + 2^{\frac{1}{2}(-5+m)} e^{2a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m \Gamma\left(\frac{1}{2}(-1-m), -\frac{2b}{x^2}\right) \\
&\quad + 2^{\frac{1}{2}(-5+m)} e^{-2a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m \Gamma\left(\frac{1}{2}(-1-m), \frac{2b}{x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx$$
$$=\frac{e^{-2a}x(ex)^m\left(-4e^{2a}+2^{\frac{1+m}{2}}e^{4a}(1+m)\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1}{2}(-1-m),-\frac{2b}{x^2}\right)+2^{\frac{1+m}{2}}(1+m)\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1}{2}(-1-\right.\right.}{8(1+m)}$$

```
[In] Integrate[(e*x)^m*Sinh[a + b/x^2]^2,x]
[Out] (x*(e*x)^m*(-4*E^(2*a) + 2^((1 + m)/2)*E^(4*a)*(1 + m)*(-(b/x^2))^((1 + m)/
2)*Gamma[(-1 - m)/2, (-2*b)/x^2] + 2^((1 + m)/2)*(1 + m)*(b/x^2)^((1 + m)/2
)*Gamma[(-1 - m)/2, (2*b)/x^2]))/(8*E^(2*a)*(1 + m))
```

Maple [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^2 dx$$

```
[In] int((e*x)^m*sinh(a+b/x^2)^2,x)
[Out] int((e*x)^m*sinh(a+b/x^2)^2,x)
```

Fricas [F]

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx=\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^2 dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^2,x, algorithm="fricas")
[Out] integral((e*x)^m*sinh((a*x^2 + b)/x^2)^2, x)
```

Sympy [F]

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx=\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate((e*x)**m*sinh(a+b/x**2)**2,x)
[Out] Integral((e*x)**m*sinh(a + b/x**2)**2, x)
```

Maxima [F]

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx = \int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^2\,dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*e^m*integrate(e^(m*log(x) + 2*a + 2*b/x^2), x) + 1/4*e^m*integrate(e^(m
*log(x) - 2*a - 2*b/x^2), x) - 1/2*(e*x)^(m + 1)/(e*(m + 1))
```

Giac [F]

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx = \int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^2\,dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx = \int \sinh\left(a+\frac{b}{x^2}\right)^2 (e\,x)^m\,dx$$

```
[In] int(sinh(a + b/x^2)^2*(e*x)^m,x)
```

```
[Out] int(sinh(a + b/x^2)^2*(e*x)^m, x)
```

3.55 $\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 87

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right) dx = \frac{1}{4}e^a\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),-\frac{b}{x^2}\right) - \frac{1}{4}e^{-a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),\frac{b}{x^2}\right)$$

```
[Out] 1/4*exp(a)*(-b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m,-b/x^2)-1/4*(b/x
^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m,b/x^2)/exp(a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5458, 5436, 2250}

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right) dx = \frac{1}{4}e^a x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}} (ex)^m\Gamma\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right) - \frac{1}{4}e^{-a}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}} (ex)^m\Gamma\left(\frac{1}{2}(-m-1),\frac{b}{x^2}\right)$$

```
[In] Int[(e*x)^m*Sinh[a + b/x^2],x]
```

```
[Out] (E^a*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, -(b/x^2)])/4 - ((b/
x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, b/x^2])/(4*E^a)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5436

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 5458

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^
p/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p]
 && ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\right)\\
&= \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-a-bx^2} x^{-2-m}\,dx, x, \frac{1}{x}\right)\\
&\quad -\frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{a+bx^2} x^{-2-m}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{4}e^a\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m), -\frac{b}{x^2}\right) - \frac{1}{4}e^{-a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m), \frac{b}{x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx = \frac{1}{4}e^{-a}x(ex)^m\left(e^{2a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1}{2}(-1-m), -\frac{b}{x^2}\right) - \left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1}{2}(-1-m), \frac{b}{x^2}\right)\right)$$

```
[In] Integrate[(e*x)^m*Sinh[a + b/x^2],x]
```

```
[Out] (x*(e*x)^m*(E^(2*a)*(-(b/x^2))^((1 + m)/2)*Gamma[(-1 - m)/2, -(b/x^2)] - (b
/x^2)^((1 + m)/2)*Gamma[(-1 - m)/2, b/x^2]))/(4*E^a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

method	result	size
meijerg	$\frac{(ex)^m b\,\text{hypergeom}\left(\left[\frac{1}{4}-\frac{m}{4}\right],\left[\frac{3}{2},\frac{5}{4}-\frac{m}{4}\right],\frac{b^2}{4x^4}\right)\cosh(a)}{(m-1)x}+\frac{(ex)^m x\,\text{hypergeom}\left(\left[-\frac{1}{4}-\frac{m}{4}\right],\left[\frac{1}{2},\frac{3}{4}-\frac{m}{4}\right],\frac{b^2}{4x^4}\right)\sinh(a)}{1+m}$	77

```
[In] int((e*x)^m*sinh(a+b/x^2),x,method=_RETURNVERBOSE)
[Out] (e*x)^m*b/(m-1)/x*hypergeom([1/4-1/4*m],[3/2,5/4-1/4*m],1/4*b^2/x^4)*cosh(a
)+(e*x)^m/(1+m)*x*hypergeom([-1/4-1/4*m],[1/2,3/4-1/4*m],1/4*b^2/x^4)*sinh(
a)
```

Fricas [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx=\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x^2),x, algorithm="fricas")
[Out] integral((e*x)^m*sinh((a*x^2 + b)/x^2), x)
```

Sympy [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx=\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate((e*x)**m*sinh(a+b/x**2),x)
[Out] Integral((e*x)**m*sinh(a + b/x**2), x)
```

Maxima [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx=\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x^2),x, algorithm="maxima")
[Out] integrate((e*x)^m*sinh(a + b/x^2), x)
```

Giac [F]

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx=\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate((e*x)^m*sinh(a+b/x^2),x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(a + b/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx=\int \sinh\left(a+\frac{b}{x^2}\right)\,(e\,x)^m\,dx$$

```
[In] int(sinh(a + b/x^2)*(e*x)^m,x)
[Out] int(sinh(a + b/x^2)*(e*x)^m, x)
```

3.56 $\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x^2}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x^2}\right) \, dx = x^{-m}(ex)^m \mathrm{Int}\left(x^m \mathrm{csch}\left(a+\frac{b}{x^2}\right),x\right)$$

```
[Out] (e*x)^m*Unintegrable(x^m*csch(a+b/x^2),x)/(x^m)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x^2}\right) \, dx = \int (ex)^m \mathrm{csch}\left(a+\frac{b}{x^2}\right) \, dx$$

```
[In] Int[(e*x)^m*Csch[a + b/x^2],x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b/x^2], x])/x^m
```

Rubi steps

$$\text{integral} = \left(x^{-m}(ex)^m\right) \int x^m \mathrm{csch}\left(a+\frac{b}{x^2}\right) \, dx$$

Mathematica [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (ex)^m \text{csch}\left(a+\frac{b}{x^2}\right)\,dx=\int (ex)^m \text{csch}\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] Integrate[(e*x)^m*Csch[a + b/x^2],x]
[Out] Integrate[(e*x)^m*Csch[a + b/x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x^2}\right)}dx$$

```
[In] int((e*x)^m/sinh(a+b/x^2),x)
[Out] int((e*x)^m/sinh(a+b/x^2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int (ex)^m \text{csch}\left(a+\frac{b}{x^2}\right)\,dx=\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x^2}\right)}\,dx$$

```
[In] integrate((e*x)^m/sinh(a+b/x^2),x, algorithm="fricas")
[Out] integral((e*x)^m/sinh((a*x^2 + b)/x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (ex)^m \text{csch}\left(a+\frac{b}{x^2}\right)\,dx=\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x^2}\right)}\,dx$$

```
[In] integrate((e*x)**m/sinh(a+b/x**2),x)
[Out] Integral((e*x)**m/sinh(a + b/x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x^2}\right)\,dx=\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x^2}\right)}\,dx$$

```
[In] integrate((e*x)^m/sinh(a+b/x^2),x, algorithm="maxima")
[Out] integrate((e*x)^m/sinh(a + b/x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x^2}\right)\,dx=\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x^2}\right)}\,dx$$

```
[In] integrate((e*x)^m/sinh(a+b/x^2),x, algorithm="giac")
[Out] integrate((e*x)^m/sinh(a + b/x^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x^2}\right)\,dx=\int \frac{(e\,x)^m}{\sinh\left(a+\frac{b}{x^2}\right)}\,dx$$

```
[In] int((e*x)^m/sinh(a + b/x^2),x)
[Out] int((e*x)^m/sinh(a + b/x^2), x)
```

3.57 $\int \frac{\sinh(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\cosh\left(\sqrt{x}\right)$$

```
[Out] 2*cosh(x^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5428, 2718}

$$\int \frac{\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\cosh\left(\sqrt{x}\right)$$

```
[In] Int[Sinh[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Cosh[Sqrt[x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
```

```
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\text{Subst}\left(\int \sinh(x)\,dx, x, \sqrt{x}\right) \\ &= 2\cosh\left(\sqrt{x}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\cosh\left(\sqrt{x}\right)$$

```
[In] Integrate[Sinh[Sqrt[x]]/Sqrt[x],x]
[Out] 2*Cosh[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\cosh\left(\sqrt{x}\right)$	7
default	$2\cosh\left(\sqrt{x}\right)$	7
meijerg	$-2\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(\sqrt{x}\right)}{\sqrt{\pi}}\right)$	19

```
[In] int(sinh(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*cosh(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\cosh\left(\sqrt{x}\right)$$

```
[In] integrate(sinh(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*cosh(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\cosh\left(\sqrt{x}\right)$$

```
[In] integrate(sinh(x**(1/2))/x**(1/2),x)
[Out] 2*cosh(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\cosh\left(\sqrt{x}\right)$$

```
[In] integrate(sinh(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*cosh(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = e^{\left(-\sqrt{x}\right)} + e^{\sqrt{x}}$$

```
[In] integrate(sinh(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] e^(-sqrt(x)) + e^sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\cosh\left(\sqrt{x}\right)$$

```
[In] int(sinh(x^(1/2))/x^(1/2),x)
[Out] 2*cosh(x^(1/2))
```

3.58 $\int x^2 \sinh(a+bx^n)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x^2 \sinh(a+bx^n)\,dx = -\frac{e^a x^3 (-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-bx^n\right)}{2n} + \frac{e^{-a} x^3 (bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},bx^n\right)}{2n}$$

```
[Out] -1/2*exp(a)*x^3*GAMMA(3/n,-b*x^n)/n/((-b*x^n)^(3/n))+1/2*x^3*GAMMA(3/n,b*x^
n)/exp(a)/n/((b*x^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5468, 2250}

$$\int x^2 \sinh(a+bx^n)\,dx = \frac{e^{-a} x^3 (bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},bx^n\right)}{2n} - \frac{e^a x^3 (-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-bx^n\right)}{2n}$$

```
[In] Int[x^2*Sinh[a + b*x^n],x]
```

```
[Out] -1/2*(E^a*x^3*Gamma[3/n, -(b*x^n)])/(n*(-(b*x^n))^(3/n)) + (x^3*Gamma[3/n,
b*x^n])/(2*E^a*n*(b*x^n)^(3/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5468

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\int e^{-a-bx^n}x^2\,dx\right) + \frac{1}{2}\int e^{a+bx^n}x^2\,dx$$
$$= -\frac{e^a x^3(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-bx^n\right)}{2n} + \frac{e^{-a}x^3(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},bx^n\right)}{2n}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int x^2\sinh\left(a+bx^n\right)\,dx = \frac{e^{-a}x^3\left(-e^{2a}(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-bx^n\right)+(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},bx^n\right)\right)}{2n}$$

```
[In] Integrate[x^2*Sinh[a + b*x^n],x]
[Out] (x^3*(-((E^(2*a)*Gamma[3/n, -(b*x^n)])/(-(b*x^n))^(3/n)) + Gamma[3/n, b*x^n
]/(b*x^n)^(3/n)))/(2*E^a*n)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{x^3\,\text{hypergeom}\left(\left[\frac{3}{2n}\right],\left[\frac{1}{2},1+\frac{3}{2n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{3}+\frac{x^{n+3}b\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{3}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{3}{2n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{n+3}$	77

```
[In] int(x^2*sinh(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*hypergeom([3/2/n],[1/2,1+3/2/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(n+3)*x^
(n+3)*b*hypergeom([1/2+3/2/n],[3/2,3/2+3/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Fricas [F]

$$\int x^2 \sinh(a + bx^n) \, dx = \int x^2 \sinh(bx^n + a) \, dx$$

```
[In] integrate(x^2*sinh(a+b*x^n),x, algorithm="fricas")
[Out] integral(x^2*sinh(b*x^n + a), x)
```

Sympy [F]

$$\int x^2 \sinh(a + bx^n) \, dx = \int x^2 \sinh(a + bx^n) \, dx$$

```
[In] integrate(x**2*sinh(a+b*x**n),x)
[Out] Integral(x**2*sinh(a + b*x**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x^2 \sinh(a + bx^n) \, dx = \frac{x^3 e^{(-a)} \Gamma\left(\frac{3}{n}, bx^n\right)}{2 (bx^n)^{\frac{3}{n}} n} - \frac{x^3 e^a \Gamma\left(\frac{3}{n}, -bx^n\right)}{2 (-bx^n)^{\frac{3}{n}} n}$$

```
[In] integrate(x^2*sinh(a+b*x^n),x, algorithm="maxima")
[Out] 1/2*x^3*e^(-a)*gamma(3/n, b*x^n)/((b*x^n)^(3/n)*n) - 1/2*x^3*e^a*gamma(3/n,
 -b*x^n)/((-b*x^n)^(3/n)*n)
```

Giac [F]

$$\int x^2 \sinh(a + bx^n) \, dx = \int x^2 \sinh(bx^n + a) \, dx$$

```
[In] integrate(x^2*sinh(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^2*sinh(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh(a + bx^n) \, dx = \int x^2 \sinh(a + b\,x^n) \, dx$$

```
[In] int(x^2*sinh(a + b*x^n),x)
[Out] int(x^2*sinh(a + b*x^n), x)
```

3.59 $\int x \sinh(a + bx^n) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 75

$$\int x \sinh(a + bx^n) \, dx = -\frac{e^a x^2 (-bx^n)^{-2/n} \Gamma\left(\frac{2}{n}, -bx^n\right)}{2n} + \frac{e^{-a} x^2 (bx^n)^{-2/n} \Gamma\left(\frac{2}{n}, bx^n\right)}{2n}$$

```
[Out] -1/2*exp(a)*x^2*GAMMA(2/n,-b*x^n)/n/((-b*x^n)^(2/n))+1/2*x^2*GAMMA(2/n,b*x^
n)/exp(a)/n/((b*x^n)^(2/n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5468, 2250}

$$\int x \sinh(a + bx^n) \, dx = \frac{e^{-a} x^2 (bx^n)^{-2/n} \Gamma\left(\frac{2}{n}, bx^n\right)}{2n} - \frac{e^a x^2 (-bx^n)^{-2/n} \Gamma\left(\frac{2}{n}, -bx^n\right)}{2n}$$

```
[In] Int[x*Sinh[a + b*x^n],x]
```

```
[Out] -1/2*(E^a*x^2*Gamma[2/n, -(b*x^n)])/(n*(-(b*x^n))^(2/n)) + (x^2*Gamma[2/n,
b*x^n])/(2*E^a*n*(b*x^n)^(2/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5468

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int e^{-a-bx^n}x\,dx\right)+\frac{1}{2}\int e^{a+bx^n}x\,dx\\ &= -\frac{e^a x^2(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-bx^n\right)}{2n}+\frac{e^{-a}x^2(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},bx^n\right)}{2n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int x\sinh\left(a+bx^n\right)\,dx=\frac{e^{-a}x^2\left(-e^{2a}(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-bx^n\right)+(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},bx^n\right)\right)}{2n}$$

```
[In] Integrate[x*Sinh[a + b*x^n],x]
[Out] (x^2*(-((E^(2*a)*Gamma[2/n, -(b*x^n)])/(-(b*x^n))^(2/n)) + Gamma[2/n, b*x^n
]/(b*x^n)^(2/n)))/(2*E^a*n)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

method	result	size
meijerg	$\frac{x^2\,\text{hypergeom}\left(\left[\frac{1}{n}\right],\left[\frac{1}{2},1+\frac{1}{n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{2}+\frac{x^{2+n}b\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{1}{n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{1}{n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{2+n}$	69

```
[In] int(x*sinh(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*hypergeom([1/n],[1/2,1+1/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(2+n)*x^(2+n
)*b*hypergeom([1/2+1/n],[3/2,3/2+1/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Fricas [F]

$$\int x\sinh\left(a+bx^n\right)\,dx=\int x\sinh\left(bx^n+a\right)\,dx$$

```
[In] integrate(x*sinh(a+b*x^n),x, algorithm="fricas")
[Out] integral(x*sinh(b*x^n + a), x)
```

Sympy [F]

$$\int x\sinh\left(a+bx^n\right)\,dx=\int x\sinh\left(a+bx^n\right)dx$$

```
[In] integrate(x*sinh(a+b*x**n),x)
[Out] Integral(x*sinh(a + b*x**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x\sinh\left(a+bx^n\right)\,dx=\frac{x^2e^{(-a)}\Gamma\left(\frac{2}{n},bx^n\right)}{2\left(bx^n\right)^{\frac{2}{n}}n}-\frac{x^2e^a\Gamma\left(\frac{2}{n},-bx^n\right)}{2\left(-bx^n\right)^{\frac{2}{n}}n}$$

```
[In] integrate(x*sinh(a+b*x^n),x, algorithm="maxima")
[Out] 1/2*x^2*e^(-a)*gamma(2/n, b*x^n)/((b*x^n)^(2/n)*n) - 1/2*x^2*e^a*gamma(2/n,
 -b*x^n)/((-b*x^n)^(2/n)*n)
```

Giac [F]

$$\int x\sinh\left(a+bx^n\right)\,dx=\int x\sinh\left(bx^n+a\right)\,dx$$

```
[In] integrate(x*sinh(a+b*x^n),x, algorithm="giac")
[Out] integrate(x*sinh(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sinh\left(a+b x^{n}\right)\,dx=\int x \sinh(a+b\,x^{n})\,dx$$

```
[In] int(x*sinh(a + b*x^n),x)
[Out] int(x*sinh(a + b*x^n), x)
```

3.60 $\int \sinh(a+bx^n)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \sinh(a+bx^n)\,dx=-\frac{e^a x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{2n}+\frac{e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{2n}$$

```
[Out] -1/2*exp(a)*x*GAMMA(1/n,-b*x^n)/n/((-b*x^n)^(1/n))+1/2*x*GAMMA(1/n,b*x^n)/e
xp(a)/n/((b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5414, 2239}

$$\int \sinh(a+bx^n)\,dx=\frac{e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{2n}-\frac{e^a x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{2n}$$

```
[In] Int[Sinh[a + b*x^n],x]
```

```
[Out] -1/2*(E^a*x*Gamma[n^(-1), -(b*x^n)])/(n*(-(b*x^n))^n^(-1)) + (x*Gamma[n^(-1
), b*x^n])/(2*E^a*n*(b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 5414

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int e^{-a-bx^n}\,dx\right)+\frac{1}{2}\int e^{a+bx^n}\,dx\\ &= -\frac{e^a x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-bx^n\right)}{2n}+\frac{e^{-a}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},bx^n\right)}{2n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \sinh\left(a+bx^n\right)\,dx = -\frac{e^a x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-bx^n\right)-e^{-a}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},bx^n\right)}{2n}$$

```
[In] Integrate[Sinh[a + b*x^n],x]
[Out] -1/2*((E^a*x*Gamma[n^(-1), -(b*x^n)])/(-(b*x^n))^n^(-1) - (x*Gamma[n^(-1),
b*x^n])/(E^a*(b*x^n)^n^(-1)))/n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

method	result	siz
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\sinh\left(a\right)+\frac{x^{n+1}b\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{n+1}$	74

```
[In] int(sinh(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] x*hypergeom([1/2/n],[1/2,1+1/2/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(n+1)*x^(n+1)*
b*hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Fricas [F]

$$\int \sinh(a+bx^n)\,dx=\int \sinh(bx^n+a)\,dx$$

```
[In] integrate(sinh(a+b*x^n),x, algorithm="fricas")
[Out] integral(sinh(b*x^n + a), x)
```

Sympy [F]

$$\int \sinh(a+bx^n)\,dx=\int \sinh(a+bx^n)\,dx$$

```
[In] integrate(sinh(a+b*x**n),x)
[Out] Integral(sinh(a + b*x**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \sinh(a+bx^n)\,dx=\frac{xe^{(-a)}\Gamma\left(\frac{1}{n},bx^n\right)}{2\,(bx^n)^{\left(\frac{1}{n}\right)}n}-\frac{xe^{a}\Gamma\left(\frac{1}{n},-bx^n\right)}{2\,(-bx^n)^{\left(\frac{1}{n}\right)}n}$$

```
[In] integrate(sinh(a+b*x^n),x, algorithm="maxima")
[Out] 1/2*x*e^(-a)*gamma(1/n, b*x^n)/((b*x^n)^(1/n)*n) - 1/2*x*e^a*gamma(1/n, -b*
x^n)/((-b*x^n)^(1/n)*n)
```

Giac [F]

$$\int \sinh(a+bx^n)\,dx=\int \sinh(bx^n+a)\,dx$$

```
[In] integrate(sinh(a+b*x^n),x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh\left(a+bx^n\right)\,dx=\int \sinh(a+b\,x^n)\,dx$$

```
[In] int(sinh(a + b*x^n),x)
[Out] int(sinh(a + b*x^n), x)
```

3.61 $\int \frac{\sinh(a+bx^n)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\sinh(a+bx^n)}{x}\,dx = \frac{\text{Chi}(bx^n)\sinh(a)}{n} + \frac{\cosh(a)\text{Shi}(bx^n)}{n}$$

```
[Out] cosh(a)*Shi(b*x^n)/n+Chi(b*x^n)*sinh(a)/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5426, 5425, 5424}

$$\int \frac{\sinh(a+bx^n)}{x}\,dx = \frac{\sinh(a)\text{Chi}(bx^n)}{n} + \frac{\cosh(a)\text{Shi}(bx^n)}{n}$$

```
[In] Int[Sinh[a + b*x^n]/x,x]
```

```
[Out] (CoshIntegral[b*x^n]*Sinh[a])/n + (Cosh[a]*SinhIntegral[b*x^n])/n
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5426

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \cosh(a)\int \frac{\sinh(bx^n)}{x}\,dx + \sinh(a)\int \frac{\cosh(bx^n)}{x}\,dx \\ &= \frac{\text{Chi}(bx^n)\sinh(a)}{n} + \frac{\cosh(a)\text{Shi}(bx^n)}{n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sinh(a+bx^n)}{x}\,dx = \frac{\text{Chi}(bx^n)\sinh(a) + \cosh(a)\text{Shi}(bx^n)}{n}$$

```
[In] Integrate[Sinh[a + b*x^n]/x,x]
[Out] (CoshIntegral[b*x^n]*Sinh[a] + Cosh[a]*SinhIntegral[b*x^n])/n
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{e^{-a}\,\text{Ei}_1(b\,x^n)}{2n} - \frac{e^{a}\,\text{Ei}_1(-b\,x^n)}{2n}$	33
meijerg	$\frac{\sqrt{\pi}\left(\frac{2\ \text{Chi}(b\,x^n)-2\ln(b\,x^n)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2n\ln(x)+2\ln(ib)}{\sqrt{\pi}}\right)\sinh(a)}{2n} + \frac{\cosh(a)\,\text{Shi}(b\,x^n)}{n}$	68

```
[In] int(sinh(a+b*x^n)/x,x,method=_RETURNVERBOSE)
[Out] 1/2/n*exp(-a)*Ei(1,b*x^n)-1/2/n*exp(a)*Ei(1,-b*x^n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(25) = 50$.

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{\sinh(a + bx^n)}{x}\,dx$$
$$= \frac{(\cosh(a) + \sinh(a))\mathrm{Ei}(b\cosh(n\log(x)) + b\sinh(n\log(x))) - (\cosh(a) - \sinh(a))\mathrm{Ei}(-b\cosh(n\log(x)) -}{2\,n}$$

```
[In] integrate(sinh(a+b*x^n)/x,x, algorithm="fricas")
[Out] 1/2*((cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) - (cosh(a)
 - sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))))/n
```

Sympy [F]

$$\int \frac{\sinh(a + bx^n)}{x}\,dx = \int \frac{\sinh(a + bx^n)}{x}\,dx$$

```
[In] integrate(sinh(a+b*x**n)/x,x)
[Out] Integral(sinh(a + b*x**n)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{\sinh(a + bx^n)}{x}\,dx = -\frac{\mathrm{Ei}(-bx^n)\,e^{(-a)}}{2\,n} + \frac{\mathrm{Ei}(bx^n)\,e^a}{2\,n}$$

```
[In] integrate(sinh(a+b*x^n)/x,x, algorithm="maxima")
[Out] -1/2*Ei(-b*x^n)*e^(-a)/n + 1/2*Ei(b*x^n)*e^a/n
```

Giac [F]

$$\int \frac{\sinh(a+bx^n)}{x}\,dx = \int \frac{\sinh(bx^n+a)}{x}\,dx$$

```
[In] integrate(sinh(a+b*x^n)/x,x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx^n)}{x}\,dx = \int \frac{\sinh(a+b\,x^n)}{x}\,dx$$

```
[In] int(sinh(a + b*x^n)/x,x)
[Out] int(sinh(a + b*x^n)/x, x)
```

3.62 $\int \frac{\sinh(a+bx^n)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 71

$$\int \frac{\sinh(a+bx^n)}{x^2}\,dx = -\frac{e^a(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-bx^n\right)}{2nx} + \frac{e^{-a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},bx^n\right)}{2nx}$$

```
[Out] -1/2*exp(a)*(-b*x^n)^(1/n)*GAMMA(-1/n,-b*x^n)/n/x+1/2*(b*x^n)^(1/n)*GAMMA(-
1/n,b*x^n)/exp(a)/n/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5468, 2250}

$$\int \frac{\sinh(a+bx^n)}{x^2}\,dx = \frac{e^{-a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},bx^n\right)}{2nx} - \frac{e^a(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-bx^n\right)}{2nx}$$

```
[In] Int[Sinh[a + b*x^n]/x^2,x]
```

```
[Out] -1/2*(E^a*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)])/(n*x) + ((b*x^n)^n^(-
1)*Gamma[-n^(-1), b*x^n])/(2*E^a*n*x)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5468

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int \frac{e^{-a-bx^n}}{x^2}\,dx\right)+\frac{1}{2}\int \frac{e^{a+bx^n}}{x^2}\,dx\\ &= -\frac{e^a(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-bx^n\right)}{2nx}+\frac{e^{-a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},bx^n\right)}{2nx}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{\sinh(a+bx^n)}{x^2}\,dx = -\frac{e^a(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-bx^n\right)-e^{-a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},bx^n\right)}{2nx}$$

```
[In] Integrate[Sinh[a + b*x^n]/x^2,x]
[Out] -1/2*(E^a*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)] - ((b*x^n)^n^(-1)*Gamm
a[-n^(-1), b*x^n])/E^a)/(n*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

method	result	size
meijerg	$-\frac{\text{hypergeom}\left(\left[-\frac{1}{2n}\right],\left[\frac{1}{2},1-\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{x}+\frac{x^{-1+n}b\,\text{hypergeom}\left(\left[\frac{1}{2}-\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}-\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{-1+n}$	77

```
[In] int(sinh(a+b*x^n)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x*hypergeom([-1/2/n],[1/2,1-1/2/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(-1+n)*x^(
-1+n)*b*hypergeom([1/2-1/2/n],[3/2,3/2-1/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Fricas [F]

$$\int \frac{\sinh(a + bx^n)}{x^2} dx = \int \frac{\sinh(bx^n + a)}{x^2} dx$$

```
[In] integrate(sinh(a+b*x^n)/x^2,x, algorithm="fricas")
[Out] integral(sinh(b*x^n + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sinh(a + bx^n)}{x^2} dx = \int \frac{\sinh(a + bx^n)}{x^2} dx$$

```
[In] integrate(sinh(a+b*x**n)/x**2,x)
[Out] Integral(sinh(a + b*x**n)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{\sinh(a + bx^n)}{x^2} dx = \frac{(bx^n)^{\left(\frac{1}{n}\right)} e^{(-a)}\Gamma\left(-\frac{1}{n}, bx^n\right)}{2 nx} - \frac{(-bx^n)^{\left(\frac{1}{n}\right)} e^{a}\Gamma\left(-\frac{1}{n}, -bx^n\right)}{2 nx}$$

```
[In] integrate(sinh(a+b*x^n)/x^2,x, algorithm="maxima")
[Out] 1/2*(b*x^n)^(1/n)*e^(-a)*gamma(-1/n, b*x^n)/(n*x) - 1/2*(-b*x^n)^(1/n)*e^a*
gamma(-1/n, -b*x^n)/(n*x)
```

Giac [F]

$$\int \frac{\sinh(a + bx^n)}{x^2} dx = \int \frac{\sinh(bx^n + a)}{x^2} dx$$

```
[In] integrate(sinh(a+b*x^n)/x^2,x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx^n)}{x^2}\,dx = \int \frac{\sinh(a+b\,x^n)}{x^2}\,dx$$

```
[In] int(sinh(a + b*x^n)/x^2,x)
[Out] int(sinh(a + b*x^n)/x^2, x)
```

3.63 $\int \frac{\sinh(a+bx^n)}{x^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int \frac{\sinh(a+bx^n)}{x^3} dx = -\frac{e^a(-bx^n)^{2/n}\Gamma\left(-\frac{2}{n},-bx^n\right)}{2nx^2} + \frac{e^{-a}(bx^n)^{2/n}\Gamma\left(-\frac{2}{n},bx^n\right)}{2nx^2}$$

```
[Out] -1/2*exp(a)*(-b*x^n)^(2/n)*GAMMA(-2/n,-b*x^n)/n/x^2+1/2*(b*x^n)^(2/n)*GAMMA
(-2/n,b*x^n)/exp(a)/n/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5468, 2250}

$$\int \frac{\sinh(a+bx^n)}{x^3} dx = \frac{e^{-a}(bx^n)^{2/n}\Gamma\left(-\frac{2}{n},bx^n\right)}{2nx^2} - \frac{e^a(-bx^n)^{2/n}\Gamma\left(-\frac{2}{n},-bx^n\right)}{2nx^2}$$

```
[In] Int[Sinh[a + b*x^n]/x^3,x]
```

```
[Out] -1/2*(E^a*(-(b*x^n))^(2/n)*Gamma[-2/n, -(b*x^n)])/(n*x^2) + ((b*x^n)^(2/n)*
Gamma[-2/n, b*x^n])/(2*E^a*n*x^2)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5468

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int \frac{e^{-a-bx^n}}{x^3}\,dx\right)+\frac{1}{2}\int \frac{e^{a+bx^n}}{x^3}\,dx\\ &= -\frac{e^a(-bx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},-bx^n\right)}{2nx^2}+\frac{e^{-a}(bx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},bx^n\right)}{2nx^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{\sinh(a+bx^n)}{x^3}\,dx = -\frac{e^a(-bx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},-bx^n\right)-e^{-a}(bx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},bx^n\right)}{2nx^2}$$

```
[In] Integrate[Sinh[a + b*x^n]/x^3,x]
[Out] -1/2*(E^a*(-(b*x^n))^(2/n)*Gamma[-2/n, -(b*x^n)] - ((b*x^n)^(2/n)*Gamma[-2/
n, b*x^n])/E^a)/(n*x^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.59 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{1}{n}\right],\left[\frac{1}{2},1-\frac{1}{n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{2x^2}+\frac{x^{-2+n}b\operatorname{hypergeom}\left(\left[\frac{1}{2}-\frac{1}{n}\right],\left[\frac{3}{2},\frac{3}{2}-\frac{1}{n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{-2+n}$	77

```
[In] int(sinh(a+b*x^n)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2/x^2*hypergeom([-1/n],[1/2,1-1/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(-2+n)*x^(
-2+n)*b*hypergeom([1/2-1/n],[3/2,3/2-1/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Fricas [F]

$$\int \frac{\sinh(a+bx^n)}{x^3}\,dx = \int \frac{\sinh(bx^n+a)}{x^3}\,dx$$

[In] integrate(sinh(a+b*x^n)/x^3,x, algorithm="fricas")

[Out] integral(sinh(b*x^n + a)/x^3, x)

Sympy [F]

$$\int \frac{\sinh(a+bx^n)}{x^3}\,dx = \int \frac{\sinh(a+bx^n)}{x^3}\,dx$$

[In] integrate(sinh(a+b*x**n)/x**3,x)

[Out] Integral(sinh(a + b*x**n)/x**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{\sinh(a+bx^n)}{x^3}\,dx = \frac{(bx^n)^{\frac{2}{n}}\,e^{(-a)}\Gamma\left(-\frac{2}{n},bx^n\right)}{2\,nx^2} - \frac{(-bx^n)^{\frac{2}{n}}\,e^{a}\Gamma\left(-\frac{2}{n},-bx^n\right)}{2\,nx^2}$$

[In] integrate(sinh(a+b*x^n)/x^3,x, algorithm="maxima")

[Out] 1/2*(b*x^n)^(2/n)*e^(-a)*gamma(-2/n, b*x^n)/(n*x^2) - 1/2*(-b*x^n)^(2/n)*e^a*gamma(-2/n, -b*x^n)/(n*x^2)

Giac [F]

$$\int \frac{\sinh(a+bx^n)}{x^3}\,dx = \int \frac{\sinh(bx^n+a)}{x^3}\,dx$$

[In] integrate(sinh(a+b*x^n)/x^3,x, algorithm="giac")

[Out] integrate(sinh(b*x^n + a)/x^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(a+bx^n\right)}{x^3}\,dx=\int \frac{\sinh(a+b\,x^n)}{x^3}\,dx$$

```
[In] int(sinh(a + b*x^n)/x^3,x)
[Out] int(sinh(a + b*x^n)/x^3, x)
```

3.64 $\int x^2 \sinh^2(a + bx^n) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int x^2 \sinh^2(a + bx^n) \, dx = -\frac{x^3}{6} - \frac{2^{-2-\frac{3}{n}} e^{2a} x^3 (-bx^n)^{-3/n} \Gamma\left(\frac{3}{n}, -2bx^n\right)}{n} - \frac{2^{-2-\frac{3}{n}} e^{-2a} x^3 (bx^n)^{-3/n} \Gamma\left(\frac{3}{n}, 2bx^n\right)}{n}$$

```
[Out] -1/6*x^3-2^(-2-3/n)*exp(2*a)*x^3*GAMMA(3/n,-2*b*x^n)/n/((-b*x^n)^(3/n))-2^(
-2-3/n)*x^3*GAMMA(3/n,2*b*x^n)/exp(2*a)/n/((b*x^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5470, 5469, 2250}

$$\int x^2 \sinh^2(a + bx^n) \, dx = -\frac{e^{2a} 2^{-\frac{3}{n}-2} x^3 (-bx^n)^{-3/n} \Gamma\left(\frac{3}{n}, -2bx^n\right)}{n} - \frac{e^{-2a} 2^{-\frac{3}{n}-2} x^3 (bx^n)^{-3/n} \Gamma\left(\frac{3}{n}, 2bx^n\right)}{n} - \frac{x^3}{6}$$

```
[In] Int[x^2*Sinh[a + b*x^n]^2,x]
```

```
[Out] -1/6*x^3 - (2^(-2 - 3/n)*E^(2*a)*x^3*Gamma[3/n, -2*b*x^n])/(n*(-(b*x^n))^(3
/n)) - (2^(-2 - 3/n)*x^3*Gamma[3/n, 2*b*x^n])/(E^(2*a)*n*(b*x^n)^(3/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5469

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{x^2}{2}+\frac{1}{2}x^2\cosh\left(2a+2bx^n\right)\right)\,dx \\
&= -\frac{x^3}{6}+\frac{1}{2}\int x^2\cosh\left(2a+2bx^n\right)\,dx \\
&= -\frac{x^3}{6}+\frac{1}{4}\int e^{-2a-2bx^n}x^2\,dx+\frac{1}{4}\int e^{2a+2bx^n}x^2\,dx \\
&= -\frac{x^3}{6}-\frac{2^{-2-\frac{3}{n}}e^{2a}x^3(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-2bx^n\right)}{n}-\frac{2^{-2-\frac{3}{n}}e^{-2a}x^3(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},2bx^n\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int x^2\sinh^2\left(a+bx^n\right)\,dx = -\frac{x^3\left(2n+3\ 8^{-1/n}e^{2a}(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-2bx^n\right)+3\ 8^{-1/n}e^{-2a}(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},2bx^n\right)\right)}{12n}$$

```
[In] Integrate[x^2*Sinh[a + b*x^n]^2,x]
[Out] -1/12*(x^3*(2*n + (3*E^(2*a)*Gamma[3/n, -2*b*x^n])/(8^n^(-1)*(-(b*x^n))^(3/
n)) + (3*Gamma[3/n, 2*b*x^n])/(8^n^(-1)*E^(2*a)*(b*x^n)^(3/n))))/n
```

Maple [F]

$$\int x^2 \sinh(a+b\,x^n)^2\,dx$$

```
[In] int(x^2*sinh(a+b*x^n)^2,x)
[Out] int(x^2*sinh(a+b*x^n)^2,x)
```

Fricas [F]

$$\int x^2 \sinh^2(a+bx^n)\ dx = \int x^2 \sinh(bx^n+a)^2\ dx$$

```
[In] integrate(x^2*sinh(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral(x^2*sinh(b*x^n + a)^2, x)
```

Sympy [F]

$$\int x^2 \sinh^2(a+bx^n)\ dx = \int x^2 \sinh^2(a+bx^n)\,dx$$

```
[In] integrate(x**2*sinh(a+b*x**n)**2,x)
[Out] Integral(x**2*sinh(a + b*x**n)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int x^2 \sinh^2(a+bx^n)\ dx = -\frac{1}{6}\,x^3 - \frac{x^3 e^{(-2\,a)}\Gamma\left(\frac{3}{n}, 2\,bx^n\right)}{4\,\left(2\,bx^n\right)^{\frac{3}{n}}\,n} - \frac{x^3 e^{(2\,a)}\Gamma\left(\frac{3}{n}, -2\,bx^n\right)}{4\,\left(-2\,bx^n\right)^{\frac{3}{n}}\,n}$$

```
[In] integrate(x^2*sinh(a+b*x^n)^2,x, algorithm="maxima")
[Out] -1/6*x^3 - 1/4*x^3*e^(-2*a)*gamma(3/n, 2*b*x^n)/((2*b*x^n)^(3/n)*n) - 1/4*x
^3*e^(2*a)*gamma(3/n, -2*b*x^n)/((-2*b*x^n)^(3/n)*n)
```

Giac [F]

$$\int x^2 \sinh^2(a+bx^n)\,dx = \int x^2 \sinh\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(x^2*sinh(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^2*sinh(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh^2(a+bx^n)\,dx = \int x^2 \sinh(a+b\,x^n)^2\,dx$$

```
[In] int(x^2*sinh(a + b*x^n)^2,x)
[Out] int(x^2*sinh(a + b*x^n)^2, x)
```

3.65 $\int x \sinh^2(a+bx^n)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 99

$$\int x \sinh^2(a+bx^n)\,dx = -\frac{x^2}{4} - \frac{4^{-1-\frac{1}{n}}e^{2a}x^2(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-2bx^n\right)}{n} - \frac{4^{-1-\frac{1}{n}}e^{-2a}x^2(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},2bx^n\right)}{n}$$

```
[Out] -1/4*x^2-4^(-1-1/n)*exp(2*a)*x^2*GAMMA(2/n,-2*b*x^n)/n/((-b*x^n)^(2/n))-4^(
-1-1/n)*x^2*GAMMA(2/n,2*b*x^n)/exp(2*a)/n/((b*x^n)^(2/n))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5470, 5469, 2250}

$$\int x \sinh^2(a+bx^n)\,dx = -\frac{e^{2a}4^{-\frac{1}{n}-1}x^2(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-2bx^n\right)}{n} - \frac{e^{-2a}4^{-\frac{1}{n}-1}x^2(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},2bx^n\right)}{n} - \frac{x^2}{4}$$

```
[In] Int[x*Sinh[a + b*x^n]^2,x]
```

```
[Out] -1/4*x^2 - (4^(-1 - n^(-1))*E^(2*a)*x^2*Gamma[2/n, -2*b*x^n])/(n*(-(b*x^n))
^(2/n)) - (4^(-1 - n^(-1))*x^2*Gamma[2/n, 2*b*x^n])/(E^(2*a)*n*(b*x^n)^(2/n
))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5469

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{x}{2}+\frac{1}{2}x\cosh\left(2a+2bx^n\right)\right)\,dx \\
&= -\frac{x^2}{4}+\frac{1}{2}\int x\cosh\left(2a+2bx^n\right)\,dx \\
&= -\frac{x^2}{4}+\frac{1}{4}\int e^{-2a-2bx^n}x\,dx+\frac{1}{4}\int e^{2a+2bx^n}x\,dx \\
&= -\frac{x^2}{4}-\frac{4^{-1-\frac{1}{n}}e^{2a}x^2(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-2bx^n\right)}{n}-\frac{4^{-1-\frac{1}{n}}e^{-2a}x^2(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},2bx^n\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int x\sinh^2\left(a+bx^n\right)\,dx = -\frac{x^2\left(n+4^{-1/n}e^{2a}(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-2bx^n\right)+4^{-1/n}e^{-2a}(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},2bx^n\right)\right)}{4n}$$

```
[In] Integrate[x*Sinh[a + b*x^n]^2,x]
[Out] -1/4*(x^2*(n + (E^(2*a)*Gamma[2/n, -2*b*x^n])/(4^n^(-1)*(-(b*x^n))^(2/n)) +
 Gamma[2/n, 2*b*x^n]/(4^n^(-1)*E^(2*a)*(b*x^n)^(2/n))))/n
```

Maple [F]

$$\int x \sinh \left(a + b\,x^{n}\right)^{2} dx$$

```
[In] int(x*sinh(a+b*x^n)^2,x)
[Out] int(x*sinh(a+b*x^n)^2,x)
```

Fricas [F]

$$\int x \sinh^{2}\left(a + bx^{n}\right)\,dx = \int x \sinh\left(bx^{n} + a\right)^{2}\,dx$$

```
[In] integrate(x*sinh(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral(x*sinh(b*x^n + a)^2, x)
```

Sympy [F]

$$\int x \sinh^{2}\left(a + bx^{n}\right)\,dx = \int x \sinh^{2}\left(a + bx^{n}\right)dx$$

```
[In] integrate(x*sinh(a+b*x**n)**2,x)
[Out] Integral(x*sinh(a + b*x**n)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int x \sinh^{2}\left(a + bx^{n}\right)\,dx = -\frac{1}{4}\,x^{2} - \frac{x^{2}e^{(-2\,a)}\Gamma\left(\frac{2}{n}, 2\,bx^{n}\right)}{4\,\left(2\,bx^{n}\right)^{\frac{2}{n}}\,n} - \frac{x^{2}e^{(2\,a)}\Gamma\left(\frac{2}{n}, -2\,bx^{n}\right)}{4\,\left(-2\,bx^{n}\right)^{\frac{2}{n}}\,n}$$

```
[In] integrate(x*sinh(a+b*x^n)^2,x, algorithm="maxima")
[Out] -1/4*x^2 - 1/4*x^2*e^(-2*a)*gamma(2/n, 2*b*x^n)/((2*b*x^n)^(2/n)*n) - 1/4*x
^2*e^(2*a)*gamma(2/n, -2*b*x^n)/((-2*b*x^n)^(2/n)*n)
```

Giac [F]

$$\int x\sinh^2(a+bx^n)\,dx = \int x\sinh(bx^n+a)^2\,dx$$

```
[In] integrate(x*sinh(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x*sinh(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sinh^2(a+bx^n)\,dx = \int x\sinh(a+b\,x^n)^2\,dx$$

```
[In] int(x*sinh(a + b*x^n)^2,x)
[Out] int(x*sinh(a + b*x^n)^2, x)
```

3.66 $\int \sinh^2(a+bx^n)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \sinh^2(a+bx^n)\,dx = -\frac{x}{2}-\frac{2^{-2-\frac{1}{n}}e^{2a}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2bx^n\right)}{n} - \frac{2^{-2-\frac{1}{n}}e^{-2a}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2bx^n\right)}{n}$$

```
[Out] -1/2*x-2^(-2-1/n)*exp(2*a)*x*GAMMA(1/n,-2*b*x^n)/n/((-b*x^n)^(1/n))-2^(-2-1
/n)*x*GAMMA(1/n,2*b*x^n)/exp(2*a)/n/((b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5416, 5415, 2239}

$$\int \sinh^2(a+bx^n)\,dx = -\frac{e^{2a}2^{-\frac{1}{n}-2}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2bx^n\right)}{n} - \frac{e^{-2a}2^{-\frac{1}{n}-2}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2bx^n\right)}{n}-\frac{x}{2}$$

```
[In] Int[Sinh[a + b*x^n]^2,x]
```

```
[Out] -1/2*x - (2^(-2 - n^(-1))*E^(2*a)*x*Gamma[n^(-1), -2*b*x^n])/(n*(-(b*x^n))^
n^(-1)) - (2^(-2 - n^(-1))*x*Gamma[n^(-1), 2*b*x^n])/(E^(2*a)*n*(b*x^n)^n^(
-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
```

```
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 5415

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rule 5416

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}+\frac{1}{2}\cosh\left(2a+2bx^n\right)\right) dx \\
&= -\frac{x}{2}+\frac{1}{2}\int \cosh\left(2a+2bx^n\right) dx \\
&= -\frac{x}{2}+\frac{1}{4}\int e^{-2a-2bx^n}\, dx+\frac{1}{4}\int e^{2a+2bx^n}\, dx \\
&= -\frac{x}{2}-\frac{2^{-2-\frac{1}{n}}e^{2a}x\left(-bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{2^{-2-\frac{1}{n}}e^{-2a}x\left(bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2bx^n\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \sinh^2\left(a+bx^n\right) dx = -\frac{x\left(2n+2^{-1/n}e^{2a}\left(-bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2bx^n\right)+2^{-1/n}e^{-2a}\left(bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2bx^n\right)\right)}{4n}$$

```
[In] Integrate[Sinh[a + b*x^n]^2,x]
[Out] -1/4*(x*(2*n + (E^(2*a)*Gamma[n^(-1), -2*b*x^n])/(2^n^(-1)*(-(b*x^n))^n^(-1
)) + Gamma[n^(-1), 2*b*x^n]/(2^n^(-1)*E^(2*a)*(b*x^n)^n^(-1))))/n
```

Maple [F]

$$\int \sinh \left(a + b\, x^{n}\right)^{2} dx$$

```
[In] int(sinh(a+b*x^n)^2,x)
```

```
[Out] int(sinh(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \sinh^{2}\left(a + bx^{n}\right)\, dx = \int \sinh\left(bx^{n} + a\right)^{2}\, dx$$

```
[In] integrate(sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)^2, x)
```

Sympy [F]

$$\int \sinh^{2}\left(a + bx^{n}\right)\, dx = \int \sinh^{2}\left(a + bx^{n}\right) dx$$

```
[In] integrate(sinh(a+b*x**n)**2,x)
```

```
[Out] Integral(sinh(a + b*x**n)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \sinh^{2}\left(a + bx^{n}\right)\, dx = -\frac{1}{2}\, x - \frac{xe^{(-2\, a)}\Gamma\left(\frac{1}{n}, 2\, bx^{n}\right)}{4\, \left(2\, bx^{n}\right)^{\left(\frac{1}{n}\right)}\, n} - \frac{xe^{(2\, a)}\Gamma\left(\frac{1}{n}, -2\, bx^{n}\right)}{4\, \left(-2\, bx^{n}\right)^{\left(\frac{1}{n}\right)}\, n}$$

```
[In] integrate(sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/4*x*e^(-2*a)*gamma(1/n, 2*b*x^n)/((2*b*x^n)^(1/n)*n) - 1/4*x*e^(
2*a)*gamma(1/n, -2*b*x^n)/((-2*b*x^n)^(1/n)*n)
```

Giac [F]

$$\int \sinh^2(a+bx^n)\,dx = \int \sinh(bx^n+a)^2\,dx$$

```
[In] integrate(sinh(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2(a+bx^n)\,dx = \int \sinh(a+b\,x^n)^2\,dx$$

```
[In] int(sinh(a + b*x^n)^2,x)
[Out] int(sinh(a + b*x^n)^2, x)
```

3.67 $\int \frac{\sinh^2(a+bx^n)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{\sinh^2(a+bx^n)}{x} dx = \frac{\cosh(2a)\mathrm{Chi}(2bx^n)}{2n} - \frac{\log(x)}{2} + \frac{\sinh(2a)\mathrm{Shi}(2bx^n)}{2n}$$

```
[Out] 1/2*Chi(2*b*x^n)*cosh(2*a)/n-1/2*ln(x)+1/2*Shi(2*b*x^n)*sinh(2*a)/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5470, 5427, 5425, 5424}

$$\int \frac{\sinh^2(a+bx^n)}{x} dx = \frac{\cosh(2a)\mathrm{Chi}(2bx^n)}{2n} + \frac{\sinh(2a)\mathrm{Shi}(2bx^n)}{2n} - \frac{\log(x)}{2}$$

```
[In] Int[Sinh[a + b*x^n]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^n])/(2*n) - Log[x]/2 + (Sinh[2*a]*SinhIntegra
l[2*b*x^n])/(2*n)
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2x}+\frac{\cosh\left(2a+2bx^n\right)}{2x}\right) dx \\
&= -\frac{\log(x)}{2}+\frac{1}{2}\int \frac{\cosh\left(2a+2bx^n\right)}{x}\, dx \\
&= -\frac{\log(x)}{2}+\frac{1}{2}\cosh(2a)\int \frac{\cosh\left(2bx^n\right)}{x}\, dx+\frac{1}{2}\sinh(2a)\int \frac{\sinh\left(2bx^n\right)}{x}\, dx \\
&= \frac{\cosh(2a)\mathrm{Chi}(2bx^n)}{2n}-\frac{\log(x)}{2}+\frac{\sinh(2a)\mathrm{Shi}(2bx^n)}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^2\left(a+bx^n\right)}{x}\, dx = -\frac{\log(x)}{2}+\frac{\cosh(2a)\mathrm{Chi}(2bx^n)+\sinh(2a)\mathrm{Shi}(2bx^n)}{2n}$$

```
[In] Integrate[Sinh[a + b*x^n]^2/x,x]
[Out] -1/2*Log[x] + (Cosh[2*a]*CoshIntegral[2*b*x^n] + Sinh[2*a]*SinhIntegral[2*b
*x^n])/(2*n)
```

Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{\ln(x)}{2}-\frac{\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2b\,x^n)}{4n}-\frac{\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2b\,x^n)}{4n}$	40

```
[In] int(sinh(a+b*x^n)^2/x,x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x)-1/4/n*exp(-2*a)*Ei(1,2*b*x^n)-1/4/n*exp(2*a)*Ei(1,-2*b*x^n)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{\sinh^2(a+bx^n)}{x}\,dx = \frac{(\cosh(2a)+\sinh(2a))\mathrm{Ei}(2b\cosh(n\log(x))+2b\sinh(n\log(x)))+(\cosh(2a)-\sinh(2a))\mathrm{Ei}(-2b\cosh(}{4n}$$

```
[In] integrate(sinh(a+b*x^n)^2/x,x, algorithm="fricas")
[Out] 1/4*((cosh(2*a) + sinh(2*a))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) +
(cosh(2*a) - sinh(2*a))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) - 2*n*
log(x))/n
```

Sympy [F]

$$\int \frac{\sinh^2(a+bx^n)}{x}\,dx = \int \frac{\sinh^2(a+bx^n)}{x}\,dx$$

```
[In] integrate(sinh(a+b*x**n)**2/x,x)
[Out] Integral(sinh(a + b*x**n)**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^2(a+bx^n)}{x}\,dx = \frac{\mathrm{Ei}(2bx^n)\,e^{(2a)}}{4n} + \frac{\mathrm{Ei}(-2bx^n)\,e^{(-2a)}}{4n} - \frac{1}{2}\log(x)$$

```
[In] integrate(sinh(a+b*x^n)^2/x,x, algorithm="maxima")
[Out] 1/4*Ei(2*b*x^n)*e^(2*a)/n + 1/4*Ei(-2*b*x^n)*e^(-2*a)/n - 1/2*log(x)
```

Giac [F]

$$\int \frac{\sinh^2(a+bx^n)}{x}\,dx = \int \frac{\sinh(bx^n+a)^2}{x}\,dx$$

```
[In] integrate(sinh(a+b*x^n)^2/x,x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx^n)}{x}\,dx = \int \frac{\sinh(a+b\,x^n)^2}{x}\,dx$$

```
[In] int(sinh(a + b*x^n)^2/x,x)
[Out] int(sinh(a + b*x^n)^2/x, x)
```

3.68 $\int \frac{\sinh^2(a+bx^n)}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{\sinh^2(a+bx^n)}{x^2} dx = \frac{1}{2x} - \frac{2^{-2+\frac{1}{n}}e^{2a}(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-2bx^n\right)}{nx} - \frac{2^{-2+\frac{1}{n}}e^{-2a}(bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},2bx^n\right)}{nx}$$

```
[Out] 1/2/x-2^(-2+1/n)*exp(2*a)*(-b*x^n)^(1/n)*GAMMA(-1/n,-2*b*x^n)/n/x-2^(-2+1/n
)*(b*x^n)^(1/n)*GAMMA(-1/n,2*b*x^n)/exp(2*a)/n/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5470, 5469, 2250}

$$\int \frac{\sinh^2(a+bx^n)}{x^2} dx = -\frac{e^{2a}2^{\frac{1}{n}-2}(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-2bx^n\right)}{nx} - \frac{e^{-2a}2^{\frac{1}{n}-2}(bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},2bx^n\right)}{nx} + \frac{1}{2x}$$

```
[In] Int[Sinh[a + b*x^n]^2/x^2,x]
```

```
[Out] 1/(2*x) - (2^(-2 + n^(-1))*E^(2*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -2*b*x^
n])/(n*x) - (2^(-2 + n^(-1))*(b*x^n)^n^(-1)*Gamma[-n^(-1), 2*b*x^n])/(E^(2*
a)*n*x)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5469

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2x^2}+\frac{\cosh(2a+2bx^n)}{2x^2}\right)dx \\
&= \frac{1}{2x}+\frac{1}{2}\int \frac{\cosh(2a+2bx^n)}{x^2}\,dx \\
&= \frac{1}{2x}+\frac{1}{4}\int \frac{e^{-2a-2bx^n}}{x^2}\,dx+\frac{1}{4}\int \frac{e^{2a+2bx^n}}{x^2}\,dx \\
&= \frac{1}{2x}-\frac{2^{-2+\frac{1}{n}}e^{2a}(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-2bx^n\right)}{nx}-\frac{2^{-2+\frac{1}{n}}e^{-2a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},2bx^n\right)}{nx}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$
\int \frac{\sinh^2(a+bx^n)}{x^2}\,dx=-\frac{-2n+2^{\frac{1}{n}}e^{2a}(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-2bx^n\right)+2^{\frac{1}{n}}e^{-2a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},2bx^n\right)}{4nx}
$$

```
[In] Integrate[Sinh[a + b*x^n]^2/x^2,x]
```

```
[Out] -1/4*(-2*n + 2^n^(-1)*E^(2*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -2*b*x^n] +
(2^n^(-1)*(b*x^n)^n^(-1)*Gamma[-n^(-1), 2*b*x^n])/E^(2*a))/(n*x)
```

Maple [F]

$$\int \frac{\sinh\left(a+b\,x^{n}\right)^{2}}{x^{2}}dx$$

```
[In] int(sinh(a+b*x^n)^2/x^2,x)
[Out] int(sinh(a+b*x^n)^2/x^2,x)
```

Fricas [F]

$$\int \frac{\sinh^{2}\left(a+bx^{n}\right)}{x^{2}}\,dx = \int \frac{\sinh\left(bx^{n}+a\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate(sinh(a+b*x^n)^2/x^2,x, algorithm="fricas")
[Out] integral(sinh(b*x^n + a)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\sinh^{2}\left(a+bx^{n}\right)}{x^{2}}\,dx = \int \frac{\sinh^{2}\left(a+bx^{n}\right)}{x^{2}}\,dx$$

```
[In] integrate(sinh(a+b*x**n)**2/x**2,x)
[Out] Integral(sinh(a + b*x**n)**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81

$$\int \frac{\sinh^{2}\left(a+bx^{n}\right)}{x^{2}}\,dx = -\frac{\left(2\,bx^{n}\right)^{\left(\frac{1}{n}\right)}e^{\left(-2\,a\right)}\Gamma\left(-\frac{1}{n},2\,bx^{n}\right)}{4\,nx} - \frac{\left(-2\,bx^{n}\right)^{\left(\frac{1}{n}\right)}e^{\left(2\,a\right)}\Gamma\left(-\frac{1}{n},-2\,bx^{n}\right)}{4\,nx} + \frac{1}{2\,x}$$

```
[In] integrate(sinh(a+b*x^n)^2/x^2,x, algorithm="maxima")
[Out] -1/4*(2*b*x^n)^(1/n)*e^(-2*a)*gamma(-1/n, 2*b*x^n)/(n*x) - 1/4*(-2*b*x^n)^(
1/n)*e^(2*a)*gamma(-1/n, -2*b*x^n)/(n*x) + 1/2/x
```

Giac [F]

$$\int \frac{\sinh^2(a+bx^n)}{x^2}\,dx = \int \frac{\sinh(bx^n+a)^2}{x^2}\,dx$$

```
[In] integrate(sinh(a+b*x^n)^2/x^2,x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx^n)}{x^2}\,dx = \int \frac{\sinh(a+b\,x^n)^2}{x^2}\,dx$$

```
[In] int(sinh(a + b*x^n)^2/x^2,x)
[Out] int(sinh(a + b*x^n)^2/x^2, x)
```

3.69 $\int x^2 \sinh^3(a+bx^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 166

$$\int x^2 \sinh^3(a+bx^n)\,dx = -\frac{3^{-3/n}e^{3a}x^3(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-3bx^n\right)}{8n} + \frac{3e^a x^3(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-bx^n\right)}{8n} - \frac{3e^{-a}x^3(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},bx^n\right)}{8n} + \frac{3^{-3/n}e^{-3a}x^3(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},3bx^n\right)}{8n}$$

```
[Out] -1/8*exp(3*a)*x^3*GAMMA(3/n,-3*b*x^n)/(3^(3/n))/n/((-b*x^n)^(3/n))+3/8*exp(
a)*x^3*GAMMA(3/n,-b*x^n)/n/((-b*x^n)^(3/n))-3/8*x^3*GAMMA(3/n,b*x^n)/exp(a)
/n/((b*x^n)^(3/n))+1/8*x^3*GAMMA(3/n,3*b*x^n)/(3^(3/n))/exp(3*a)/n/((b*x^n)
^(3/n))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5470, 5468, 2250}

$$\int x^2 \sinh^3(a+bx^n)\,dx = -\frac{e^{3a}3^{-3/n}x^3(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-3bx^n\right)}{8n} + \frac{3e^a x^3(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-bx^n\right)}{8n} - \frac{3e^{-a}x^3(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},bx^n\right)}{8n} + \frac{e^{-3a}3^{-3/n}x^3(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},3bx^n\right)}{8n}$$

```
[In] Int[x^2*Sinh[a + b*x^n]^3,x]
```

```
[Out] -1/8*(E^(3*a)*x^3*Gamma[3/n, -3*b*x^n])/(3^(3/n)*n*(-(b*x^n))^(3/n)) + (3*E
^a*x^3*Gamma[3/n, -(b*x^n)])/(8*n*(-(b*x^n))^(3/n)) - (3*x^3*Gamma[3/n, b*x
^n])/(8*E^a*n*(b*x^n)^(3/n)) + (x^3*Gamma[3/n, 3*b*x^n])/(8*3^(3/n)*E^(3*a)
*n*(b*x^n)^(3/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5468

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{4}x^2\sinh\left(a+bx^n\right)+\frac{1}{4}x^2\sinh\left(3a+3bx^n\right)\right)\,dx\\
&= \frac{1}{4}\int x^2\sinh\left(3a+3bx^n\right)\,dx-\frac{3}{4}\int x^2\sinh\left(a+bx^n\right)\,dx\\
&= -\left(\frac{1}{8}\int e^{-3a-3bx^n}x^2\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}x^2\,dx+\frac{3}{8}\int e^{-a-bx^n}x^2\,dx-\frac{3}{8}\int e^{a+bx^n}x^2\,dx\\
&= -\frac{3^{-3/n}e^{3a}x^3(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-3bx^n\right)}{8n}+\frac{3e^ax^3(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-bx^n\right)}{8n}\\
&\quad -\frac{3e^{-a}x^3(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},bx^n\right)}{8n}+\frac{3^{-3/n}e^{-3a}x^3(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},3bx^n\right)}{8n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int x^2 \sinh^3\left(a+bx^n\right)\,dx = \frac{27^{-1/n}e^{-3a}x^3(-b^2x^{2n})^{-3/n}\left(-e^{6a}(bx^n)^{3/n}\,\Gamma\left(\frac{3}{n},-3bx^n\right)+3^{\frac{3+n}{n}}e^{4a}(bx^n)^{3/n}\,\Gamma\left(\frac{3}{n},-bx^n\right)+(-bx^n)^{3/n}\left(-3^{\frac{3+n}{n}}e\right.\right.}{8n}$$

```
[In] Integrate[x^2*Sinh[a + b*x^n]^3,x]
```

```
[Out] (x^3*(-(E^(6*a)*(b*x^n)^(3/n)*Gamma[3/n, -3*b*x^n]) + 3^((3 + n)/n)*E^(4*a)
*(b*x^n)^(3/n)*Gamma[3/n, -(b*x^n)] + (-(b*x^n))^(3/n)*(-(3^((3 + n)/n)*E^(
2*a)*Gamma[3/n, b*x^n]) + Gamma[3/n, 3*b*x^n])))/(8*27^n^(-1)*E^(3*a)*n*(-(
b^2*x^(2*n)))^(3/n))
```

Maple [F]

$$\int x^2 \sinh\left(a+b\,x^n\right)^3\,dx$$

```
[In] int(x^2*sinh(a+b*x^n)^3,x)
```

```
[Out] int(x^2*sinh(a+b*x^n)^3,x)
```

Fricas [F]

$$\int x^2 \sinh^3\left(a+bx^n\right)\,dx = \int x^2 \sinh\left(bx^n+a\right)^3\,dx$$

```
[In] integrate(x^2*sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh(b*x^n + a)^3, x)
```

Sympy [F]

$$\int x^2 \sinh^3\left(a+bx^n\right)\,dx = \int x^2 \sinh^3\left(a+bx^n\right)dx$$

```
[In] integrate(x**2*sinh(a+b*x**n)**3,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**n)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int x^2 \sinh^3(a+bx^n)\,dx = \frac{x^3 e^{(-3\,a)}\Gamma\left(\frac{3}{n}, 3\,bx^n\right)}{8\,(3\,bx^n)^{\frac{3}{n}}\,n} - \frac{3\,x^3 e^{(-a)}\Gamma\left(\frac{3}{n}, bx^n\right)}{8\,(bx^n)^{\frac{3}{n}}\,n} + \frac{3\,x^3 e^{a}\Gamma\left(\frac{3}{n}, -bx^n\right)}{8\,(-bx^n)^{\frac{3}{n}}\,n} - \frac{x^3 e^{(3\,a)}\Gamma\left(\frac{3}{n}, -3\,bx^n\right)}{8\,(-3\,bx^n)^{\frac{3}{n}}\,n}$$

```
[In] integrate(x^2*sinh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/8*x^3*e^(-3*a)*gamma(3/n, 3*b*x^n)/((3*b*x^n)^(3/n)*n) - 3/8*x^3*e^(-a)*g
amma(3/n, b*x^n)/((b*x^n)^(3/n)*n) + 3/8*x^3*e^a*gamma(3/n, -b*x^n)/((-b*x^
n)^(3/n)*n) - 1/8*x^3*e^(3*a)*gamma(3/n, -3*b*x^n)/((-3*b*x^n)^(3/n)*n)
```

Giac [F]

$$\int x^2 \sinh^3(a+bx^n)\,dx = \int x^2 \sinh\left(bx^n + a\right)^3\,dx$$

```
[In] integrate(x^2*sinh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh^3(a+bx^n)\,dx = \int x^2\,\sinh(a+b\,x^n)^3\,dx$$

```
[In] int(x^2*sinh(a + b*x^n)^3,x)
```

```
[Out] int(x^2*sinh(a + b*x^n)^3, x)
```

3.70 $\int x \sinh^3(a + bx^n)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 166

$$\int x \sinh^3(a+bx^n)\, dx = -\frac{9^{-1/n}e^{3a}x^2(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-3bx^n\right)}{8n}+\frac{3e^a x^2(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-bx^n\right)}{8n} - \frac{3e^{-a}x^2(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},bx^n\right)}{8n}+\frac{9^{-1/n}e^{-3a}x^2(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},3bx^n\right)}{8n}$$

```
[Out] -1/8*exp(3*a)*x^2*GAMMA(2/n,-3*b*x^n)/(9^(1/n))/n/((-b*x^n)^(2/n))+3/8*exp(
a)*x^2*GAMMA(2/n,-b*x^n)/n/((-b*x^n)^(2/n))-3/8*x^2*GAMMA(2/n,b*x^n)/exp(a)
/n/((b*x^n)^(2/n))+1/8*x^2*GAMMA(2/n,3*b*x^n)/(9^(1/n))/exp(3*a)/n/((b*x^n)
^(2/n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5470, 5468, 2250}

$$\int x \sinh^3(a+bx^n)\, dx = -\frac{e^{3a}9^{-1/n}x^2(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-3bx^n\right)}{8n}+\frac{3e^a x^2(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-bx^n\right)}{8n} - \frac{3e^{-a}x^2(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},bx^n\right)}{8n}+\frac{e^{-3a}9^{-1/n}x^2(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},3bx^n\right)}{8n}$$

```
[In] Int[x*Sinh[a + b*x^n]^3,x]
```

```
[Out] -1/8*(E^(3*a)*x^2*Gamma[2/n, -3*b*x^n])/(9^n^(-1)*n*(-(b*x^n))^(2/n)) + (3*
E^a*x^2*Gamma[2/n, -(b*x^n)])/(8*n*(-(b*x^n))^(2/n)) - (3*x^2*Gamma[2/n, b*
x^n])/(8*E^a*n*(b*x^n)^(2/n)) + (x^2*Gamma[2/n, 3*b*x^n])/(8*9^n^(-1)*E^(3*
a)*n*(b*x^n)^(2/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5468

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{4}x\sinh\left(a+bx^n\right)+\frac{1}{4}x\sinh\left(3a+3bx^n\right)\right)\,dx \\
&= \frac{1}{4}\int x\sinh\left(3a+3bx^n\right)\,dx-\frac{3}{4}\int x\sinh\left(a+bx^n\right)\,dx \\
&= -\left(\frac{1}{8}\int e^{-3a-3bx^n}x\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}x\,dx+\frac{3}{8}\int e^{-a-bx^n}x\,dx-\frac{3}{8}\int e^{a+bx^n}x\,dx \\
&= -\frac{9^{-1/n}e^{3a}x^2\left(-bx^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-3bx^n\right)}{8n}+\frac{3e^{a}x^2\left(-bx^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-bx^n\right)}{8n} \\
&\quad -\frac{3e^{-a}x^2\left(bx^n\right)^{-2/n}\Gamma\left(\frac{2}{n},bx^n\right)}{8n}+\frac{9^{-1/n}e^{-3a}x^2\left(bx^n\right)^{-2/n}\Gamma\left(\frac{2}{n},3bx^n\right)}{8n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int x\sinh^3\left(a+bx^n\right)\,dx$$

$$=\frac{9^{-1/n}e^{-3a}x^2\left(-b^2x^{2n}\right)^{-2/n}\left(-e^{6a}\left(bx^n\right)^{2/n}\Gamma\left(\frac{2}{n},-3bx^n\right)+3^{\frac{2+n}{n}}e^{4a}\left(bx^n\right)^{2/n}\Gamma\left(\frac{2}{n},-bx^n\right)+\left(-bx^n\right)^{2/n}\left(-3^{\frac{2+n}{n}}\right.\right.}{8n}$$

```
[In] Integrate[x*Sinh[a + b*x^n]^3,x]
```

```
[Out] (x^2*(-(E^(6*a)*(b*x^n)^(2/n)*Gamma[2/n, -3*b*x^n]) + 3^((2 + n)/n)*E^(4*a)
*(b*x^n)^(2/n)*Gamma[2/n, -(b*x^n)] + (-(b*x^n))^(2/n)*(-(3^((2 + n)/n)*E^(
2*a)*Gamma[2/n, b*x^n]) + Gamma[2/n, 3*b*x^n])))/(8*9^n^(-1)*E^(3*a)*n*(-(b
^2*x^(2*n)))^(2/n))
```

Maple [F]

$$\int x\sinh\left(a+b\,x^{n}\right)^{3}dx$$

```
[In] int(x*sinh(a+b*x^n)^3,x)
[Out] int(x*sinh(a+b*x^n)^3,x)
```

Fricas [F]

$$\int x\sinh^3\left(a+bx^n\right)\,dx=\int x\sinh\left(bx^n+a\right)^3\,dx$$

```
[In] integrate(x*sinh(a+b*x^n)^3,x, algorithm="fricas")
[Out] integral(x*sinh(b*x^n + a)^3, x)
```

Sympy [F]

$$\int x\sinh^3\left(a+bx^n\right)\,dx=\int x\sinh^3\left(a+bx^n\right)dx$$

```
[In] integrate(x*sinh(a+b*x**n)**3,x)
[Out] Integral(x*sinh(a + b*x**n)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int x\sinh^3\left(a+bx^n\right)\,dx=\frac{x^2e^{(-3\,a)}\Gamma\left(\frac{2}{n},3\,bx^n\right)}{8\,\left(3\,bx^n\right)^{\frac{2}{n}}\,n}-\frac{3\,x^2e^{(-a)}\Gamma\left(\frac{2}{n},bx^n\right)}{8\,\left(bx^n\right)^{\frac{2}{n}}\,n}+\frac{3\,x^2e^{a}\Gamma\left(\frac{2}{n},-bx^n\right)}{8\,\left(-bx^n\right)^{\frac{2}{n}}\,n}-\frac{x^2e^{(3\,a)}\Gamma\left(\frac{2}{n},-3\,bx^n\right)}{8\,\left(-3\,bx^n\right)^{\frac{2}{n}}\,n}$$

```
[In] integrate(x*sinh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/8*x^2*e^(-3*a)*gamma(2/n, 3*b*x^n)/((3*b*x^n)^(2/n)*n) - 3/8*x^2*e^(-a)*g
amma(2/n, b*x^n)/((b*x^n)^(2/n)*n) + 3/8*x^2*e^a*gamma(2/n, -b*x^n)/((-b*x^
n)^(2/n)*n) - 1/8*x^2*e^(3*a)*gamma(2/n, -3*b*x^n)/((-3*b*x^n)^(2/n)*n)
```

Giac [F]

$$\int x \sinh^3(a+bx^n)\,dx = \int x \sinh(bx^n+a)^3\,dx$$

```
[In] integrate(x*sinh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sinh^3(a+bx^n)\,dx = \int x \sinh(a+b\,x^n)^3\,dx$$

```
[In] int(x*sinh(a + b*x^n)^3,x)
```

```
[Out] int(x*sinh(a + b*x^n)^3, x)
```

3.71 $\int \sinh^3(a+bx^n)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 150

$$\int \sinh^3(a+bx^n)\,dx = -\frac{3^{-1/n}e^{3a}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3bx^n\right)}{8n}+\frac{3e^ax(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{8n} - \frac{3e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{8n}+\frac{3^{-1/n}e^{-3a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},3bx^n\right)}{8n}$$

```
[Out] -1/8*exp(3*a)*x*GAMMA(1/n,-3*b*x^n)/(3^(1/n))/n/((-b*x^n)^(1/n))+3/8*exp(a)
*x*GAMMA(1/n,-b*x^n)/n/((-b*x^n)^(1/n))-3/8*x*GAMMA(1/n,b*x^n)/exp(a)/n/((b
*x^n)^(1/n))+1/8*x*GAMMA(1/n,3*b*x^n)/(3^(1/n))/exp(3*a)/n/((b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5416, 5414, 2239}

$$\int \sinh^3(a+bx^n)\,dx = -\frac{e^{3a}3^{-1/n}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3bx^n\right)}{8n}+\frac{3e^ax(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{8n} - \frac{3e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{8n}+\frac{e^{-3a}3^{-1/n}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},3bx^n\right)}{8n}$$

```
[In] Int[Sinh[a + b*x^n]^3,x]
```

```
[Out] -1/8*(E^(3*a)*x*Gamma[n^(-1), -3*b*x^n])/(3^n^(-1)*n*(-(b*x^n))^n^(-1)) + (
3*E^a*x*Gamma[n^(-1), -(b*x^n)])/(8*n*(-(b*x^n))^n^(-1)) - (3*x*Gamma[n^(-1
), b*x^n])/(8*E^a*n*(b*x^n)^n^(-1)) + (x*Gamma[n^(-1), 3*b*x^n])/(8*3^n^(-1
)*E^(3*a)*n*(b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 5414

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rule 5416

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{4}\sinh(a+bx^n)+\frac{1}{4}\sinh(3a+3bx^n)\right) dx\\
&= \frac{1}{4}\int \sinh(3a+3bx^n)\,dx-\frac{3}{4}\int \sinh(a+bx^n)\,dx\\
&= -\left(\frac{1}{8}\int e^{-3a-3bx^n}\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}\,dx+\frac{3}{8}\int e^{-a-bx^n}\,dx-\frac{3}{8}\int e^{a+bx^n}\,dx\\
&= -\frac{3^{-1/n}e^{3a}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-3bx^n\right)}{8n}+\frac{3e^a x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-bx^n\right)}{8n}\\
&\quad-\frac{3e^{-a}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},bx^n\right)}{8n}+\frac{3^{-1/n}e^{-3a}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},3bx^n\right)}{8n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int \sinh^3(a+bx^n)\,dx$$
$$=\frac{3^{-1/n}e^{-3a}x(-b^2x^{2n})^{-1/n}\left(-e^{6a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-3bx^n\right)+3^{1+\frac{1}{n}}e^{4a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-bx^n\right)+(-bx^n)^{\frac{1}{n}}\left(-3^{1+\frac{1}{n}}e^{2a}\Gamma\right.\right.}{8n}$$

```
[In] Integrate[Sinh[a + b*x^n]^3,x]
```

```
[Out] (x*(-(E^(6*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -3*b*x^n]) + 3^(1 + n^(-1))*E^(4
*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -(b*x^n)] + (-(b*x^n))^n^(-1)*(-(3^(1 + n^
(-1))*E^(2*a)*Gamma[n^(-1), b*x^n]) + Gamma[n^(-1), 3*b*x^n])))/(8*3^n^(-1)
*E^(3*a)*n*(-(b^2*x^(2*n)))^n^(-1))
```

Maple [F]

$$\int \sinh\left(a+b\,x^{n}\right)^{3}dx$$

[In] int(sinh(a+b*x^n)^3,x)

[Out] int(sinh(a+b*x^n)^3,x)

Fricas [F]

$$\int \sinh^{3}\left(a+bx^{n}\right)\,dx=\int \sinh\left(bx^{n}+a\right)^{3}\,dx$$

[In] integrate(sinh(a+b*x^n)^3,x, algorithm="fricas")

[Out] integral(sinh(b*x^n + a)^3, x)

Sympy [F]

$$\int \sinh^{3}\left(a+bx^{n}\right)\,dx=\int \sinh^{3}\left(a+bx^{n}\right)dx$$

[In] integrate(sinh(a+b*x**n)**3,x)

[Out] Integral(sinh(a + b*x**n)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int \sinh^{3}\left(a+bx^{n}\right)\,dx=\frac{xe^{(-3\,a)}\Gamma\left(\frac{1}{n},3\,bx^{n}\right)}{8\,\left(3\,bx^{n}\right)^{\left(\frac{1}{n}\right)}n}-\frac{3\,xe^{(-a)}\Gamma\left(\frac{1}{n},bx^{n}\right)}{8\,\left(bx^{n}\right)^{\left(\frac{1}{n}\right)}n}+\frac{3\,xe^{a}\Gamma\left(\frac{1}{n},-bx^{n}\right)}{8\,\left(-bx^{n}\right)^{\left(\frac{1}{n}\right)}n}-\frac{xe^{(3\,a)}\Gamma\left(\frac{1}{n},-3\,bx^{n}\right)}{8\,\left(-3\,bx^{n}\right)^{\left(\frac{1}{n}\right)}n}$$

[In] integrate(sinh(a+b*x^n)^3,x, algorithm="maxima")

[Out] 1/8*x*e^(-3*a)*gamma(1/n, 3*b*x^n)/((3*b*x^n)^(1/n)*n) - 3/8*x*e^(-a)*gamma(1/n, b*x^n)/((b*x^n)^(1/n)*n) + 3/8*x*e^a*gamma(1/n, -b*x^n)/((-b*x^n)^(1/n)*n) - 1/8*x*e^(3*a)*gamma(1/n, -3*b*x^n)/((-3*b*x^n)^(1/n)*n)

Giac [F]

$$\int \sinh^3(a+bx^n)\,dx = \int \sinh(bx^n+a)^3\,dx$$

```
[In] integrate(sinh(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3(a+bx^n)\,dx = \int \sinh(a+b\,x^n)^3\,dx$$

```
[In] int(sinh(a + b*x^n)^3,x)
[Out] int(sinh(a + b*x^n)^3, x)
```

3.72 $\int \frac{\sinh^3(a+bx^n)}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{\sinh^3(a+bx^n)}{x} \, dx = -\frac{3\text{Chi}(bx^n)\sinh(a)}{4n} + \frac{\text{Chi}(3bx^n)\sinh(3a)}{4n} - \frac{3\cosh(a)\text{Shi}(bx^n)}{4n} + \frac{\cosh(3a)\text{Shi}(3bx^n)}{4n}$$

```
[Out] -3/4*cosh(a)*Shi(b*x^n)/n+1/4*cosh(3*a)*Shi(3*b*x^n)/n-3/4*Chi(b*x^n)*sinh(
a)/n+1/4*Chi(3*b*x^n)*sinh(3*a)/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5470, 5426, 5425, 5424}

$$\int \frac{\sinh^3(a+bx^n)}{x} \, dx = -\frac{3\sinh(a)\text{Chi}(bx^n)}{4n} + \frac{\sinh(3a)\text{Chi}(3bx^n)}{4n} - \frac{3\cosh(a)\text{Shi}(bx^n)}{4n} + \frac{\cosh(3a)\text{Shi}(3bx^n)}{4n}$$

```
[In] Int[Sinh[a + b*x^n]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^n]*Sinh[a])/(4*n) + (CoshIntegral[3*b*x^n]*Sinh[3*a])/
(4*n) - (3*Cosh[a]*SinhIntegral[b*x^n])/(4*n) + (Cosh[3*a]*SinhIntegral[3*b
*x^n])/(4*n)
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5426

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3\sinh(a+bx^n)}{4x}+\frac{\sinh(3a+3bx^n)}{4x}\right)dx\\
&= \frac{1}{4}\int \frac{\sinh(3a+3bx^n)}{x}\,dx-\frac{3}{4}\int \frac{\sinh(a+bx^n)}{x}\,dx\\
&= -\left(\frac{1}{4}(3\cosh(a))\int \frac{\sinh(bx^n)}{x}\,dx\right)+\frac{1}{4}\cosh(3a)\int \frac{\sinh(3bx^n)}{x}\,dx\\
&\quad -\frac{1}{4}(3\sinh(a))\int \frac{\cosh(bx^n)}{x}\,dx+\frac{1}{4}\sinh(3a)\int \frac{\cosh(3bx^n)}{x}\,dx\\
&= -\frac{3\text{Chi}(bx^n)\sinh(a)}{4n}+\frac{\text{Chi}(3bx^n)\sinh(3a)}{4n}-\frac{3\cosh(a)\text{Shi}(bx^n)}{4n}+\frac{\cosh(3a)\text{Shi}(3bx^n)}{4n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \frac{\sinh^3(a+bx^n)}{x}\,dx = \frac{-3\text{Chi}(bx^n)\sinh(a)+\text{Chi}(3bx^n)\sinh(3a)-3\cosh(a)\text{Shi}(bx^n)+\cosh(3a)\text{Shi}(3bx^n)}{4n}$$

```
[In] Integrate[Sinh[a + b*x^n]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^n]*Sinh[a] + CoshIntegral[3*b*x^n]*Sinh[3*a] - 3*Cosh[
a]*SinhIntegral[b*x^n] + Cosh[3*a]*SinhIntegral[3*b*x^n])/(4*n)
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3b\,x^n)}{8n}-\frac{3\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(b\,x^n)}{8n}+\frac{3\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-b\,x^n)}{8n}-\frac{\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3b\,x^n)}{8n}$	67

```
[In] int(sinh(a+b*x^n)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/n*exp(-3*a)*Ei(1,3*b*x^n)-3/8/n*exp(-a)*Ei(1,b*x^n)+3/8/n*exp(a)*Ei(1,-
b*x^n)-1/8/n*exp(3*a)*Ei(1,-3*b*x^n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.72

$$\int \frac{\sinh^3(a+bx^n)}{x}\,dx = \frac{(\cosh(3a)+\sinh(3a))\mathrm{Ei}(3b\cosh(n\log(x))+3b\sinh(n\log(x)))-3(\cosh(a)+\sinh(a))\mathrm{Ei}(b\cosh(n\log$$

```
[In] integrate(sinh(a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] 1/8*((cosh(3*a) + sinh(3*a))*Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) -
3*(cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + 3*(cosh(a)
- sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))) - (cosh(3*a) - sinh(3*a
))*Ei(-3*b*cosh(n*log(x)) - 3*b*sinh(n*log(x))))/n
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx^n)}{x}\,dx = \int \frac{\sinh^3(a+bx^n)}{x}\,dx$$

```
[In] integrate(sinh(a+b*x**n)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x**n)**3/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{\sinh^3(a+bx^n)}{x}\,dx = \frac{\text{Ei}(3\,bx^n)\,e^{(3\,a)}}{8\,n} + \frac{3\,\text{Ei}(-bx^n)\,e^{(-a)}}{8\,n} - \frac{\text{Ei}(-3\,bx^n)\,e^{(-3\,a)}}{8\,n} - \frac{3\,\text{Ei}(bx^n)\,e^{a}}{8\,n}$$

```
[In] integrate(sinh(a+b*x^n)^3/x,x, algorithm="maxima")
[Out] 1/8*Ei(3*b*x^n)*e^(3*a)/n + 3/8*Ei(-b*x^n)*e^(-a)/n - 1/8*Ei(-3*b*x^n)*e^(-
3*a)/n - 3/8*Ei(b*x^n)*e^a/n
```

Giac [F]

$$\int \frac{\sinh^3(a+bx^n)}{x}\,dx = \int \frac{\sinh(bx^n+a)^3}{x}\,dx$$

```
[In] integrate(sinh(a+b*x^n)^3/x,x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx^n)}{x}\,dx = \int \frac{\sinh(a+b\,x^n)^3}{x}\,dx$$

```
[In] int(sinh(a + b*x^n)^3/x,x)
[Out] int(sinh(a + b*x^n)^3/x, x)
```

3.73 $\int \frac{\sinh^3(a+bx^n)}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 154

$$\int \frac{\sinh^3(a+bx^n)}{x^2} dx = -\frac{3^{\frac{1}{n}}e^{3a}(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-3bx^n\right)}{8nx} + \frac{3e^a(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-bx^n\right)}{8nx} - \frac{3e^{-a}(bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},bx^n\right)}{8nx} + \frac{3^{\frac{1}{n}}e^{-3a}(bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},3bx^n\right)}{8nx}$$

```
[Out] -1/8*3^(1/n)*exp(3*a)*(-b*x^n)^(1/n)*GAMMA(-1/n,-3*b*x^n)/n/x+3/8*exp(a)*(-
b*x^n)^(1/n)*GAMMA(-1/n,-b*x^n)/n/x-3/8*(b*x^n)^(1/n)*GAMMA(-1/n,b*x^n)/exp
(a)/n/x+1/8*3^(1/n)*(b*x^n)^(1/n)*GAMMA(-1/n,3*b*x^n)/exp(3*a)/n/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5470, 5468, 2250}

$$\int \frac{\sinh^3(a+bx^n)}{x^2} dx = -\frac{e^{3a}3^{\frac{1}{n}}(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-3bx^n\right)}{8nx} + \frac{3e^a(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-bx^n\right)}{8nx} - \frac{3e^{-a}(bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},bx^n\right)}{8nx} + \frac{e^{-3a}3^{\frac{1}{n}}(bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},3bx^n\right)}{8nx}$$

```
[In] Int[Sinh[a + b*x^n]^3/x^2,x]
```

```
[Out] -1/8*(3^n^(-1)*E^(3*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -3*b*x^n])/(n*x) +
(3*E^a*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)])/(8*n*x) - (3*(b*x^n)^n^(
-1)*Gamma[-n^(-1), b*x^n])/(8*E^a*n*x) + (3^n^(-1)*(b*x^n)^n^(-1)*Gamma[-n^
(-1), 3*b*x^n])/(8*E^(3*a)*n*x)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5468

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3\sinh(a+bx^n)}{4x^2}+\frac{\sinh(3a+3bx^n)}{4x^2}\right)dx \\
&= \frac{1}{4}\int \frac{\sinh(3a+3bx^n)}{x^2}\,dx-\frac{3}{4}\int \frac{\sinh(a+bx^n)}{x^2}\,dx \\
&= -\left(\frac{1}{8}\int \frac{e^{-3a-3bx^n}}{x^2}\,dx\right)+\frac{1}{8}\int \frac{e^{3a+3bx^n}}{x^2}\,dx+\frac{3}{8}\int \frac{e^{-a-bx^n}}{x^2}\,dx-\frac{3}{8}\int \frac{e^{a+bx^n}}{x^2}\,dx \\
&= -\frac{3^{\frac{1}{n}}e^{3a}(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-3bx^n\right)}{8nx}+\frac{3e^a(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-bx^n\right)}{8nx} \\
&\quad -\frac{3e^{-a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},bx^n\right)}{8nx}+\frac{3^{\frac{1}{n}}e^{-3a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},3bx^n\right)}{8nx}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \frac{\sinh^3(a+bx^n)}{x^2}\,dx = \frac{e^{-3a}\left(-3^{\frac{1}{n}}e^{6a}(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-3bx^n\right)+3e^{4a}(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-bx^n\right)+(bx^n)^{\frac{1}{n}}\left(-3e^{2a}\Gamma\left(-\frac{1}{n},bx^n\right)+3^{\frac{1}{n}}\Gamma\left(-\right.\right.\right.}{8nx}$$

```
[In] Integrate[Sinh[a + b*x^n]^3/x^2,x]
```

```
[Out] (-(3^n^(-1)*E^(6*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -3*b*x^n]) + 3*E^(4*a)
*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)] + (b*x^n)^n^(-1)*(-3*E^(2*a)*Ga
mma[-n^(-1), b*x^n] + 3^n^(-1)*Gamma[-n^(-1), 3*b*x^n]))/(8*E^(3*a)*n*x)
```

Maple [F]

$$\int \frac{\sinh\left(a+b\,x^{n}\right)^{3}}{x^{2}}dx$$

```
[In] int(sinh(a+b*x^n)^3/x^2,x)
[Out] int(sinh(a+b*x^n)^3/x^2,x)
```

Fricas [F]

$$\int \frac{\sinh^3(a+bx^n)}{x^2}\,dx = \int \frac{\sinh\left(bx^n+a\right)^3}{x^2}\,dx$$

```
[In] integrate(sinh(a+b*x^n)^3/x^2,x, algorithm="fricas")
[Out] integral(sinh(b*x^n + a)^3/x^2, x)
```

Sympy [F]

$$\int \frac{\sinh^3(a+bx^n)}{x^2}\,dx = \int \frac{\sinh^3\left(a+bx^n\right)}{x^2}\,dx$$

```
[In] integrate(sinh(a+b*x**n)**3/x**2,x)
[Out] Integral(sinh(a + b*x**n)**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^3(a+bx^n)}{x^2}\,dx = \frac{\left(3\,bx^n\right)^{\left(\frac{1}{n}\right)}e^{\left(-3\,a\right)}\Gamma\left(-\frac{1}{n},3\,bx^n\right)}{8\,nx} - \frac{3\left(bx^n\right)^{\left(\frac{1}{n}\right)}e^{\left(-a\right)}\Gamma\left(-\frac{1}{n},bx^n\right)}{8\,nx} + \frac{3\left(-bx^n\right)^{\left(\frac{1}{n}\right)}e^{a}\Gamma\left(-\frac{1}{n},-bx^n\right)}{8\,nx} - \frac{\left(-3\,bx^n\right)^{\left(\frac{1}{n}\right)}e^{\left(3\,a\right)}\Gamma\left(-\frac{1}{n},-3\,bx^n\right)}{8\,nx}$$

```
[In] integrate(sinh(a+b*x^n)^3/x^2,x, algorithm="maxima")
[Out] 1/8*(3*b*x^n)^(1/n)*e^(-3*a)*gamma(-1/n, 3*b*x^n)/(n*x) - 3/8*(b*x^n)^(1/n)
*e^(-a)*gamma(-1/n, b*x^n)/(n*x) + 3/8*(-b*x^n)^(1/n)*e^a*gamma(-1/n, -b*x^
n)/(n*x) - 1/8*(-3*b*x^n)^(1/n)*e^(3*a)*gamma(-1/n, -3*b*x^n)/(n*x)
```

Giac [F]

$$\int \frac{\sinh^3(a+bx^n)}{x^2}\,dx = \int \frac{\sinh(bx^n+a)^3}{x^2}\,dx$$

```
[In] integrate(sinh(a+b*x^n)^3/x^2,x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx^n)}{x^2}\,dx = \int \frac{\sinh(a+b\,x^n)^3}{x^2}\,dx$$

```
[In] int(sinh(a + b*x^n)^3/x^2,x)
[Out] int(sinh(a + b*x^n)^3/x^2, x)
```

3.74 $\int (ex)^m (b \sinh (c+dx^n))^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (ex)^m (b \sinh (c+dx^n))^p \, dx = \text{Int}((ex)^m (b \sinh (c+dx^n))^p , x)$$

```
[Out] Unintegrable((e*x)^m*(b*sinh(c+d*x^n))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (b \sinh (c+dx^n))^p \, dx = \int (ex)^m (b \sinh (c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^m*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(b*Sinh[c + d*x^n])^p, x]
```

Rubi steps

$$\text{integral} = \int (ex)^m (b \sinh (c+dx^n))^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 3.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sinh (c+dx^n))^p \, dx = \int (ex)^m (b\sinh (c+dx^n))^p \, dx$$

```
[In] Integrate[(e*x)^m*(b*Sinh[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(b*Sinh[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex)^m (b\sinh (c+d\,x^n))^p dx$$

```
[In] int((e*x)^m*(b*sinh(c+d*x^n))^p,x)
[Out] int((e*x)^m*(b*sinh(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sinh (c+dx^n))^p \, dx = \int (ex)^m (b\sinh (dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^m*(b*sinh(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*sinh(d*x^n + c))^p, x)
```

Sympy [N/A]

Not integrable

Time = 10.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (ex)^m (b\sinh (c+dx^n))^p \, dx = \int (b\sinh (c+dx^n))^p (ex)^m \, dx$$

```
[In] integrate((e*x)**m*(b*sinh(c+d*x**n))**p,x)
[Out] Integral((b*sinh(c + d*x**n))**p*(e*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sinh(c+dx^n))^p \, dx = \int (ex)^m (b\sinh(dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^m*(b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c))^p, x)
```

Giac [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sinh(c+dx^n))^p \, dx = \int (ex)^m (b\sinh(dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^m*(b*sinh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c))^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\sinh(c+dx^n))^p \, dx = \int (b\,\mathrm{sinh}(c+d\,x^n))^p\,(e\,x)^m \, dx$$

```
[In] int((b*sinh(c + d*x^n))^p*(e*x)^m,x)
[Out] int((b*sinh(c + d*x^n))^p*(e*x)^m, x)
```

3.75 $\int (ex)^m (a+b\sinh(c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a+b\sinh(c+dx^n))^p \, dx = \text{Int}((ex)^m (a+b\sinh(c+dx^n))^p, x)$$

```
[Out] Unintegrable((e*x)^m*(a+b*sinh(c+d*x^n))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a+b\sinh(c+dx^n))^p \, dx = \int (ex)^m (a+b\sinh(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^m*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sinh[c + d*x^n])^p, x]
```

Rubi steps

$$\text{integral} = \int (ex)^m (a+b\sinh(c+dx^n))^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 6.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\sinh(c+dx^n))^p \, dx = \int (ex)^m (a+b\sinh(c+dx^n))^p \, dx$$

```
[In] Integrate[(e*x)^m*(a + b*Sinh[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Sinh[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a+b\sinh(c+d\,x^n))^p dx$$

```
[In] int((e*x)^m*(a+b*sinh(c+d*x^n))^p,x)
[Out] int((e*x)^m*(a+b*sinh(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\sinh(c+dx^n))^p \, dx = \int (ex)^m (b\sinh(dx^n+c)+a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sinh(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*sinh(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 37.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a+b\sinh(c+dx^n))^p \, dx = \int (ex)^m (a+b\sinh(c+dx^n))^p \, dx$$

```
[In] integrate((e*x)**m*(a+b*sinh(c+d*x**n))**p,x)
[Out] Integral((e*x)**m*(a + b*sinh(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\sinh (c + dx^n))^p \, dx = \int (ex)^m (b\sinh (dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 7.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\sinh (c + dx^n))^p \, dx = \int (ex)^m (b\sinh (dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sinh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\sinh (c + dx^n))^p \, dx = \int (e\,x)^m \,(a + b\sinh(c + d\,x^n))^p \, dx$$

```
[In] int((e*x)^m*(a + b*sinh(c + d*x^n))^p,x)
[Out] int((e*x)^m*(a + b*sinh(c + d*x^n))^p, x)
```

3.76 $\int (ex)^{-1+n} (b \sinh (c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 94

$$\int (ex)^{-1+n} (b \sinh (c+dx^n))^p \, dx = \frac{x^{-n}(ex)^n \cosh (c+dx^n) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},-\sinh^2(c+dx^n)\right) (b \sinh (c+dx^n))^{1+p}}{bden(1+p)\sqrt{\cosh^2(c+dx^n)}}$$

```
[Out] (e*x)^n*cosh(c+d*x^n)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],-sinh(c+d*x^n)
^2)*(b*sinh(c+d*x^n))^(p+1)/b/d/e/n/(p+1)/(x^n)/(cosh(c+d*x^n)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5430, 5428, 2722}

$$\int (ex)^{-1+n} (b \sinh (c+dx^n))^p \, dx = \frac{x^{-n}(ex)^n \cosh (c+dx^n) (b \sinh (c+dx^n))^{p+1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},-\sinh^2(dx^n+c)\right)}{bden(p+1)\sqrt{\cosh^2(c+dx^n)}}$$

```
[In] Int[(e*x)^(-1 + n)*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*Cosh[c + d*x^n]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, -Sinh
[c + d*x^n]^2]*(b*Sinh[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n*Sqrt[Cosh[
c + d*x^n]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5430

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && In
tegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{(x^{-n}(ex)^n)\int x^{-1+n}(b\sinh(c+dx^n))^p\,dx}{e}$$
$$= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int (b\sinh(c+dx))^p\,dx, x, x^n\right)}{en}$$
$$= \frac{x^{-n}(ex)^n\cosh(c+dx^n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},-\sinh^2(c+dx^n)\right)(b\sinh(c+dx^n))^{1+p}}{bden(1+p)\sqrt{\cosh^2(c+dx^n)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

$$\int (ex)^{-1+n}(b\sinh(c+dx^n))^p\,dx$$
$$= \frac{x^{1-n}(ex)^{-1+n}\sqrt{\cosh^2(c+dx^n)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},-\sinh^2(c+dx^n)\right)(b\sinh(c+dx^n))^p\tan}{dn(1+p)}$$

```
[In] Integrate[(e*x)^(-1 + n)*(b*Sinh[c + d*x^n])^p,x]
[Out] (x^(1 - n)*(e*x)^(-1 + n)*Sqrt[Cosh[c + d*x^n]^2]*Hypergeometric2F1[1/2, (1
 + p)/2, (3 + p)/2, -Sinh[c + d*x^n]^2]*(b*Sinh[c + d*x^n])^p*Tanh[c + d*x^
n])/(d*n*(1 + p))
```

Maple [F]

$$\int (ex)^{-1+n} (b\sinh (c+d\,x^n))^p dx$$

```
[In] int((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n} (b\sinh (c+dx^n))^p \, dx = \int (ex)^{n-1} (b\sinh (dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(n - 1)*(b*sinh(d*x^n + c))^p, x)
```

Sympy [F]

$$\int (ex)^{-1+n} (b\sinh (c+dx^n))^p \, dx = \int (b\sinh (c+dx^n))^p (ex)^{n-1} \, dx$$

```
[In] integrate((e*x)**(-1+n)*(b*sinh(c+d*x**n))**p,x)
[Out] Integral((b*sinh(c + d*x**n))**p*(e*x)**(n - 1), x)
```

Maxima [F]

$$\int (ex)^{-1+n} (b\sinh (c+dx^n))^p \, dx = \int (ex)^{n-1} (b\sinh (dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1+n}(b\sinh(c+dx^n))^p\,dx = \int (ex)^{n-1}(b\sinh(dx^n+c))^p\,dx$$

```
[In] integrate((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n}(b\sinh(c+dx^n))^p\,dx = \int (b\sinh(c+d\,x^n))^p\,(e\,x)^{n-1}\,dx$$

```
[In] int((b*sinh(c + d*x^n))^p*(e*x)^(n - 1),x)
[Out] int((b*sinh(c + d*x^n))^p*(e*x)^(n - 1), x)
```

3.77 $\int (ex)^{-1+2n} (b\sinh (c+dx^n))^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (ex)^{-1+2n} (b\sinh (c+dx^n))^p \, dx = \frac{x^{-2n}(ex)^{2n}\text{Int}(x^{-1+2n}(b\sinh (c+dx^n))^p\,,x)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(b*sinh(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^{-1+2n} (b\sinh (c+dx^n))^p \, dx = \int (ex)^{-1+2n} (b\sinh (c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p, x])/(e*x^(2*n))
```

Rubi steps

$$\text{integral} = \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n}(b\sinh (c+dx^n))^p \, dx}{e}$$

Mathematica [N/A]

Not integrable

Time = 4.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sinh(c+dx^n))^p\,dx=\int (ex)^{-1+2n}(b\sinh(c+dx^n))^p\,dx$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (ex)^{-1+2n}(b\sinh(c+d\,x^n))^p dx$$

```
[In] int((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\sinh(c+dx^n))^p\,dx=\int (ex)^{2\,n-1}(b\sinh(dx^n+c))^p\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(2*n - 1)*(b*sinh(d*x^n + c))^p, x)
```

Sympy [N/A]

Not integrable

Time = 9.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (ex)^{-1+2n}(b\sinh(c+dx^n))^p\,dx=\int (b\sinh(c+dx^n))^p(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(b*sinh(c+d*x**n))**p,x)
[Out] Integral((b*sinh(c + d*x**n))**p*(e*x)**(2*n - 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} \left(b\sinh\left(c+dx^n\right)\right)^p \, dx = \int (ex)^{2n-1} \left(b\sinh\left(dx^n+c\right)\right)^p \, dx$$

```
[In] integrate((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c))^p, x)
```

Giac [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} \left(b\sinh\left(c+dx^n\right)\right)^p \, dx = \int (ex)^{2n-1} \left(b\sinh\left(dx^n+c\right)\right)^p \, dx$$

```
[In] integrate((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c))^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} \left(b\sinh\left(c+dx^n\right)\right)^p \, dx = \int \left(b\sinh(c+d\,x^n)\right)^p\,{\left(e\,x\right)}^{2\,n-1} \,dx$$

```
[In] int((b*sinh(c + d*x^n))^p*(e*x)^(2*n - 1),x)
[Out] int((b*sinh(c + d*x^n))^p*(e*x)^(2*n - 1), x)
```

3.78 $\int (ex)^{-1+n} (a+b\sinh(c+dx^n))^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int (ex)^{-1+n} (a+b\sinh(c+dx^n))^p\,dx = \frac{i\sqrt{2}x^{-n}(ex)^n \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-i\sinh(c+dx^n)),\frac{b(1-i\sinh(c+dx^n))}{ia+b}\right)\cosh(c+dx^n)(a+b\sinh(c
}{den\sqrt{1+i\sinh(c+dx^n)}}$$

```
[Out] I*(e*x)^n*AppellF1(1/2,-p,1/2,3/2,b*(1-I*sinh(c+d*x^n))/(I*a+b),1/2-1/2*I*s
inh(c+d*x^n))*cosh(c+d*x^n)*(a+b*sinh(c+d*x^n))^p*2^(1/2)/d/e/n/(x^n)/(((a+
b*sinh(c+d*x^n))/(a-I*b))^p)/(1+I*sinh(c+d*x^n))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5430, 5428, 2744, 144, 143}

$$\int (ex)^{-1+n} (a+b\sinh(c+dx^n))^p\,dx = \frac{i\sqrt{2}x^{-n}(ex)^n\cosh(c+dx^n)(a+b\sinh(c+dx^n))^p\left(\frac{a+b\sinh(c+dx^n)}{a-ib}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-i\sinh
}{den\sqrt{1+i\sinh(c+dx^n)}}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] (I*Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - I*Sinh[c + d*x^n])/2, (
b*(1 - I*Sinh[c + d*x^n]))/(I*a + b)]*Cosh[c + d*x^n]*(a + b*Sinh[c + d*x^n
])^p)/(d*e*n*x^n*Sqrt[1 + I*Sinh[c + d*x^n]]*((a + b*Sinh[c + d*x^n])/(a -
I*b))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5430

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && In
tegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{(x^{-n}(ex)^n)\int x^{-1+n}(a+b\sinh(c+dx^n))^p\,dx}{e}$$

$$= \frac{(x^{-n}(ex)^n)\operatorname{Subst}\left(\int (a+b\sinh(c+dx))^p\,dx, x, x^n\right)}{en}$$

$$= -\frac{(ix^{-n}(ex)^n\cosh(c+dx^n))\operatorname{Subst}\left(\int \frac{(a-ibx)^p}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, i\sinh(c+dx^n)\right)}{den\sqrt{1-i\sinh(c+dx^n)}\sqrt{1+i\sinh(c+dx^n)}}$$

$$= -\frac{\left(ix^{-n}(ex)^n\cosh(c+dx^n)(a+b\sinh(c+dx^n))^p\left(-\frac{a+b\sinh(c+dx^n)}{-a+ib}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a+ib}+\frac{ibx}{-a+ib}\right.}{\sqrt{1-x}\sqrt{1+x}}\right.}{den\sqrt{1-i\sinh(c+dx^n)}\sqrt{1+i\sinh(c+dx^n)}}$$

$$= \frac{i\sqrt{2}x^{-n}(ex)^n\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-i\sinh(c+dx^n)),\frac{b(1-i\sinh(c+dx^n))}{ia+b}\right)\cosh(c+dx^n)(a+}{den\sqrt{1+i\sinh(c+dx^n)}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11

$$\int (ex)^{-1+n}(a+b\sinh(c+dx^n))^p\,dx$$

$$= \frac{x^{-n}(ex)^n\operatorname{AppellF1}\left(1+p,\frac{1}{2},\frac{1}{2},2+p,\frac{a+b\sinh(c+dx^n)}{a+ib},\frac{a+b\sinh(c+dx^n)}{a-ib}\right)\operatorname{sech}(c+dx^n)\sqrt{\frac{b(1-i\sinh(c+dx^n))}{ia+b}}\sqrt{\frac{b(1+}{}}}{bden(1+p)}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Sinh[c + d*x^n])/(a + I*b)
, (a + b*Sinh[c + d*x^n])/(a - I*b)]*Sech[c + d*x^n]*Sqrt[(b*(1 - I*Sinh[c
+ d*x^n]))/(I*a + b)]*Sqrt[(b*(1 + I*Sinh[c + d*x^n]))/((-I)*a + b)]*(a + b
*Sinh[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n)
```

Maple [F]

$$\int (ex)^{-1+n}(a+b\sinh(c+d\,x^n))^p\,dx$$

```
[In] int((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n}(a+b\sinh (c+dx^n))^p\,dx=\int (ex)^{n-1}(b\sinh (dx^n+c)+a)^p\,dx$$

[In] integrate((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="fricas")

[Out] integral((e*x)^(n - 1)*(b*sinh(d*x^n + c) + a)^p, x)

Sympy [F]

$$\int (ex)^{-1+n}(a+b\sinh (c+dx^n))^p\,dx=\int (ex)^{n-1}(a+b\sinh (c+dx^n))^p\,dx$$

[In] integrate((e*x)**(-1+n)*(a+b*sinh(c+d*x**n))**p,x)

[Out] Integral((e*x)**(n - 1)*(a + b*sinh(c + d*x**n))**p, x)

Maxima [F]

$$\int (ex)^{-1+n}(a+b\sinh (c+dx^n))^p\,dx=\int (ex)^{n-1}(b\sinh (dx^n+c)+a)^p\,dx$$

[In] integrate((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="maxima")

[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c) + a)^p, x)

Giac [F]

$$\int (ex)^{-1+n}(a+b\sinh (c+dx^n))^p\,dx=\int (ex)^{n-1}(b\sinh (dx^n+c)+a)^p\,dx$$

[In] integrate((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="giac")

[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c) + a)^p, x)

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (a+b\sinh (c+dx^n))^p \, dx = \int (e\,x)^{n-1}\, (a+b\sinh(c+d\,x^n))^p\, dx$$

```
[In] int((e*x)^(n - 1)*(a + b*sinh(c + d*x^n))^p,x)
[Out] int((e*x)^(n - 1)*(a + b*sinh(c + d*x^n))^p, x)
```

3.79 $\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx = \frac{x^{-2n}(ex)^{2n}\text{Int}(x^{-1+2n}(a+b\sinh(c+dx^n))^p, x)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx = \int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p, x])/(e*x^(2
*n))
```

Rubi steps

$$\text{integral} = \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n}(a+b\sinh(c+dx^n))^p \, dx}{e}$$

Mathematica [N/A]

Not integrable

Time = 6.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}(a+b\sinh(c+dx^n))^p\,dx=\int (ex)^{-1+2n}(a+b\sinh(c+dx^n))^p\,dx$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ex)^{-1+2n}(a+b\sinh(c+d\,x^n))^p dx$$

```
[In] int((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}(a+b\sinh(c+dx^n))^p\,dx=\int (ex)^{2\,n-1}(b\sinh(dx^n+c)+a)^p\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(2*n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 28.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (ex)^{-1+2n}(a+b\sinh(c+dx^n))^p\,dx=\int (ex)^{2n-1}(a+b\sinh(c+dx^n))^p\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sinh(c+d*x**n))**p,x)
[Out] Integral((e*x)**(2*n - 1)*(a + b*sinh(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx = \int (ex)^{2n-1} (b\sinh(dx^n+c)+a)^p \, dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 7.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx = \int (ex)^{2n-1} (b\sinh(dx^n+c)+a)^p \, dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx = \int (e\,x)^{2\,n-1}\,(a+b\,\sinh(c+d\,x^n))^p \, dx$$

```
[In] int((e*x)^(2*n - 1)*(a + b*sinh(c + d*x^n))^p,x)
[Out] int((e*x)^(2*n - 1)*(a + b*sinh(c + d*x^n))^p, x)
```

3.80 $\int (ex)^m \sinh^3(a+bx^n)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 220

$$\int (ex)^m \sinh^3(a+bx^n)\,dx = -\frac{3^{-\frac{1+m}{n}} e^{3a}(ex)^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3bx^n\right)}{8en} + \frac{3e^a(ex)^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)}{8en} - \frac{3e^{-a}(ex)^{1+m}(bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},bx^n\right)}{8en} + \frac{3^{-\frac{1+m}{n}} e^{-3a}(ex)^{1+m}(bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},3bx^n\right)}{8en}$$

```
[Out] -1/8*exp(3*a)*(e*x)^(1+m)*GAMMA((1+m)/n,-3*b*x^n)/(3^((1+m)/n))/e/n/((-b*x^
n)^((1+m)/n))+3/8*exp(a)*(e*x)^(1+m)*GAMMA((1+m)/n,-b*x^n)/e/n/((-b*x^n)^((
1+m)/n))-3/8*(e*x)^(1+m)*GAMMA((1+m)/n,b*x^n)/e/exp(a)/n/((b*x^n)^((1+m)/n)
)+1/8*(e*x)^(1+m)*GAMMA((1+m)/n,3*b*x^n)/(3^((1+m)/n))/e/exp(3*a)/n/((b*x^n
)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used

$= \{5470, 5468, 2250\}$

$$\int (ex)^m \sinh^3\left(a+bx^n\right)\,dx = -\frac{e^{3a}3^{-\frac{m+1}{n}}(ex)^{m+1}\left(-bx^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-3bx^n\right)}{8en} + \frac{3e^a(ex)^{m+1}\left(-bx^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-bx^n\right)}{8en} - \frac{3e^{-a}(ex)^{m+1}\left(bx^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},bx^n\right)}{8en} + \frac{e^{-3a}3^{-\frac{m+1}{n}}(ex)^{m+1}\left(bx^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},3bx^n\right)}{8en}$$

```
[In] Int[(e*x)^m*Sinh[a + b*x^n]^3,x]
```

```
[Out] -1/8*(E^(3*a)*(e*x)^(1 + m)*Gamma[(1 + m)/n, -3*b*x^n])/(3^((1 + m)/n)*e*n*
(-(b*x^n))^((1 + m)/n)) + (3*E^a*(e*x)^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/
(8*e*n*(-(b*x^n))^((1 + m)/n)) - (3*(e*x)^(1 + m)*Gamma[(1 + m)/n, b*x^n])/
(8*e*E^a*n*(b*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*Gamma[(1 + m)/n, 3*b*x^n])
/(8*3^((1 + m)/n)*e*E^(3*a)*n*(b*x^n)^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5468

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{3}{4}(ex)^m \sinh\left(a+bx^n\right)+\frac{1}{4}(ex)^m \sinh\left(3a+3bx^n\right)\right)\,dx$$

$$= \frac{1}{4}\int (ex)^m \sinh\left(3a+3bx^n\right)\,dx - \frac{3}{4}\int (ex)^m \sinh\left(a+bx^n\right)\,dx$$

$$= -\left(\frac{1}{8}\int e^{-3a-3bx^n}(ex)^m\,dx\right) + \frac{1}{8}\int e^{3a+3bx^n}(ex)^m\,dx + \frac{3}{8}\int e^{-a-bx^n}(ex)^m\,dx - \frac{3}{8}\int e^{a+bx^n}(ex)^m\,dx$$

$$= -\frac{3^{-\frac{1+m}{n}}e^{3a}(ex)^{1+m}\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3bx^n\right)}{8en} + \frac{3e^a(ex)^{1+m}\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)}{8en} - \frac{3e^{-a}(ex)^{1+m}\left(bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},bx^n\right)}{8en} + \frac{3^{-\frac{1+m}{n}}e^{-3a}(ex)^{1+m}\left(bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},3bx^n\right)}{8en}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.84

$$\int (ex)^m \sinh^3\left(a+bx^n\right)\,dx$$
$$= \frac{3^{-\frac{1+m}{n}}e^{-3a}x(ex)^m\left(-b^2x^{2n}\right)^{-\frac{1+m}{n}}\left(-e^{6a}(bx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3bx^n\right)+3^{\frac{1+m+n}{n}}e^{4a}(bx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)+(-\right.}{8n}$$

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^n]^3,x]
```

```
[Out] (x*(e*x)^m*(-(E^(6*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -3*b*x^n]) + 3^(
(1 + m + n)/n)*E^(4*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -(b*x^n)] + (-(
b*x^n))^((1 + m)/n)*(-(3^((1 + m + n)/n)*E^(2*a)*Gamma[(1 + m)/n, b*x^n]) +
 Gamma[(1 + m)/n, 3*b*x^n])))/(8*3^((1 + m)/n)*E^(3*a)*n*(-(b^2*x^(2*n)))^(
(1 + m)/n))
```

Maple [F]

$$\int (ex)^m \sinh\left(a+b\,x^n\right)^3 dx$$

```
[In] int((e*x)^m*sinh(a+b*x^n)^3,x)
```

```
[Out] int((e*x)^m*sinh(a+b*x^n)^3,x)
```

Fricas [F]

$$\int (ex)^m \sinh^3\left(a+bx^n\right)\,dx = \int (ex)^m \sinh\left(bx^n+a\right)^3\,dx$$

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh(b*x^n + a)^3, x)
```

Sympy [F]

$$\int (ex)^m \sinh^3(a+bx^n)\, dx = \int (ex)^m \sinh^3(a+bx^n)\, dx$$

```
[In] integrate((e*x)**m*sinh(a+b*x**n)**3,x)
[Out] Integral((e*x)**m*sinh(a + b*x**n)**3, x)
```

Maxima [F]

$$\int (ex)^m \sinh^3(a+bx^n)\, dx = \int (ex)^m \sinh(bx^n+a)^3\, dx$$

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate((e*x)^m*sinh(b*x^n + a)^3, x)
```

Giac [F]

$$\int (ex)^m \sinh^3(a+bx^n)\, dx = \int (ex)^m \sinh(bx^n+a)^3\, dx$$

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh^3(a+bx^n)\, dx = \int \sinh(a+b\,x^n)^3\,(e\,x)^m\, dx$$

```
[In] int(sinh(a + b*x^n)^3*(e*x)^m,x)
[Out] int(sinh(a + b*x^n)^3*(e*x)^m, x)
```

3.81 $\int (ex)^m \sinh^2(a+bx^n)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 143

$$\int (ex)^m \sinh^2(a+bx^n)\,dx = -\frac{(ex)^{1+m}}{2e(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2a}(ex)^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2bx^n\right)}{en} - \frac{2^{-\frac{1+m+2n}{n}} e^{-2a}(ex)^{1+m}(bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},2bx^n\right)}{en}$$

```
[Out] -1/2*(e*x)^(1+m)/e/(1+m)-exp(2*a)*(e*x)^(1+m)*GAMMA((1+m)/n,-2*b*x^n)/(2^((
1+m+2*n)/n))/e/n/((-b*x^n)^((1+m)/n))-(e*x)^(1+m)*GAMMA((1+m)/n,2*b*x^n)/(2
^((1+m+2*n)/n))/e/exp(2*a)/n/((b*x^n)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5470, 5469, 2250}

$$\int (ex)^m \sinh^2(a+bx^n)\,dx = -\frac{e^{2a}2^{-\frac{m+2n+1}{n}}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-2bx^n\right)}{en} - \frac{e^{-2a}2^{-\frac{m+2n+1}{n}}(ex)^{m+1}(bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},2bx^n\right)}{en} - \frac{(ex)^{m+1}}{2e(m+1)}$$

```
[In] Int[(e*x)^m*Sinh[a + b*x^n]^2,x]
```

```
[Out] -1/2*(e*x)^(1 + m)/(e*(1 + m)) - (E^(2*a)*(e*x)^(1 + m)*Gamma[(1 + m)/n, -2
*b*x^n])/(2^((1 + m + 2*n)/n)*e*n*(-(b*x^n))^((1 + m)/n)) - ((e*x)^(1 + m)*
Gamma[(1 + m)/n, 2*b*x^n])/(2^((1 + m + 2*n)/n)*e*E^(2*a)*n*(b*x^n)^((1 + m
)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5469

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}(ex)^m + \frac{1}{2}(ex)^m \cosh\left(2a+2bx^n\right)\right) dx \\
&= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{2}\int (ex)^m \cosh\left(2a+2bx^n\right) dx \\
&= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{4}\int e^{-2a-2bx^n}(ex)^m\, dx + \frac{1}{4}\int e^{2a+2bx^n}(ex)^m\, dx \\
&= -\frac{(ex)^{1+m}}{2e(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2a}(ex)^{1+m}\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n}, -2bx^n\right)}{en} \\
&\quad - \frac{2^{-\frac{1+m+2n}{n}} e^{-2a}(ex)^{1+m}\left(bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n}, 2bx^n\right)}{en}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.82

$$\int (ex)^m \sinh^2\left(a+bx^n\right) dx =$$

$$-\frac{x(ex)^m\left(2n + 2^{-\frac{1+m}{n}} e^{2a}(1+m)\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n}, -2bx^n\right) + 2^{-\frac{1+m}{n}} e^{-2a}(1+m)\left(bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n}, 2bx\right.}{4(1+m)n}$$

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^n]^2,x]
```

[Out] -1/4*(x*(e*x)^m*(2*n + (E^(2*a)*(1 + m)*Gamma[(1 + m)/n, -2*b*x^n])/(2^((1 + m)/n)*(-(b*x^n))^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, 2*b*x^n])/(2^((1 + m)/n)*E^(2*a)*(b*x^n)^((1 + m)/n))))/((1 + m)*n)

Maple [F]

$$\int (ex)^m \sinh\left(a+b\,x^n\right)^2 dx$$

[In] int((e*x)^m*sinh(a+b*x^n)^2,x)

[Out] int((e*x)^m*sinh(a+b*x^n)^2,x)

Fricas [F]

$$\int (ex)^m \sinh^2\left(a+bx^n\right)\,dx = \int (ex)^m \sinh\left(bx^n+a\right)^2\,dx$$

[In] integrate((e*x)^m*sinh(a+b*x^n)^2,x, algorithm="fricas")

[Out] integral((e*x)^m*sinh(b*x^n + a)^2, x)

Sympy [F]

$$\int (ex)^m \sinh^2\left(a+bx^n\right)\,dx = \int (ex)^m \sinh^2\left(a+bx^n\right) dx$$

[In] integrate((e*x)**m*sinh(a+b*x**n)**2,x)

[Out] Integral((e*x)**m*sinh(a + b*x**n)**2, x)

Maxima [F]

$$\int (ex)^m \sinh^2\left(a+bx^n\right)\,dx = \int (ex)^m \sinh\left(bx^n+a\right)^2\,dx$$

[In] integrate((e*x)^m*sinh(a+b*x^n)^2,x, algorithm="maxima")

[Out] 1/4*e^m*integrate(e^(2*b*x^n + m*log(x) + 2*a), x) + 1/4*e^m*integrate(e^(-2*b*x^n + m*log(x) - 2*a), x) - 1/2*(e*x)^(m + 1)/(e*(m + 1))

Giac [F]

$$\int (ex)^m \sinh^2(a+bx^n)\ dx = \int (ex)^m \sinh(bx^n+a)^2\ dx$$

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh^2(a+bx^n)\ dx = \int \sinh(a+b\,x^n)^2\,(e\,x)^m\,dx$$

```
[In] int(sinh(a + b*x^n)^2*(e*x)^m,x)
[Out] int(sinh(a + b*x^n)^2*(e*x)^m, x)
```

3.82 $\int (ex)^m \sinh(a+bx^n)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int (ex)^m \sinh(a+bx^n)\, dx = -\frac{e^a (ex)^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -bx^n\right)}{2en} + \frac{e^{-a} (ex)^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, bx^n\right)}{2en}$$

```
[Out] -1/2*exp(a)*(e*x)^(1+m)*GAMMA((1+m)/n,-b*x^n)/e/n/((-b*x^n)^((1+m)/n))+1/2*
(e*x)^(1+m)*GAMMA((1+m)/n,b*x^n)/e/exp(a)/n/((b*x^n)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5468, 2250}

$$\int (ex)^m \sinh(a+bx^n)\, dx = \frac{e^{-a} (ex)^{m+1} (bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, bx^n\right)}{2en} - \frac{e^a (ex)^{m+1} (-bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -bx^n\right)}{2en}$$

```
[In] Int[(e*x)^m*Sinh[a + b*x^n],x]
```

```
[Out] -1/2*(E^a*(e*x)^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(e*n*(-(b*x^n))^((1 + m
)/n)) + ((e*x)^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(2*e*E^a*n*(b*x^n)^((1 + m)
/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5468

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int e^{-a-bx^n}(ex)^m\,dx\right)+\frac{1}{2}\int e^{a+bx^n}(ex)^m\,dx\\ &= -\frac{e^a(ex)^{1+m}\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)}{2en}+\frac{e^{-a}(ex)^{1+m}\left(bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},bx^n\right)}{2en}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\begin{aligned}&\int (ex)^m\sinh\left(a+bx^n\right)\,dx\\ &= \frac{-e^a x(ex)^m\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)+e^{-a}x(ex)^m\left(bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},bx^n\right)}{2n}\end{aligned}$$

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^n],x]
```

```
[Out] (-((E^a*x*(e*x)^m*Gamma[(1 + m)/n, -(b*x^n)])/(-(b*x^n))^((1 + m)/n)) + (x*
(e*x)^m*Gamma[(1 + m)/n, b*x^n])/(E^a*(b*x^n)^((1 + m)/n)))/(2*n)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.16

method	result
meijerg	$\frac{(ex)^m x \,\mathrm{hypergeom}\left(\left[\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{1+m}+\frac{(ex)^m x^{n+1} b\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2n}+\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\cosh}{n+m+1}$

```
[In] int((e*x)^m*sinh(a+b*x^n),x,method=_RETURNVERBOSE)
```

[Out] (e*x)^m/(1+m)*x*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],1/4*x^(2*n)
*b^2)*sinh(a)+(e*x)^m/(n+m+1)*x^(n+1)*b*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,
3/2+1/2/n*m+1/2/n],1/4*x^(2*n)*b^2)*cosh(a)

Fricas [F]

$$\int (ex)^m \sinh \left(a+bx^n\right) \, dx = \int (ex)^m \sinh \left(bx^n+a\right) \, dx$$

[In] integrate((e*x)^m*sinh(a+b*x^n),x, algorithm="fricas")

[Out] integral((e*x)^m*sinh(b*x^n + a), x)

Sympy [F]

$$\int (ex)^m \sinh \left(a+bx^n\right) \, dx = \int (ex)^m \sinh \left(a+bx^n\right) dx$$

[In] integrate((e*x)**m*sinh(a+b*x**n),x)

[Out] Integral((e*x)**m*sinh(a + b*x**n), x)

Maxima [F]

$$\int (ex)^m \sinh \left(a+bx^n\right) \, dx = \int (ex)^m \sinh \left(bx^n+a\right) \, dx$$

[In] integrate((e*x)^m*sinh(a+b*x^n),x, algorithm="maxima")

[Out] integrate((e*x)^m*sinh(b*x^n + a), x)

Giac [F]

$$\int (ex)^m \sinh \left(a+bx^n\right) \, dx = \int (ex)^m \sinh \left(bx^n+a\right) \, dx$$

[In] integrate((e*x)^m*sinh(a+b*x^n),x, algorithm="giac")

[Out] integrate((e*x)^m*sinh(b*x^n + a), x)

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sinh (a + bx^n) \, dx = \int \sinh(a + b\,x^n) \, (e\,x)^m \, dx$$

```
[In] int(sinh(a + b*x^n)*(e*x)^m,x)
[Out] int(sinh(a + b*x^n)*(e*x)^m, x)
```

3.83 $\int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx = x^{-m}(ex)^m \operatorname{Int}\left(x^m \operatorname{csch}^2(a+bx^n),x\right)$$

```
[Out] (e*x)^m*Unintegrable(x^m*csch(a+b*x^n)^2,x)/(x^m)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx = \int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx$$

```
[In] Int[(e*x)^m*Csch[a + b*x^n]^2,x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b*x^n]^2, x])/x^m
```

Rubi steps

$$\text{integral} = \left(x^{-m}(ex)^m\right)\int x^m \operatorname{csch}^2(a+bx^n)\,dx$$

Mathematica [N/A]

Not integrable

Time = 18.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx = \int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx$$

```
[In] Integrate[(e*x)^m*Csch[a + b*x^n]^2,x]
[Out] Integrate[(e*x)^m*Csch[a + b*x^n]^2, x]
```

Maple [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m}{\sinh\left(a+b\,x^n\right)^2}dx$$

```
[In] int((e*x)^m/sinh(a+b*x^n)^2,x)
[Out] int((e*x)^m/sinh(a+b*x^n)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx = \int \frac{(ex)^m}{\sinh\left(bx^n+a\right)^2}\,dx$$

```
[In] integrate((e*x)^m/sinh(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral((e*x)^m/sinh(b*x^n + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx = \int \frac{(ex)^m}{\sinh^2\left(a+bx^n\right)}\,dx$$

```
[In] integrate((e*x)**m/sinh(a+b*x**n)**2,x)
[Out] Integral((e*x)**m/sinh(a + b*x**n)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 7.69

$$\int (ex)^m \text{csch}^2(a+bx^n)\,dx = \int \frac{(ex)^m}{\sinh(bx^n+a)^2}\,dx$$

```
[In] integrate((e*x)^m/sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -4*e^m*(m - n + 1)*integrate(1/4*x^m/(b*n*x^n + b*n*e^(b*x^n + n*log(x) + a
)), x) + 4*e^m*(m - n + 1)*integrate(-1/4*x^m/(b*n*x^n - b*n*e^(b*x^n + n*l
og(x) + a)), x) + 2*e^m*x*x^m/(b*n*x^n - b*n*e^(2*b*x^n + n*log(x) + 2*a))
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (ex)^m \text{csch}^2(a+bx^n)\,dx = \int \frac{(ex)^m}{\sinh(bx^n+a)^2}\,dx$$

```
[In] integrate((e*x)^m/sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/sinh(b*x^n + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (ex)^m \text{csch}^2(a+bx^n)\,dx = \int \frac{(e\,x)^m}{\sinh(a+b\,x^n)^2}\,dx$$

```
[In] int((e*x)^m/sinh(a + b*x^n)^2,x)
```

```
[Out] int((e*x)^m/sinh(a + b*x^n)^2, x)
```

3.84 $\int x^{-1-n} \sinh(a+bx^n)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x^{-1-n} \sinh(a+bx^n)\,dx = \frac{b\cosh(a)\text{Chi}(bx^n)}{n} - \frac{x^{-n}\sinh(a+bx^n)}{n} + \frac{b\sinh(a)\text{Shi}(bx^n)}{n}$$

```
[Out] b*Chi(b*x^n)*cosh(a)/n+b*Shi(b*x^n)*sinh(a)/n-sinh(a+b*x^n)/n/(x^n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5428, 3378, 3384, 3379, 3382}

$$\int x^{-1-n} \sinh(a+bx^n)\,dx = \frac{b\cosh(a)\text{Chi}(bx^n)}{n} + \frac{b\sinh(a)\text{Shi}(bx^n)}{n} - \frac{x^{-n}\sinh(a+bx^n)}{n}$$

```
[In] Int[x^(-1 - n)*Sinh[a + b*x^n],x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^n])/n - Sinh[a + b*x^n]/(n*x^n) + (b*Sinh[a]*Si
nhIntegral[b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx, x, x^n\right)}{n} \\
&= -\frac{x^{-n}\sinh\left(a+bx^n\right)}{n} + \frac{b\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx, x, x^n\right)}{n} \\
&= -\frac{x^{-n}\sinh\left(a+bx^n\right)}{n} + \frac{(b\cosh(a))\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, x^n\right)}{n} \\
&\quad + \frac{(b\sinh(a))\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, x^n\right)}{n} \\
&= \frac{b\cosh(a)\text{Chi}(bx^n)}{n} - \frac{x^{-n}\sinh\left(a+bx^n\right)}{n} + \frac{b\sinh(a)\text{Shi}(bx^n)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x^{-1-n}\sinh\left(a+bx^n\right)\,dx = \frac{x^{-n}(bx^n\cosh(a)\text{Chi}(bx^n)-\sinh\left(a+bx^n\right)+bx^n\sinh(a)\text{Shi}(bx^n))}{n}$$

```
[In] Integrate[x^(-1 - n)*Sinh[a + b*x^n],x]
[Out] (b*x^n*Cosh[a]*CoshIntegral[b*x^n] - Sinh[a + b*x^n] + b*x^n*Sinh[a]*SinhIn
tegral[b*x^n])/(n*x^n)
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

method	result	size
risch	$\frac{\left(-b\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(b\,x^n)x^n-b\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-b\,x^n)x^n+\mathrm{e}^{-a-b\,x^n}-\mathrm{e}^{a+b\,x^n}\right)x^{-n}}{2n}$	66

```
[In] int(x^(-1-n)*sinh(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] 1/2*(-b*exp(-a)*Ei(1,b*x^n)*x^n-b*exp(a)*Ei(1,-b*x^n)*x^n+exp(-a-b*x^n)-exp
(a+b*x^n))/n/(x^n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(45) = 90.

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.09

$$\int x^{-1-n}\sinh\left(a+bx^n\right)\,dx = \frac{((b\cosh(a)+b\sinh(a))\cosh(n\log(x))+(b\cosh(a)+b\sinh(a))\sinh(n\log(x)))\text{Ei}(b\cosh(n\log(x))+b\sinh}{}$$

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n),x, algorithm="fricas")
[Out] 1/2*(((b*cosh(a) + b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh
(n*log(x)))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + ((b*cosh(a) - b*sinh(
a))*cosh(n*log(x)) + (b*cosh(a) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*l
og(x)) - b*sinh(n*log(x))) - 2*sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a
))/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

Sympy [F]

$$\int x^{-1-n}\sinh\left(a+bx^n\right)\,dx=\int x^{-n-1}\sinh\left(a+bx^n\right)dx$$

```
[In] integrate(x**(-1-n)*sinh(a+b*x**n),x)
[Out] Integral(x**(-n - 1)*sinh(a + b*x**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int x^{-1-n}\sinh\left(a+bx^n\right)\,dx=\frac{be^{(-a)}\Gamma(-1,bx^n)}{2\,n}+\frac{be^{a}\Gamma(-1,-bx^n)}{2\,n}$$

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n),x, algorithm="maxima")
[Out] 1/2*b*e^(-a)*gamma(-1, b*x^n)/n + 1/2*b*e^a*gamma(-1, -b*x^n)/n
```

Giac [F]

$$\int x^{-1-n}\sinh\left(a+bx^n\right)\,dx=\int x^{-n-1}\sinh\left(bx^n+a\right)\,dx$$

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(-n - 1)*sinh(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sinh\left(a+bx^n\right)\,dx=\int\frac{\sinh(a+b\,x^n)}{x^{n+1}}\,dx$$

```
[In] int(sinh(a + b*x^n)/x^(n + 1),x)
[Out] int(sinh(a + b*x^n)/x^(n + 1), x)
```

3.85 $\int x^{-1-n}\sinh^2(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int x^{-1-n}\sinh^2(a+bx^n)\,dx=\frac{x^{-n}}{2n}-\frac{x^{-n}\cosh(2(a+bx^n))}{2n}+\frac{b\text{Chi}(2bx^n)\sinh(2a)}{n}+\frac{b\cosh(2a)\text{Shi}(2bx^n)}{n}$$

```
[Out] 1/2/n/(x^n)-1/2*cosh(2*a+2*b*x^n)/n/(x^n)+b*cosh(2*a)*Shi(2*b*x^n)/n+b*Chi(
2*b*x^n)*sinh(2*a)/n
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5470, 5429, 3378, 3384, 3379, 3382}

$$\int x^{-1-n}\sinh^2(a+bx^n)\,dx=\frac{b\sinh(2a)\text{Chi}(2bx^n)}{n}+\frac{b\cosh(2a)\text{Shi}(2bx^n)}{n}-\frac{x^{-n}\cosh(2(a+bx^n))}{2n}+\frac{x^{-n}}{2n}$$

```
[In] Int[x^(-1 - n)*Sinh[a + b*x^n]^2,x]
```

```
[Out] 1/(2*n*x^n) - Cosh[2*(a + b*x^n)]/(2*n*x^n) + (b*CoshIntegral[2*b*x^n]*Sinh
[2*a])/n + (b*Cosh[2*a]*SinhIntegral[2*b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}x^{-1-n}+\frac{1}{2}x^{-1-n}\cosh\left(2a+2bx^n\right)\right)\,dx \\
&= \frac{x^{-n}}{2n}+\frac{1}{2}\int x^{-1-n}\cosh\left(2a+2bx^n\right)\,dx \\
&= \frac{x^{-n}}{2n}+\frac{\text{Subst}\left(\int\frac{\cosh(2a+2bx)}{x^2}\,dx,x,x^n\right)}{2n}
\end{aligned}$$

$$= \frac{x^{-n}}{2n} - \frac{x^{-n}\cosh\left(2(a+bx^n)\right)}{2n} + \frac{b\text{Subst}\left(\int \frac{\sinh(2a+2bx)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^{-n}}{2n} - \frac{x^{-n}\cosh\left(2(a+bx^n)\right)}{2n} + \frac{(b\cosh(2a))\text{Subst}\left(\int \frac{\sinh(2bx)}{x}\,dx, x, x^n\right)}{n} + \frac{(b\sinh(2a))\text{Subst}\left(\int \frac{\cosh(2bx)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^{-n}}{2n} - \frac{x^{-n}\cosh\left(2(a+bx^n)\right)}{2n} + \frac{b\text{Chi}(2bx^n)\sinh(2a)}{n} + \frac{b\cosh(2a)\text{Shi}(2bx^n)}{n}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int x^{-1-n}\sinh^2\left(a+bx^n\right)\,dx = \frac{x^{-n}\left(bx^n\text{Chi}(2bx^n)\sinh(2a) - \sinh^2\left(a+bx^n\right) + bx^n\cosh(2a)\text{Shi}(2bx^n)\right)}{n}$$

```
[In] Integrate[x^(-1 - n)*Sinh[a + b*x^n]^2,x]
```

```
[Out] (b*x^n*CoshIntegral[2*b*x^n]*Sinh[2*a] - Sinh[a + b*x^n]^2 + b*x^n*Cosh[2*a
]*SinhIntegral[2*b*x^n])/(n*x^n)
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{\left(2b\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2b\,x^n)x^n - 2b\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2b\,x^n)x^n - \mathrm{e}^{-2a-2b\,x^n} - \mathrm{e}^{2a+2b\,x^n} + 2\right)x^{-n}}{4n}$	75

```
[In] int(x^(-1-n)*sinh(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*b*exp(-2*a)*Ei(1,2*b*x^n)*x^n-2*b*exp(2*a)*Ei(1,-2*b*x^n)*x^n-exp(-2
*a-2*b*x^n)-exp(2*a+2*b*x^n)+2)/(x^n)/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.72

$$\int x^{-1-n}\sinh^2\left(a+bx^n\right)\,dx$$
$$= \frac{((b\cosh(2a) + b\sinh(2a))\cosh(n\log(x)) + (b\cosh(2a) + b\sinh(2a))\sinh(n\log(x)))\mathrm{Ei}(2b\cosh(n\log}{}$$

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((b*cosh(2*a) + b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) + b*sinh(2*
a))*sinh(n*log(x)))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) - ((b*cosh(
2*a) - b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) - b*sinh(2*a))*sinh(n*log
(x)))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) - cosh(b*cosh(n*log(x))
+ b*sinh(n*log(x)) + a)^2 - sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^2
 + 1)/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

Sympy [F]

$$\int x^{-1-n}\sinh^2\left(a+bx^n\right)\,dx = \int x^{-n-1}\sinh^2\left(a+bx^n\right)dx$$

```
[In] integrate(x**(-1-n)*sinh(a+b*x**n)**2,x)
```

```
[Out] Integral(x**(-n - 1)*sinh(a + b*x**n)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int x^{-1-n}\sinh^2\left(a+bx^n\right)\,dx = -\frac{be^{(-2a)}\Gamma(-1,2bx^n)}{2n} + \frac{be^{(2a)}\Gamma(-1,-2bx^n)}{2n} + \frac{1}{2nx^n}$$

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*e^(-2*a)*gamma(-1, 2*b*x^n)/n + 1/2*b*e^(2*a)*gamma(-1, -2*b*x^n)/n
+ 1/2/(n*x^n)
```

Giac [F]

$$\int x^{-1-n}\sinh^2\left(a+bx^n\right)\,dx=\int x^{-n-1}\sinh\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^(-n - 1)*sinh(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sinh^2\left(a+bx^n\right)\,dx=\int \frac{\sinh(a+b\,x^n)^2}{x^{n+1}}\,dx$$

```
[In] int(sinh(a + b*x^n)^2/x^(n + 1),x)
[Out] int(sinh(a + b*x^n)^2/x^(n + 1), x)
```

3.86 $\int x^{-1-n}\sinh^3(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int x^{-1-n}\sinh^3(a+bx^n)\,dx = -\frac{3b\cosh(a)\text{Chi}(bx^n)}{4n}+\frac{3b\cosh(3a)\text{Chi}(3bx^n)}{4n}+\frac{3x^{-n}\sinh(a+bx^n)}{4n}-\frac{x^{-n}\sinh(3(a+bx^n))}{4n}-\frac{3b\sinh(a)\text{Shi}(bx^n)}{4n}+\frac{3b\sinh(3a)\text{Shi}(3bx^n)}{4n}$$

```
[Out] -3/4*b*Chi(b*x^n)*cosh(a)/n+3/4*b*Chi(3*b*x^n)*cosh(3*a)/n-3/4*b*Shi(b*x^n)
*sinh(a)/n+3/4*b*Shi(3*b*x^n)*sinh(3*a)/n+3/4*sinh(a+b*x^n)/n/(x^n)-1/4*sin
h(3*a+3*b*x^n)/n/(x^n)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5470, 5428, 3378, 3384, 3379, 3382}

$$\int x^{-1-n}\sinh^3(a+bx^n)\,dx = -\frac{3b\cosh(a)\text{Chi}(bx^n)}{4n}+\frac{3b\cosh(3a)\text{Chi}(3bx^n)}{4n}-\frac{3b\sinh(a)\text{Shi}(bx^n)}{4n}+\frac{3b\sinh(3a)\text{Shi}(3bx^n)}{4n}+\frac{3x^{-n}\sinh(a+bx^n)}{4n}-\frac{x^{-n}\sinh(3(a+bx^n))}{4n}$$

```
[In] Int[x^(-1 - n)*Sinh[a + b*x^n]^3,x]
```

```
[Out] (-3*b*Cosh[a]*CoshIntegral[b*x^n])/(4*n) + (3*b*Cosh[3*a]*CoshIntegral[3*b*
x^n])/(4*n) + (3*Sinh[a + b*x^n])/(4*n*x^n) - Sinh[3*(a + b*x^n)]/(4*n*x^n)
```

```
 - (3*b*Sinh[a]*SinhIntegral[b*x^n])/(4*n) + (3*b*Sinh[3*a]*SinhIntegral[3*
b*x^n])/(4*n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5470

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{3}{4}x^{-1-n}\sinh\left(a+bx^n\right)+\frac{1}{4}x^{-1-n}\sinh\left(3a+3bx^n\right)\right)\,dx$$

$$= \frac{1}{4}\int x^{-1-n}\sinh\left(3a+3bx^n\right)\,dx - \frac{3}{4}\int x^{-1-n}\sinh\left(a+bx^n\right)\,dx$$

$$= \frac{\operatorname{Subst}\left(\int \frac{\sinh(3a+3bx)}{x^2}\,dx,x,x^n\right)}{4n} - \frac{3\operatorname{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx,x,x^n\right)}{4n}$$

$$= \frac{3x^{-n}\sinh\left(a+bx^n\right)}{4n} - \frac{x^{-n}\sinh\left(3(a+bx^n)\right)}{4n} - \frac{(3b)\operatorname{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx,x,x^n\right)}{4n} + \frac{(3b)\operatorname{Subst}\left(\int \frac{\cosh(3a+3bx)}{x}\,dx,x,x^n\right)}{4n}$$

$$= \frac{3x^{-n}\sinh\left(a+bx^n\right)}{4n} - \frac{x^{-n}\sinh\left(3(a+bx^n)\right)}{4n} - \frac{(3b\cosh(a))\operatorname{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx,x,x^n\right)}{4n} + \frac{(3b\cosh(3a))\operatorname{Subst}\left(\int \frac{\cosh(3bx)}{x}\,dx,x,x^n\right)}{4n} - \frac{(3b\sinh(a))\operatorname{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx,x,x^n\right)}{4n} + \frac{(3b\sinh(3a))\operatorname{Subst}\left(\int \frac{\sinh(3bx)}{x}\,dx,x,x^n\right)}{4n}$$

$$= -\frac{3b\cosh(a)\operatorname{Chi}(bx^n)}{4n} + \frac{3b\cosh(3a)\operatorname{Chi}(3bx^n)}{4n} + \frac{3x^{-n}\sinh\left(a+bx^n\right)}{4n} - \frac{x^{-n}\sinh\left(3(a+bx^n)\right)}{4n} - \frac{3b\sinh(a)\operatorname{Shi}(bx^n)}{4n} + \frac{3b\sinh(3a)\operatorname{Shi}(3bx^n)}{4n}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int x^{-1-n}\sinh^3\left(a+bx^n\right)\,dx = -\frac{x^{-n}(3bx^n\cosh(a)\operatorname{Chi}(bx^n) - 3bx^n\cosh(3a)\operatorname{Chi}(3bx^n) - 3\sinh\left(a+bx^n\right) + \sinh\left(3(a+bx^n)\right) + 3bx^n \sin}{4n}$$

```
[In] Integrate[x^(-1 - n)*Sinh[a + b*x^n]^3,x]
```

```
[Out] -1/4*(3*b*x^n*Cosh[a]*CoshIntegral[b*x^n] - 3*b*x^n*Cosh[3*a]*CoshIntegral[
3*b*x^n] - 3*Sinh[a + b*x^n] + Sinh[3*(a + b*x^n)] + 3*b*x^n*Sinh[a]*SinhIn
tegral[b*x^n] - 3*b*x^n*Sinh[3*a]*SinhIntegral[3*b*x^n])/(n*x^n)
```

Maple [A] (verified)

Time = 5.77 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

method	result
risch	$\frac{\left(3b\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(b\,x^n)x^n+3b\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-b\,x^n)x^n-3b\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3b\,x^n)x^n-3b\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3b\,x^n)x^n+3\,\mathrm{e}^{a+b\,x^n}-\mathrm{e}^{3a+3b\,x^n}+\mathrm{e}^{-3a-3b\,x^n}-3\,\mathrm{e}^{-a-b}\right.}{8n}$

[In] int(x^(-1-n)*sinh(a+b*x^n)^3,x,method=_RETURNVERBOSE)

[Out] 1/8*(3*b*exp(-a)*Ei(1,b*x^n)*x^n+3*b*exp(a)*Ei(1,-b*x^n)*x^n-3*b*exp(3*a)*Ei(1,-3*b*x^n)*x^n-3*b*exp(-3*a)*Ei(1,3*b*x^n)*x^n+3*exp(a+b*x^n)-exp(3*a+3*b*x^n)+exp(-3*a-3*b*x^n)-3*exp(-a-b*x^n))/(x^n)/n

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(102) = 204.

Time = 0.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.68

$$\int x^{-1-n}\sinh^3\left(a+bx^n\right)\,dx =$$

$$-\frac{2\,\sinh\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)^{3}-3\left(\left(b\cosh\left(3\,a\right)+b\sinh\left(3\,a\right)\right)\cosh\left(n\log\left(x\right)\right)+\left(b\cosh\right.\right.}{}$$

[In] integrate(x^(-1-n)*sinh(a+b*x^n)^3,x, algorithm="fricas")

[Out] -1/8*(2*sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^3 - 3*((b*cosh(3*a) + b*sinh(3*a))*cosh(n*log(x)) + (b*cosh(3*a) + b*sinh(3*a))*sinh(n*log(x)))*Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) + 3*((b*cosh(a) + b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh(n*log(x)))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + 3*((b*cosh(a) - b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))) - 3*((b*cosh(3*a) - b*sinh(3*a))*cosh(n*log(x)) + (b*cosh(3*a) - b*sinh(3*a))*sinh(n*log(x)))*Ei(-3*b*cosh(n*log(x)) - 3*b*sinh(n*log(x))) + 6*(cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^2 - 1)*sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a))/(n*cosh(n*log(x)) + n*sinh(n*log(x)))

Sympy [F]

$$\int x^{-1-n}\sinh^3(a+bx^n)\,dx=\int x^{-n-1}\sinh^3(a+bx^n)\,dx$$

```
[In] integrate(x**(-1-n)*sinh(a+b*x**n)**3,x)
[Out] Integral(x**(-n - 1)*sinh(a + b*x**n)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int x^{-1-n}\sinh^3(a+bx^n)\,dx=\frac{3\,be^{(-3\,a)}\Gamma(-1,3\,bx^n)}{8\,n}-\frac{3\,be^{(-a)}\Gamma(-1,bx^n)}{8\,n}-\frac{3\,be^{a}\Gamma(-1,-bx^n)}{8\,n}+\frac{3\,be^{(3\,a)}\Gamma(-1,-3\,bx^n)}{8\,n}$$

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^3,x, algorithm="maxima")
[Out] 3/8*b*e^(-3*a)*gamma(-1, 3*b*x^n)/n - 3/8*b*e^(-a)*gamma(-1, b*x^n)/n - 3/8
*b*e^a*gamma(-1, -b*x^n)/n + 3/8*b*e^(3*a)*gamma(-1, -3*b*x^n)/n
```

Giac [F]

$$\int x^{-1-n}\sinh^3(a+bx^n)\,dx=\int x^{-n-1}\sinh(bx^n+a)^3\,dx$$

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(x^(-n - 1)*sinh(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sinh^3(a+bx^n)\,dx=\int\frac{\sinh(a+b\,x^n)^3}{x^{n+1}}\,dx$$

```
[In] int(sinh(a + b*x^n)^3/x^(n + 1),x)
[Out] int(sinh(a + b*x^n)^3/x^(n + 1), x)
```

3.87 $\int x^{-1+\frac{n}{2}} \sinh(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int x^{-1+\frac{n}{2}} \sinh(a+bx^n)\,dx = -\frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n} + \frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n}$$

```
[Out] -1/2*erf(x^(1/2*n)*b^(1/2))*Pi^(1/2)/exp(a)/n/b^(1/2)+1/2*exp(a)*erfi(x^(1/
2*n)*b^(1/2))*Pi^(1/2)/n/b^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5464, 5406, 2235, 2236}

$$\int x^{-1+\frac{n}{2}} \sinh(a+bx^n)\,dx = \frac{\sqrt{\pi}e^{a}\text{erfi}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n} - \frac{\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n}$$

```
[In] Int[x^(-1 + n/2)*Sinh[a + b*x^n],x]
```

```
[Out] -1/2*(Sqrt[Pi]*Erf[Sqrt[b]*x^(n/2)])/(Sqrt[b]*E^a*n) + (E^a*Sqrt[Pi]*Erfi[S
qrt[b]*x^(n/2)])/(2*Sqrt[b]*n)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5464

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/(m + 1), Subst[Int[(a + b*Sinh[c + d*x^Simplify[n/(m + 1)]])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegerQ[p] && N
eQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx, x, x^{n/2}\right)}{n} \\
&= -\frac{\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, x^{n/2}\right)}{n} + \frac{\text{Subst}\left(\int e^{a+bx^2}\,dx, x, x^{n/2}\right)}{n} \\
&= -\frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n} + \frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int x^{-1+\frac{n}{2}}\sinh\left(a+bx^n\right)\,dx = \frac{e^{-a}\sqrt{\pi}\left(-\text{erf}\left(\sqrt{b}x^{n/2}\right)+e^{2a}\text{erfi}\left(\sqrt{b}x^{n/2}\right)\right)}{2\sqrt{b}n}$$

```
[In] Integrate[x^(-1 + n/2)*Sinh[a + b*x^n],x]
[Out] (Sqrt[Pi]*(-Erf[Sqrt[b]*x^(n/2)] + E^(2*a)*Erfi[Sqrt[b]*x^(n/2)]))/(2*Sqrt[
b]*E^a*n)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result	si
risch	$-\frac{e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2n\sqrt{b}}+\frac{e^{a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}\,x^{\frac{n}{2}}\right)}{2n\sqrt{-b}}$	5
meijerg	$\frac{\sqrt{2}\,\sqrt{\pi}\left(\frac{\sqrt{ib}\,\sqrt{2}\,\mathrm{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2\sqrt{b}}+\frac{\sqrt{ib}\,\sqrt{2}\,\mathrm{erfi}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2\sqrt{b}}\right)\sinh(a)}{2\sqrt{ib}\,n}-\frac{i\sqrt{2}\,\sqrt{\pi}\left(-\frac{\sqrt{2}\,(ib)^{\frac{3}{2}}\,\mathrm{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{2}\,(ib)^{\frac{3}{2}}\,\mathrm{erfi}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2b^{\frac{3}{2}}}\right)\cosh(a)}{2\sqrt{ib}\,n}$	1

```
[In] int(x^(-1+1/2*n)*sinh(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/n*exp(-a)*Pi^(1/2)/b^(1/2)*erf(x^(1/2*n)*b^(1/2))+1/2/n*exp(a)*Pi^(1/2
)/(-b)^(1/2)*erf((-b)^(1/2)*x^(1/2*n))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int x^{-1+\frac{n}{2}}\sinh\left(a+bx^n\right)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-b}(\cosh(a)+\sinh(a))\,\mathrm{erf}\left(\sqrt{-b}x\cosh\left(\frac{1}{2}(n-2)\log(x)\right)+\sqrt{-b}x\sinh\left(\frac{1}{2}(n-2)\log(x)\right)\right)+\sqrt{\pi}}{2\,bn}$$

```
[In] integrate(x^(-1+1/2*n)*sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x*cosh(1/2*(n - 2)
*log(x)) + sqrt(-b)*x*sinh(1/2*(n - 2)*log(x))) + sqrt(pi)*sqrt(b)*(cosh(a)
 - sinh(a))*erf(sqrt(b)*x*cosh(1/2*(n - 2)*log(x)) + sqrt(b)*x*sinh(1/2*(n
- 2)*log(x))))/(b*n)
```

Sympy [F]

$$\int x^{-1+\frac{n}{2}}\sinh\left(a+bx^n\right)\,dx=\int x^{\frac{n}{2}-1}\sinh\left(a+bx^n\right)dx$$

```
[In] integrate(x**(-1+1/2*n)*sinh(a+b*x**n),x)
```

```
[Out] Integral(x**(n/2 - 1)*sinh(a + b*x**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int x^{-1+\frac{n}{2}}\sinh(a+bx^n)\,dx = -\frac{\sqrt{\pi}x^{\frac{1}{2}n}\left(\operatorname{erf}\left(\sqrt{bx^n}\right)-1\right)e^{(-a)}}{2\sqrt{bx^n}n} + \frac{\sqrt{\pi}x^{\frac{1}{2}n}\left(\operatorname{erf}\left(\sqrt{-bx^n}\right)-1\right)e^{a}}{2\sqrt{-bx^n}n}$$

```
[In] integrate(x^(-1+1/2*n)*sinh(a+b*x^n),x, algorithm="maxima")
[Out] -1/2*sqrt(pi)*x^(1/2*n)*(erf(sqrt(b*x^n)) - 1)*e^(-a)/(sqrt(b*x^n)*n) + 1/2
*sqrt(pi)*x^(1/2*n)*(erf(sqrt(-b*x^n)) - 1)*e^a/(sqrt(-b*x^n)*n)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int x^{-1+\frac{n}{2}}\sinh(a+bx^n)\,dx = \frac{\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}\sqrt{x^n}\right)e^{(-a)}}{\sqrt{b}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-b}\sqrt{x^n}\right)e^{a}}{\sqrt{-b}}}{2n}$$

```
[In] integrate(x^(-1+1/2*n)*sinh(a+b*x^n),x, algorithm="giac")
[Out] 1/2*(sqrt(pi)*erf(-sqrt(b)*sqrt(x^n))*e^(-a)/sqrt(b) - sqrt(pi)*erf(-sqrt(-
b)*sqrt(x^n))*e^a/sqrt(-b))/n
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+\frac{n}{2}}\sinh(a+bx^n)\,dx = \int x^{\frac{n}{2}-1}\sinh(a+b\,x^n)\,dx$$

```
[In] int(x^(n/2 - 1)*sinh(a + b*x^n),x)
[Out] int(x^(n/2 - 1)*sinh(a + b*x^n), x)
```

3.88 $\int x^2 \sinh\left((a+bx)^2\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 113

$$\int x^2 \sinh\left((a+bx)^2\right) dx = -\frac{a\cosh\left((a+bx)^2\right)}{b^3} + \frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3} - \frac{\sqrt{\pi}\operatorname{erf}(a+bx)}{8b^3} - \frac{a^2\sqrt{\pi}\operatorname{erf}(a+bx)}{4b^3} - \frac{\sqrt{\pi}\operatorname{erfi}(a+bx)}{8b^3} + \frac{a^2\sqrt{\pi}\operatorname{erfi}(a+bx)}{4b^3}$$

```
[Out] -a*cosh((b*x+a)^2)/b^3+1/2*(b*x+a)*cosh((b*x+a)^2)/b^3-1/8*erf(b*x+a)*Pi^(1
/2)/b^3-1/4*a^2*erf(b*x+a)*Pi^(1/2)/b^3-1/8*erfi(b*x+a)*Pi^(1/2)/b^3+1/4*a^
2*erfi(b*x+a)*Pi^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5472, 6874, 5406, 2235, 2236, 5428, 2718, 5432, 5407}

$$\int x^2 \sinh\left((a+bx)^2\right) dx = -\frac{\sqrt{\pi}a^2\operatorname{erf}(a+bx)}{4b^3} + \frac{\sqrt{\pi}a^2\operatorname{erfi}(a+bx)}{4b^3} - \frac{\sqrt{\pi}\operatorname{erf}(a+bx)}{8b^3} - \frac{\sqrt{\pi}\operatorname{erfi}(a+bx)}{8b^3} - \frac{a\cosh\left((a+bx)^2\right)}{b^3} + \frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3}$$

```
[In] Int[x^2*Sinh[(a + b*x)^2],x]
```

```
[Out] -((a*Cosh[(a + b*x)^2])/b^3) + ((a + b*x)*Cosh[(a + b*x)^2])/(2*b^3) - (Sqr
t[Pi]*Erf[a + b*x])/(8*b^3) - (a^2*Sqrt[Pi]*Erf[a + b*x])/(4*b^3) - (Sqrt[P
i]*Erfi[a + b*x])/(8*b^3) + (a^2*Sqrt[Pi]*Erfi[a + b*x])/(4*b^3)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5472

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
```

```
0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int(-a+x)^2\sinh\left(x^2\right)\,dx,x,a+bx\right)}{b^3}\\
&= \frac{\text{Subst}\left(\int\left(a^2\sinh\left(x^2\right)-2ax\sinh\left(x^2\right)+x^2\sinh\left(x^2\right)\right)\,dx,x,a+bx\right)}{b^3}\\
&= \frac{\text{Subst}\left(\int x^2\sinh\left(x^2\right)\,dx,x,a+bx\right)}{b^3}-\frac{(2a)\text{Subst}\left(\int x\sinh\left(x^2\right)\,dx,x,a+bx\right)}{b^3}\\
&\quad+\frac{a^2\text{Subst}\left(\int\sinh\left(x^2\right)\,dx,x,a+bx\right)}{b^3}\\
&= \frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3}-\frac{\text{Subst}\left(\int\cosh\left(x^2\right)\,dx,x,a+bx\right)}{2b^3}\\
&\quad-\frac{a\text{Subst}\left(\int\sinh(x)\,dx,x,(a+bx)^2\right)}{b^3}\\
&\quad-\frac{a^2\text{Subst}\left(\int e^{-x^2}\,dx,x,a+bx\right)}{2b^3}+\frac{a^2\text{Subst}\left(\int e^{x^2}\,dx,x,a+bx\right)}{2b^3}\\
&= -\frac{a\cosh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3}-\frac{a^2\sqrt{\pi}\text{erf}(a+bx)}{4b^3}\\
&\quad+\frac{a^2\sqrt{\pi}\text{erfi}(a+bx)}{4b^3}-\frac{\text{Subst}\left(\int e^{-x^2}\,dx,x,a+bx\right)}{4b^3}-\frac{\text{Subst}\left(\int e^{x^2}\,dx,x,a+bx\right)}{4b^3}\\
&= -\frac{a\cosh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3}-\frac{\sqrt{\pi}\text{erf}(a+bx)}{8b^3}\\
&\quad-\frac{a^2\sqrt{\pi}\text{erf}(a+bx)}{4b^3}-\frac{\sqrt{\pi}\text{erfi}(a+bx)}{8b^3}+\frac{a^2\sqrt{\pi}\text{erfi}(a+bx)}{4b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.56

$$\int x^2 \sinh \left((a+bx)^2\right) \, dx = \frac{-4(a-bx)\cosh \left((a+bx)^2\right)-\left(1+2a^2\right)\sqrt{\pi}\text{erf}(a+bx)+\left(-1+2a^2\right)\sqrt{\pi}\text{erfi}(a+bx)}{8b^3}$$

```
[In] Integrate[x^2*Sinh[(a + b*x)^2],x]
```

```
[Out] (-4*(a - b*x)*Cosh[(a + b*x)^2] - (1 + 2*a^2)*Sqrt[Pi]*Erf[a + b*x] + (-1 +
 2*a^2)*Sqrt[Pi]*Erfi[a + b*x])/(8*b^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.20

method	result
risch	$\frac{x\,e^{-(bx+a)^2}}{4b^2}-\frac{a\,e^{-(bx+a)^2}}{4b^3}-\frac{a^2\operatorname{erf}(bx+a)\sqrt{\pi}}{4b^3}-\frac{\operatorname{erf}(bx+a)\sqrt{\pi}}{8b^3}+\frac{x\,e^{(bx+a)^2}}{4b^2}-\frac{a\,e^{(bx+a)^2}}{4b^3}-\frac{ia^2\sqrt{\pi}\,\operatorname{erf}(ibx+ia)}{4b^3}+\frac{i\sqrt{\pi}\,\text{e}}{}$

```
[In] int(x^2*sinh((b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*x*exp(-(b*x+a)^2)-1/4*a/b^3*exp(-(b*x+a)^2)-1/4*a^2*erf(b*x+a)*Pi^(
1/2)/b^3-1/8*erf(b*x+a)*Pi^(1/2)/b^3+1/4/b^2*x*exp((b*x+a)^2)-1/4*a/b^3*exp
((b*x+a)^2)-1/4*I*a^2/b^3*Pi^(1/2)*erf(I*a+I*b*x)+1/8*I/b^3*Pi^(1/2)*erf(I*
a+I*b*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(95) = 190.

Time = 0.27 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.85

$$\int x^2 \sinh \left((a+bx)^2\right) \, dx = \frac{2\,b^2x+2\left(b^2x-ab\right)\cosh\left(b^2x^2+2\,abx+a^2\right)^2-\sqrt{\pi}\sqrt{-b^2}\left(\left(2\,a^2-1\right)\cosh\left(b^2x^2+2\,abx+a^2\right)+\left(2\,a^2-1\right)\right.}{}$$

```
[In] integrate(x^2*sinh((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^2*x + 2*(b^2*x - a*b)*cosh(b^2*x^2 + 2*a*b*x + a^2)^2 - sqrt(pi)*s
qrt(-b^2)*((2*a^2 - 1)*cosh(b^2*x^2 + 2*a*b*x + a^2) + (2*a^2 - 1)*sinh(b^2
```

```
*x^2 + 2*a*b*x + a^2))*erf(sqrt(-b^2)*(b*x + a)/b) - sqrt(pi)*sqrt(b^2)*((2
*a^2 + 1)*cosh(b^2*x^2 + 2*a*b*x + a^2) + (2*a^2 + 1)*sinh(b^2*x^2 + 2*a*b*
x + a^2))*erf(sqrt(b^2)*(b*x + a)/b) + 4*(b^2*x - a*b)*cosh(b^2*x^2 + 2*a*b
*x + a^2)*sinh(b^2*x^2 + 2*a*b*x + a^2) + 2*(b^2*x - a*b)*sinh(b^2*x^2 + 2*
a*b*x + a^2)^2 - 2*a*b)/(b^4*cosh(b^2*x^2 + 2*a*b*x + a^2) + b^4*sinh(b^2*x
^2 + 2*a*b*x + a^2))
```

Sympy [F]

$$\int x^2 \sinh\left((a+bx)^2\right)\,dx = \int x^2 \sinh\left(a^2 + 2abx + b^2x^2\right)dx$$

```
[In] integrate(x**2*sinh((b*x+a)**2),x)
[Out] Integral(x**2*sinh(a**2 + 2*a*b*x + b**2*x**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. 2(95) = 190.

Time = 0.45 (sec) , antiderivative size = 817, normalized size of antiderivative = 7.23

$$\int x^2 \sinh\left((a+bx)^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*sinh((b*x+a)^2),x, algorithm="maxima")
[Out] 1/3*x^3*sinh((b*x + a)^2) + 1/6*((sqrt(pi)*(b^2*x + a*b)*a^3*b^4*(erf(sqrt(
(b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(7/2)) - 3*(b^2*x +
a*b)^3*a*b^4*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2
)^(7/2)) + 3*a^2*b^4*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(7/2) + b^4*gamma(2, (
b^2*x + a*b)^2/b^2)/(-b^2)^(7/2))*a/sqrt(-b^2) + (sqrt(pi)*(b^2*x + a*b)*a^
4*b^5*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(9/2
)) - 6*(b^2*x + a*b)^3*a^2*b^5*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a
*b)^2)^(3/2)*(-b^2)^(9/2)) + 4*a^3*b^5*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(9/2
) - (b^2*x + a*b)^5*b^5*gamma(5/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^
(5/2)*(-b^2)^(9/2)) + 4*a*b^5*gamma(2, (b^2*x + a*b)^2/b^2)/(-b^2)^(9/2))*b
/sqrt(-b^2) + a*(sqrt(pi)*(b^2*x + a*b)*a^3*(erf(sqrt(-(b^2*x + a*b)^2/b^2)
) - 1)/(b^4*sqrt(-(b^2*x + a*b)^2/b^2)) - 3*a^2*e^((b^2*x + a*b)^2/b^2)/b^3
 + gamma(2, -(b^2*x + a*b)^2/b^2)/b^3 - 3*(b^2*x + a*b)^3*a*gamma(3/2, -(b^
2*x + a*b)^2/b^2)/(b^6*(-(b^2*x + a*b)^2/b^2)^(3/2)))/b - sqrt(pi)*(b^2*x +
 a*b)*a^4*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/(b^5*sqrt(-(b^2*x + a*b)^2/
b^2)) + 4*a^3*e^((b^2*x + a*b)^2/b^2)/b^4 - 4*a*gamma(2, -(b^2*x + a*b)^2/b
^2)/b^4 + 6*(b^2*x + a*b)^3*a^2*gamma(3/2, -(b^2*x + a*b)^2/b^2)/(b^7*(-(b^
2*x + a*b)^2/b^2)^(3/2)) + (b^2*x + a*b)^5*gamma(5/2, -(b^2*x + a*b)^2/b^2)
/(b^9*(-(b^2*x + a*b)^2/b^2)^(5/2)))*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int x^2 \sinh\left((a+bx)^2\right) dx = -\frac{\frac{i\sqrt{\pi}(2a^2-1)\operatorname{erf}\left(ib\left(x+\frac{a}{b}\right)\right)}{b} - \frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(b^2x^2+2abx+a^2\right)}}{b}}{8b^2} + \frac{\frac{\sqrt{\pi}(2a^2+1)\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} + \frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(-b^2x^2-2abx-a^2\right)}}{b}}{8b^2}$$

```
[In] integrate(x^2*sinh((b*x+a)^2),x, algorithm="giac")
[Out] -1/8*(I*sqrt(pi)*(2*a^2 - 1)*erf(I*b*(x + a/b))/b - 2*(b*(x + a/b) - 2*a)*e
^(b^2*x^2 + 2*a*b*x + a^2)/b)/b^2 + 1/8*(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a
/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh\left((a+bx)^2\right) dx = \int x^2 \sinh\left((a+b\,x)^2\right) dx$$

```
[In] int(x^2*sinh((a + b*x)^2),x)
[Out] int(x^2*sinh((a + b*x)^2), x)
```

3.89 $\int x \sinh \left((a+bx)^2\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int x \sinh \left((a+bx)^2\right) dx = \frac{\cosh\left((a+bx)^2\right)}{2b^2} + \frac{a\sqrt{\pi}\operatorname{erf}(a+bx)}{4b^2} - \frac{a\sqrt{\pi}\operatorname{erfi}(a+bx)}{4b^2}$$

```
[Out] 1/2*cosh((b*x+a)^2)/b^2+1/4*a*erf(b*x+a)*Pi^(1/2)/b^2-1/4*a*erfi(b*x+a)*Pi^
(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5472, 6874, 5406, 2235, 2236, 5428, 2718}

$$\int x \sinh \left((a+bx)^2\right) dx = \frac{\sqrt{\pi}a\operatorname{erf}(a+bx)}{4b^2} - \frac{\sqrt{\pi}a\operatorname{erfi}(a+bx)}{4b^2} + \frac{\cosh\left((a+bx)^2\right)}{2b^2}$$

```
[In] Int[x*Sinh[(a + b*x)^2],x]
```

```
[Out] Cosh[(a + b*x)^2]/(2*b^2) + (a*Sqrt[Pi]*Erf[a + b*x])/(4*b^2) - (a*Sqrt[Pi]
*Erfi[a + b*x])/(4*b^2)
```

<u>Rule 2235</u>

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

<u>Rule 2236</u>

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5428

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5472

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (-a+x)\sinh\left(x^2\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int \left(-a\sinh\left(x^2\right)+x\sinh\left(x^2\right)\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int x\sinh\left(x^2\right)\,dx,x,a+bx\right)}{b^2}-\frac{a\text{Subst}\left(\int \sinh\left(x^2\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int \sinh(x)\,dx,x,(a+bx)^2\right)}{2b^2}\\
&\quad+\frac{a\text{Subst}\left(\int e^{-x^2}\,dx,x,a+bx\right)}{2b^2}-\frac{a\text{Subst}\left(\int e^{x^2}\,dx,x,a+bx\right)}{2b^2}
\end{aligned}$$

$$= \frac{\cosh \left((a+bx)^2\right)}{2b^2} + \frac{a\sqrt{\pi}\text{erf}(a+bx)}{4b^2} - \frac{a\sqrt{\pi}\text{erfi}(a+bx)}{4b^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x \sinh \left((a+bx)^2\right) \, dx = \frac{\cosh \left((a+bx)^2\right)}{2b^2} - \frac{a\sqrt{\pi}(-\text{erf}(a+bx) + \text{erfi}(a+bx))}{4b^2}$$

```
[In] Integrate[x*Sinh[(a + b*x)^2],x]
[Out] Cosh[(a + b*x)^2]/(2*b^2) - (a*Sqrt[Pi]*(-Erf[a + b*x] + Erfi[a + b*x]))/(4
*b^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

method	result	size
risch	$\frac{e^{-(bx+a)^2}}{4b^2} + \frac{a \,\text{erf}(bx+a)\sqrt{\pi}}{4b^2} + \frac{e^{(bx+a)^2}}{4b^2} + \frac{ia\sqrt{\pi}\, \text{erf}(ibx+ia)}{4b^2}$	66

```
[In] int(x*sinh((b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] 1/4/b^2*exp(-(b*x+a)^2)+1/4*a*erf(b*x+a)*Pi^(1/2)/b^2+1/4/b^2*exp((b*x+a)^2
)+1/4*I*a/b^2*Pi^(1/2)*erf(I*a+I*b*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(44) = 88.

Time = 0.25 (sec) , antiderivative size = 258, normalized size of antiderivative = 4.78

$$\int x \sinh \left((a+bx)^2\right) \, dx$$

$$= \frac{b\cosh \left(b^2x^2 + 2\,abx + a^2\right)^2 + \sqrt{\pi}\sqrt{-b^2}\left(a\cosh \left(b^2x^2 + 2\,abx + a^2\right) + a\sinh \left(b^2x^2 + 2\,abx + a^2\right)\right)\text{erf}\left(\frac{\sqrt{-b^2}(bx}{b}\right.}{}$$

```
[In] integrate(x*sinh((b*x+a)^2),x, algorithm="fricas")
[Out] 1/4*(b*cosh(b^2*x^2 + 2*a*b*x + a^2)^2 + sqrt(pi)*sqrt(-b^2)*(a*cosh(b^2*x^
2 + 2*a*b*x + a^2) + a*sinh(b^2*x^2 + 2*a*b*x + a^2))*erf(sqrt(-b^2)*(b*x +
 a)/b) + sqrt(pi)*sqrt(b^2)*(a*cosh(b^2*x^2 + 2*a*b*x + a^2) + a*sinh(b^2*x
```

```
^2 + 2*a*b*x + a^2))*erf(sqrt(b^2)*(b*x + a)/b) + 2*b*cosh(b^2*x^2 + 2*a*b*
x + a^2)*sinh(b^2*x^2 + 2*a*b*x + a^2) + b*sinh(b^2*x^2 + 2*a*b*x + a^2)^2
+ b)/(b^3*cosh(b^2*x^2 + 2*a*b*x + a^2) + b^3*sinh(b^2*x^2 + 2*a*b*x + a^2)
)
```

Sympy [F]

$$\int x\sinh\left((a+bx)^2\right)\,dx=\int x\sinh\left(a^2+2abx+b^2x^2\right)dx$$

```
[In] integrate(x*sinh((b*x+a)**2),x)
```

```
[Out] Integral(x*sinh(a**2 + 2*a*b*x + b**2*x**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(44) = 88.

Time = 0.42 (sec) , antiderivative size = 649, normalized size of antiderivative = 12.02

$$\int x\sinh\left((a+bx)^2\right)\,dx=\frac{1}{2}x^2\sinh\left((bx+a)^2\right)+\frac{1}{4}\left(\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a^2b^3\left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}(-b^2)^{\frac{5}{2}}}-\frac{(b^2x+ab)^3b^3\Gamma\left(\frac{3}{2},\frac{(b^2x+ab)^2}{b^2}\right)}{\left((b^2x+ab)^2\right)^{\frac{3}{2}}(-b^2)^{\frac{5}{2}}}+\frac{2ab^3e^{\left(-\frac{(b^2x+ab)^2}{b^2}\right)}}{(-b^2)^{\frac{5}{2}}}\right)a}{\sqrt{-b^2}}+\frac{\left(\frac{\sqrt{\pi}(b^2x+ab\cdots}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(x*sinh((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sinh((b*x + a)^2) + 1/4*((sqrt(pi)*(b^2*x + a*b)*a^2*b^3*(erf(sqrt(
(b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(5/2)) - (b^2*x + a*
b)^3*b^3*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(5
/2)) + 2*a*b^3*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(5/2))*a/sqrt(-b^2) + (sqrt(
pi)*(b^2*x + a*b)*a^3*b^4*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x +
 a*b)^2)*(-b^2)^(7/2)) - 3*(b^2*x + a*b)^3*a*b^4*gamma(3/2, (b^2*x + a*b)^2
/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(7/2)) + 3*a^2*b^4*e^(-(b^2*x + a*b)^
2/b^2)/(-b^2)^(7/2) + b^4*gamma(2, (b^2*x + a*b)^2/b^2)/(-b^2)^(7/2))*b/sqr
t(-b^2) - a*(sqrt(pi)*(b^2*x + a*b)*a^2*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) -
1)/(b^3*sqrt(-(b^2*x + a*b)^2/b^2)) - 2*a*e^((b^2*x + a*b)^2/b^2)/b^2 - (b^
```

```
2*x + a*b)^3*gamma(3/2, -(b^2*x + a*b)^2/b^2)/(b^5*(-(b^2*x + a*b)^2/b^2)^(
3/2)))/b + sqrt(pi)*(b^2*x + a*b)*a^3*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)
/(b^4*sqrt(-(b^2*x + a*b)^2/b^2)) - 3*a^2*e^((b^2*x + a*b)^2/b^2)/b^3 + gam
ma(2, -(b^2*x + a*b)^2/b^2)/b^3 - 3*(b^2*x + a*b)^3*a*gamma(3/2, -(b^2*x +
a*b)^2/b^2)/(b^6*(-(b^2*x + a*b)^2/b^2)^(3/2)))*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.83

$$\int x\sinh\left((a+bx)^2\right)\,dx = -\frac{-\frac{i\sqrt{\pi}a\operatorname{erf}\left(ib\left(x+\frac{a}{b}\right)\right)}{b}-\frac{e^{\left(b^2x^2+2\,abx+a^2\right)}}{b}}{4\,b}-\frac{\frac{\sqrt{\pi}a\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}-\frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{4\,b}$$

```
[In] integrate(x*sinh((b*x+a)^2),x, algorithm="giac")
[Out] -1/4*(-I*sqrt(pi)*a*erf(I*b*(x + a/b))/b - e^(b^2*x^2 + 2*a*b*x + a^2)/b)/b
 - 1/4*(sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b
```

Mupad [F(-1)]

Timed out.

$$\int x\sinh\left((a+bx)^2\right)\,dx = \int x\,\mathrm{sinh}\left({\left(a+b\,x\right)}^2\right) \,d x$$

```
[In] int(x*sinh((a + b*x)^2),x)
[Out] int(x*sinh((a + b*x)^2), x)
```

3.90 $\int \sinh\left((a+bx)^2\right) dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \sinh\left((a+bx)^2\right) dx = -\frac{\sqrt{\pi}\,\text{erf}(a+bx)}{4b} + \frac{\sqrt{\pi}\,\text{erfi}(a+bx)}{4b}$$

```
[Out] -1/4*erf(b*x+a)*Pi^(1/2)/b+1/4*erfi(b*x+a)*Pi^(1/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5418, 5406, 2235, 2236}

$$\int \sinh\left((a+bx)^2\right) dx = \frac{\sqrt{\pi}\,\text{erfi}(a+bx)}{4b} - \frac{\sqrt{\pi}\,\text{erf}(a+bx)}{4b}$$

```
[In] Int[Sinh[(a + b*x)^2],x]
```

```
[Out] -1/4*(Sqrt[Pi]*Erf[a + b*x])/b + (Sqrt[Pi]*Erfi[a + b*x])/(4*b)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5418

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \sinh\left(x^2\right)\, dx, x, a+bx\right)}{b}\\ &= -\frac{\text{Subst}\left(\int e^{-x^2}\, dx, x, a+bx\right)}{2b} + \frac{\text{Subst}\left(\int e^{x^2}\, dx, x, a+bx\right)}{2b}\\ &= -\frac{\sqrt{\pi}\text{erf}(a+bx)}{4b} + \frac{\sqrt{\pi}\text{erfi}(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \sinh\left((a+bx)^2\right)\, dx = \frac{\sqrt{\pi}(-\text{erf}(a+bx)+\text{erfi}(a+bx))}{4b}$$

```
[In] Integrate[Sinh[(a + b*x)^2],x]
[Out] (Sqrt[Pi]*(-Erf[a + b*x] + Erfi[a + b*x]))/(4*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
risch	$-\frac{\text{erf}(bx+a)\sqrt{\pi}}{4b}-\frac{i\sqrt{\pi}\,\text{erf}(ibx+ia)}{4b}$	36

```
[In] int(sinh((b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] -1/4*erf(b*x+a)*Pi^(1/2)/b-1/4*I*Pi^(1/2)/b*erf(I*a+I*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \sinh\left((a+bx)^2\right)\,dx = -\frac{\sqrt{\pi}\sqrt{-b^2}\operatorname{erf}\left(\frac{\sqrt{-b^2}(bx+a)}{b}\right)+\sqrt{\pi}\sqrt{b^2}\operatorname{erf}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{4\,b^2}$$

```
[In] integrate(sinh((b*x+a)^2),x, algorithm="fricas")
[Out] -1/4*(sqrt(pi)*sqrt(-b^2)*erf(sqrt(-b^2)*(b*x + a)/b) + sqrt(pi)*sqrt(b^2)*
erf(sqrt(b^2)*(b*x + a)/b))/b^2
```

Sympy [F]

$$\int \sinh\left((a+bx)^2\right)\,dx = \int \sinh\left((a+bx)^2\right)dx$$

```
[In] integrate(sinh((b*x+a)**2),x)
[Out] Integral(sinh((a + b*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(29) = 58.

Time = 0.38 (sec) , antiderivative size = 477, normalized size of antiderivative = 12.89

$$\int \sinh\left((a+bx)^2\right)\,dx$$
$$= \frac{1}{2}\left(\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)ab^2\left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}(-b^2)^{\frac{3}{2}}}+\frac{b^2e^{\left(-\frac{(b^2x+ab)^2}{b^2}\right)}}{(-b^2)^{\frac{3}{2}}}\right)a}{\sqrt{-b^2}}+\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a^2b^3\left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}(-b^2)^{\frac{5}{2}}}-\frac{(b^2x+\cdots}{(\cdots}\cdots\right.}{\sqrt{-\cdots}}\right.$$
$$+ x\sinh\left((bx+a)^2\right)$$

```
[In] integrate(sinh((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((sqrt(pi)*(b^2*x + a*b)*a*b^2*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqrt
((b^2*x + a*b)^2)*(-b^2)^(3/2)) + b^2*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(3/2)
)*a/sqrt(-b^2) + (sqrt(pi)*(b^2*x + a*b)*a^2*b^3*(erf(sqrt((b^2*x + a*b)^2)
/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(5/2)) - (b^2*x + a*b)^3*b^3*gamma(3
/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(5/2)) + 2*a*b^3*e
^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(5/2))*b/sqrt(-b^2) + a*(sqrt(pi)*(b^2*x + a
*b)*a*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/(b^2*sqrt(-(b^2*x + a*b)^2/b^2)
) - e^((b^2*x + a*b)^2/b^2)/b)/b - sqrt(pi)*(b^2*x + a*b)*a^2*(erf(sqrt(-(b
^2*x + a*b)^2/b^2)) - 1)/(b^3*sqrt(-(b^2*x + a*b)^2/b^2)) + 2*a*e^((b^2*x +
 a*b)^2/b^2)/b^2 + (b^2*x + a*b)^3*gamma(3/2, -(b^2*x + a*b)^2/b^2)/(b^5*(-
(b^2*x + a*b)^2/b^2)^(3/2)))*b + x*sinh((b*x + a)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \sinh \left((a+bx)^2\right) \, dx = -\frac{i\sqrt{\pi}\,\mathrm{erf}\left(i\,b\left(x+\frac{a}{b}\right)\right)}{4\,b} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{4\,b}$$

```
[In] integrate(sinh((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(pi)*erf(I*b*(x + a/b))/b + 1/4*sqrt(pi)*erf(-b*(x + a/b))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sinh \left((a+bx)^2\right) \, dx = \int \sinh\left({\left(a+b\,x\right)}^2\right) \,d x$$

```
[In] int(sinh((a + b*x)^2),x)
```

```
[Out] int(sinh((a + b*x)^2), x)
```

3.91 $\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx = b\mathrm{Int}\left(\frac{\sinh\left((a+bx)^2\right)}{bx},x\right)$$

```
[Out] b*CannotIntegrate(sinh((b*x+a)^2)/b/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx = \int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx$$

```
[In] Int[Sinh[(a + b*x)^2]/x,x]
```

```
[Out] Defer[Subst][Defer[Int][Sinh[x^2]/(-a + x), x], x, a + b*x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\sinh\left(x^2\right)}{-a+x}\,dx,x,a+bx\right)$$

Mathematica [N/A]

Not integrable

Time = 5.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh \left((a+bx)^2\right)}{x}\,dx = \int \frac{\sinh \left((a+bx)^2\right)}{x}\,dx$$

```
[In] Integrate[Sinh[(a + b*x)^2]/x,x]
[Out] Integrate[Sinh[(a + b*x)^2]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sinh \left((bx+a)^2\right)}{x}dx$$

```
[In] int(sinh((b*x+a)^2)/x,x)
[Out] int(sinh((b*x+a)^2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\sinh \left((a+bx)^2\right)}{x}\,dx = \int \frac{\sinh \left((bx+a)^2\right)}{x}\,dx$$

```
[In] integrate(sinh((b*x+a)^2)/x,x, algorithm="fricas")
[Out] integral(sinh(b^2*x^2 + 2*a*b*x + a^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\sinh \left((a+bx)^2\right)}{x}\,dx = \int \frac{\sinh \left(a^2+2abx+b^2x^2\right)}{x}\,dx$$

```
[In] integrate(sinh((b*x+a)**2)/x,x)
[Out] Integral(sinh(a**2 + 2*a*b*x + b**2*x**2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh((a+bx)^2)}{x} dx = \int \frac{\sinh\left((bx+a)^2\right)}{x} dx$$

```
[In] integrate(sinh((b*x+a)^2)/x,x, algorithm="maxima")
[Out] integrate(sinh((b*x + a)^2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh((a+bx)^2)}{x} dx = \int \frac{\sinh\left((bx+a)^2\right)}{x} dx$$

```
[In] integrate(sinh((b*x+a)^2)/x,x, algorithm="giac")
[Out] integrate(sinh((b*x + a)^2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh((a+bx)^2)}{x} dx = \int \frac{\sinh\left((a+b\,x)^2\right)}{x} dx$$

```
[In] int(sinh((a + b*x)^2)/x,x)
[Out] int(sinh((a + b*x)^2)/x, x)
```

3.92 $\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx = \text{Int}\left(\frac{\sinh\left((a+bx)^2\right)}{x^2}, x\right)$$

```
[Out] Unintegrable(sinh((b*x+a)^2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx = \int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx$$

```
[In] Int[Sinh[(a + b*x)^2]/x^2,x]
```

```
[Out] b*Defer[Subst][Defer[Int][Sinh[x^2]/(-a + x)^2, x], x, a + b*x]
```

Rubi steps

$$\text{integral} = b\text{Subst}\left(\int \frac{\sinh\left(x^2\right)}{(-a+x)^2} dx, x, a+bx\right)$$

Mathematica [N/A]

Not integrable

Time = 6.94 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2}\,dx = \int \frac{\sinh\left((a+bx)^2\right)}{x^2}\,dx$$

```
[In] Integrate[Sinh[(a + b*x)^2]/x^2,x]
[Out] Integrate[Sinh[(a + b*x)^2]/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left((bx+a)^2\right)}{x^2}dx$$

```
[In] int(sinh((b*x+a)^2)/x^2,x)
[Out] int(sinh((b*x+a)^2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2}\,dx = \int \frac{\sinh\left((bx+a)^2\right)}{x^2}\,dx$$

```
[In] integrate(sinh((b*x+a)^2)/x^2,x, algorithm="fricas")
[Out] integral(sinh(b^2*x^2 + 2*a*b*x + a^2)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2}\,dx = \int \frac{\sinh\left(a^2+2abx+b^2x^2\right)}{x^2}\,dx$$

```
[In] integrate(sinh((b*x+a)**2)/x**2,x)
[Out] Integral(sinh(a**2 + 2*a*b*x + b**2*x**2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2}\,dx = \int \frac{\sinh\left((bx+a)^2\right)}{x^2}\,dx$$

```
[In] integrate(sinh((b*x+a)^2)/x^2,x, algorithm="maxima")
[Out] integrate(sinh((b*x + a)^2)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2}\,dx = \int \frac{\sinh\left((bx+a)^2\right)}{x^2}\,dx$$

```
[In] integrate(sinh((b*x+a)^2)/x^2,x, algorithm="giac")
[Out] integrate(sinh((b*x + a)^2)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2}\,dx = \int \frac{\sinh\left((a+b\,x)^2\right)}{x^2}\,dx$$

```
[In] int(sinh((a + b*x)^2)/x^2,x)
[Out] int(sinh((a + b*x)^2)/x^2, x)
```

3.93 $\int x^2 \sinh\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 346

$$\begin{aligned}\int x^2 \sinh\left(a+b\sqrt{c+dx}\right)\,dx &= \frac{240\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^5d^3} \\ &- \frac{24c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} \\ &+ \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} \\ &+ \frac{40(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} \\ &- \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} \\ &+ \frac{2(c+dx)^{5/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} \\ &- \frac{240\sinh\left(a+b\sqrt{c+dx}\right)}{b^6d^3} \\ &+ \frac{24c\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^3} - \frac{2c^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\ &- \frac{120(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^3} \\ &+ \frac{12c(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\ &- \frac{10(c+dx)^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}\end{aligned}$$

```
[Out] 40*(d*x+c)^(3/2)*cosh(a+b*(d*x+c)^(1/2))/b^3/d^3-4*c*(d*x+c)^(3/2)*cosh(a+b
*(d*x+c)^(1/2))/b/d^3+2*(d*x+c)^(5/2)*cosh(a+b*(d*x+c)^(1/2))/b/d^3-240*sin
h(a+b*(d*x+c)^(1/2))/b^6/d^3+24*c*sinh(a+b*(d*x+c)^(1/2))/b^4/d^3-2*c^2*sin
```

```
h(a+b*(d*x+c)^(1/2))/b^2/d^3-120*(d*x+c)*sinh(a+b*(d*x+c)^(1/2))/b^4/d^3+12
*c*(d*x+c)*sinh(a+b*(d*x+c)^(1/2))/b^2/d^3-10*(d*x+c)^2*sinh(a+b*(d*x+c)^(1
/2))/b^2/d^3+240*cosh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^5/d^3-24*c*cosh(a+
b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^3+2*c^2*cosh(a+b*(d*x+c)^(1/2))*(d*x+c
)^(1/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5472, 5394, 3377, 2717}

$$\begin{aligned}\int x^2 \sinh\left(a+b\sqrt{c+dx}\right)\,dx = &-\frac{240\sinh\left(a+b\sqrt{c+dx}\right)}{b^6d^3}\\
&+\frac{240\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^5d^3}\\
&-\frac{120(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^3}\\
&+\frac{24c\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^3}\\
&+\frac{40(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3}\\
&-\frac{24c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3}\\
&-\frac{2c^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}\\
&-\frac{10(c+dx)^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}\\
&+\frac{12c(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}\\
&+\frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3}\\
&+\frac{2(c+dx)^{5/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3}\\
&-\frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3}\end{aligned}$$

```
[In] Int[x^2*Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (240*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b^5*d^3) - (24*c*Sqrt[c + d*
x]*Cosh[a + b*Sqrt[c + d*x]])/(b^3*d^3) + (2*c^2*Sqrt[c + d*x]*Cosh[a + b*S
qrt[c + d*x]])/(b*d^3) + (40*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c + d*x]])/(b^
3*d^3) - (4*c*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c + d*x]])/(b*d^3) + (2*(c +
```

```
d*x)^(5/2)*Cosh[a + b*Sqrt[c + d*x]])/(b*d^3) - (240*Sinh[a + b*Sqrt[c + d*
x]])/(b^6*d^3) + (24*c*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^3) - (2*c^2*Sinh[a
 + b*Sqrt[c + d*x]])/(b^2*d^3) - (120*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/
(b^4*d^3) + (12*c*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^2*d^3) - (10*(c +
 d*x)^2*Sinh[a + b*Sqrt[c + d*x]])/(b^2*d^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5394

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5472

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (-c+x)^2 \sinh\left(a+b\sqrt{x}\right)\,dx, x, c+dx\right)}{d^3} \\
&= \frac{2\text{Subst}\left(\int x\left(c-x^2\right)^2 \sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3} \\
&= \frac{2\text{Subst}\left(\int \left(c^2 x \sinh(a+bx) - 2cx^3 \sinh(a+bx) + x^5 \sinh(a+bx)\right)\,dx, x, \sqrt{c+dx}\right)}{d^3} \\
&= \frac{2\text{Subst}\left(\int x^5 \sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3} \\
&\quad - \frac{(4c)\text{Subst}\left(\int x^3 \sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3} \\
&\quad + \frac{\left(2c^2\right)\text{Subst}\left(\int x \sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3}
\end{aligned}
$$

$$= \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{10\text{Subst}\left(\int x^4\cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd^3} + \frac{(12c)\text{Subst}\left(\int x^2\cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd^3} - \frac{(2c^2)\,\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd^3}$$

$$= \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{2c^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{12c(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{10(c+dx)^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{40\text{Subst}\left(\int x^3\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^2d^3} - \frac{(24c)\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^2d^3}$$

$$= -\frac{24c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{40(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{2c^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{12c(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{10(c+dx)^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{120\text{Subst}\left(\int x^2\cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^3d^3} + \frac{(24c)\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^3d^3}$$

$$= -\frac{24c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{40(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{24c\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^3} - \frac{2c^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{120(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^3} + \frac{12c(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{10(c+dx)^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{240\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^4d^3}$$

$$= \frac{240\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^5d^3} - \frac{24c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{40(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{24c\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^3} - \frac{2c^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{120(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^3} + \frac{12c(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{10(c+dx)^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{240\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^5d^3}$$

$$= \frac{240\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^5d^3} - \frac{24c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{40(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{240\sinh\left(a+b\sqrt{c+dx}\right)}{b^6d^3} + \frac{24c\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^3} - \frac{2c^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{120(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^3} + \frac{12c(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{10(c+dx)^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.60

$$\int x^2 \sinh\left(a + b\sqrt{c + dx}\right) dx$$
$$= \frac{e^{-a-b\sqrt{c+dx}}\left(120 + 120b\sqrt{c+dx} + b^5d^2x^2\sqrt{c+dx} + 4b^3\sqrt{c+dx}(2c+5dx) + 12b^2(4c+5dx) + b^4dx(4c+\right.}{}$$

[In] Integrate[x^2*Sinh[a + b*Sqrt[c + d*x]],x]

[Out] (E^(-a - b*Sqrt[c + d*x])*(120 + 120*b*Sqrt[c + d*x] + b^5*d^2*x^2*Sqrt[c + d*x] + 4*b^3*Sqrt[c + d*x]*(2*c + 5*d*x) + 12*b^2*(4*c + 5*d*x) + b^4*d*x*(4*c + 5*d*x) + E^(2*(a + b*Sqrt[c + d*x]))*(-120 + 120*b*Sqrt[c + d*x] + b^5*d^2*x^2*Sqrt[c + d*x] + 4*b^3*Sqrt[c + d*x]*(2*c + 5*d*x) - 12*b^2*(4*c + 5*d*x) - b^4*d*x*(4*c + 5*d*x))))/(b^6*d^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. 2(310) = 620.

Time = 2.35 (sec) , antiderivative size = 831, normalized size of antiderivative = 2.40

method	result
derivativedivides	$\frac{\frac{10a^4((a+b\sqrt{dx+c})\cosh(a+b\sqrt{dx+c})-\sinh(a+b\sqrt{dx+c}))}{b^4}-\frac{2a^5\cosh(a+b\sqrt{dx+c})}{b^4}-\frac{20a^3\left((a+b\sqrt{dx+c})^2\cosh(a+b\sqrt{dx+c})-2(a+b\sqrt{dx\right.}}{b^4}}{}$
default	$\frac{\frac{10a^4((a+b\sqrt{dx+c})\cosh(a+b\sqrt{dx+c})-\sinh(a+b\sqrt{dx+c}))}{b^4}-\frac{2a^5\cosh(a+b\sqrt{dx+c})}{b^4}-\frac{20a^3\left((a+b\sqrt{dx+c})^2\cosh(a+b\sqrt{dx+c})-2(a+b\sqrt{dx\right.}}{b^4}}{}$
parts	$\frac{2x^2\sqrt{dx+c}\,\cosh(a+b\sqrt{dx+c})}{db}-\frac{2x^2\sinh(a+b\sqrt{dx+c})}{d\,b^2}+\frac{-\frac{48a^2\left((a+b\sqrt{dx+c})^2\sinh(a+b\sqrt{dx+c})-2(a+b\sqrt{dx+c})\cosh(a+b\right.}{b^2}}{}$

[In] int(x^2*sinh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)

[Out] 2/d^3/b^2*(5/b^4*a^4*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2)))-1/b^4*a^5*cosh(a+b*(d*x+c)^(1/2))-10/b^4*a^3*((a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))+2*cosh(a+b*(d*x+c)^(1/2)))+10/b^4*a^2*((a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))-3*(a+b*(d*x+c)^(1/2))^2*sinh(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-6*sinh(a+b*(d*x+c)^(1/2)))-6/b^2*a^2*c*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2)))+2/b^2*a^3*c*cosh(a+b*(d*x+c)^(1/2))-5/b^4*a*((a+b*(d*x+c)^(1/2))^4*cosh(a+b*(d*x+c)^(1/2))-4*(a+b*(d*x+c)^(1/2))^3*sinh(a+b*(d*x+c)^(1/2))+12*(a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))-24*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))+24*cosh(a+b*(d*x+c)^(1/2)))+6/b^2*a*c*((a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))+2*cosh(a+b*(d*x+c

```
)^(1/2)))+1/b^4*((a+b*(d*x+c)^(1/2))^5*cosh(a+b*(d*x+c)^(1/2))-5*(a+b*(d*x+
c)^(1/2))^4*sinh(a+b*(d*x+c)^(1/2))+20*(a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+
c)^(1/2))-60*(a+b*(d*x+c)^(1/2))^2*sinh(a+b*(d*x+c)^(1/2))+120*(a+b*(d*x+c)
^(1/2))*cosh(a+b*(d*x+c)^(1/2))-120*sinh(a+b*(d*x+c)^(1/2)))-2/b^2*c*((a+b*
(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))-3*(a+b*(d*x+c)^(1/2))^2*sinh(a+b*(
d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-6*sinh(a+b*(d*x
+c)^(1/2)))+c^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+
c)^(1/2)))-c^2*a*cosh(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.30

$$\int x^2 \sinh\left(a+b\sqrt{c+dx}\right)\,dx = \frac{2\left(\left(b^5d^2x^2+20\,b^3dx+8\,b^3c+120\,b\right)\sqrt{dx+c}\cosh\left(\sqrt{dx+c}b+a\right)-\left(5\,b^4d^2x^2+48\,b^2c+4\left(b^4c+15\,b^2\right)d\right.}{b^6d^3}$$

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^5*d^2*x^2 + 20*b^3*d*x + 8*b^3*c + 120*b)*sqrt(d*x + c)*cosh(sqrt(d*x
 + c)*b + a) - (5*b^4*d^2*x^2 + 48*b^2*c + 4*(b^4*c + 15*b^2)*d*x + 120)*si
nh(sqrt(d*x + c)*b + a))/(b^6*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.78

$$\int x^2 \sinh\left(a+b\sqrt{c+dx}\right)\,dx = \begin{cases} \frac{x^3\sinh(a)}{3} \\ \frac{x^3\sinh\left(a+b\sqrt{c}\right)}{3} \\ \frac{2x^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{8cx\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{10x^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{16c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3}+\frac{40x\sqrt{c+dx}\cosh\left(a\right.}{b^3d^2} \end{cases}$$

```
[In] integrate(x**2*sinh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**3*sinh(a)/3, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**3*sinh(a
+ b*sqrt(c))/3, Eq(d, 0)), (2*x**2*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/
(b*d) - 8*c*x*sinh(a + b*sqrt(c + d*x))/(b**2*d**2) - 10*x**2*sinh(a + b*sq
rt(c + d*x))/(b**2*d) + 16*c*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**3*
d**3) + 40*x*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**3*d**2) - 96*c*sin
h(a + b*sqrt(c + d*x))/(b**4*d**3) - 120*x*sinh(a + b*sqrt(c + d*x))/(b**4*
d**2) + 240*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**5*d**3) - 240*sinh(
a + b*sqrt(c + d*x))/(b**6*d**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.40

$$\int x^2 \sinh\left(a+b\sqrt{c+dx}\right)\,dx$$

$$= \frac{2\,d^3x^3\sinh\left(\sqrt{dx+c}b+a\right)+\left(\frac{c^3e^{\left(\sqrt{dx+c}b+a\right)}}{b}-\frac{c^3e^{\left(-\sqrt{dx+c}b-a\right)}}{b}-\frac{3\left((dx+c)b^2e^a-2\sqrt{dx+c}be^a+2\,e^a\right)c^2e^{\left(\sqrt{dx+c}b\right)}}{b^3}+\frac{3\left((dx+c\right.}{}\right.}{}$$

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*d^3*x^3*sinh(sqrt(d*x + c)*b + a) + (c^3*e^(sqrt(d*x + c)*b + a)/b -
 c^3*e^(-sqrt(d*x + c)*b - a)/b - 3*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*
e^a + 2*e^a)*c^2*e^(sqrt(d*x + c)*b)/b^3 + 3*((d*x + c)*b^2 + 2*sqrt(d*x +
c)*b + 2)*c^2*e^(-sqrt(d*x + c)*b - a)/b^3 + 3*((d*x + c)^2*b^4*e^a - 4*(d*
x + c)^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a + 24*e
^a)*c*e^(sqrt(d*x + c)*b)/b^5 - 3*((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2)*b^3
+ 12*(d*x + c)*b^2 + 24*sqrt(d*x + c)*b + 24)*c*e^(-sqrt(d*x + c)*b - a)/b^
5 - ((d*x + c)^3*b^6*e^a - 6*(d*x + c)^(5/2)*b^5*e^a + 30*(d*x + c)^2*b^4*e
^a - 120*(d*x + c)^(3/2)*b^3*e^a + 360*(d*x + c)*b^2*e^a - 720*sqrt(d*x + c
)*b*e^a + 720*e^a)*e^(sqrt(d*x + c)*b)/b^7 + ((d*x + c)^3*b^6 + 6*(d*x + c)
^(5/2)*b^5 + 30*(d*x + c)^2*b^4 + 120*(d*x + c)^(3/2)*b^3 + 360*(d*x + c)*b
^2 + 720*sqrt(d*x + c)*b + 720)*e^(-sqrt(d*x + c)*b - a)/b^7)*b)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 914 vs. 2(310) = 620.

Time = 0.30 (sec) , antiderivative size = 914, normalized size of antiderivative = 2.64

$$\int x^2 \sinh\left(a+b\sqrt{c+dx}\right)\,dx$$

$$= \frac{\left(\left(\sqrt{dx+c}b+a\right)b^4c^2-ab^4c^2-2\left(\sqrt{dx+c}b+a\right)^3b^2c+6\left(\sqrt{dx+c}b+a\right)^2ab^2c-6\left(\sqrt{dx+c}b+a\right)a^2b^2c+2\,a^3b^2c-b^4c^2+\left(\sqrt{dx+c}b+a\right)^5-5\left(\sqrt{dx+c}b+a\right)^4a+\right.}{}$$

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] (((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b + a)^3*b^2
*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*a^2*b^2*c
+ 2*a^3*b^2*c - b^4*c^2 + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a)
^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5*
(sqrt(d*x + c)*b + a)*a^4 - a^5 + 6*(sqrt(d*x + c)*b + a)^2*b^2*c - 12*(sqr
t(d*x + c)*b + a)*a*b^2*c + 6*a^2*b^2*c - 5*(sqrt(d*x + c)*b + a)^4 + 20*(s
```

```
qrt(d*x + c)*b + a)^3*a - 30*(sqrt(d*x + c)*b + a)^2*a^2 + 20*(sqrt(d*x + c
)*b + a)*a^3 - 5*a^4 - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a*b^2*c + 20*(sq
rt(d*x + c)*b + a)^3 - 60*(sqrt(d*x + c)*b + a)^2*a + 60*(sqrt(d*x + c)*b +
 a)*a^2 - 20*a^3 + 12*b^2*c - 60*(sqrt(d*x + c)*b + a)^2 + 120*(sqrt(d*x +
c)*b + a)*a - 60*a^2 + 120*sqrt(d*x + c)*b - 120)*e^(sqrt(d*x + c)*b + a)/(
b^5*d^2) + ((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b
+ a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*
a^2*b^2*c + 2*a^3*b^2*c + b^4*c^2 + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x +
 c)*b + a)^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^
2*a^3 + 5*(sqrt(d*x + c)*b + a)*a^4 - a^5 - 6*(sqrt(d*x + c)*b + a)^2*b^2*c
 + 12*(sqrt(d*x + c)*b + a)*a*b^2*c - 6*a^2*b^2*c + 5*(sqrt(d*x + c)*b + a)
^4 - 20*(sqrt(d*x + c)*b + a)^3*a + 30*(sqrt(d*x + c)*b + a)^2*a^2 - 20*(sq
rt(d*x + c)*b + a)*a^3 + 5*a^4 - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a*b^2*
c + 20*(sqrt(d*x + c)*b + a)^3 - 60*(sqrt(d*x + c)*b + a)^2*a + 60*(sqrt(d*
x + c)*b + a)*a^2 - 20*a^3 - 12*b^2*c + 60*(sqrt(d*x + c)*b + a)^2 - 120*(s
qrt(d*x + c)*b + a)*a + 60*a^2 + 120*sqrt(d*x + c)*b + 120)*e^(-sqrt(d*x +
c)*b - a)/(b^5*d^2))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh \left(a+b\sqrt{c+dx}\right)\, dx = \int x^2 \sinh\left(a+b\,\sqrt{c+d\,x}\right)\, dx$$

```
[In] int(x^2*sinh(a + b*(c + d*x)^(1/2)),x)
[Out] int(x^2*sinh(a + b*(c + d*x)^(1/2)), x)
```

3.94 $\int x \sinh \left(a+b\sqrt{c+dx}\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 167

$$\int x \sinh \left(a+b\sqrt{c+dx}\right) \, dx = \frac{12\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{12\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2} + \frac{2c\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{6(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}$$

```
[Out] 2*(d*x+c)^(3/2)*cosh(a+b*(d*x+c)^(1/2))/b/d^2-12*sinh(a+b*(d*x+c)^(1/2))/b^
4/d^2+2*c*sinh(a+b*(d*x+c)^(1/2))/b^2/d^2-6*(d*x+c)*sinh(a+b*(d*x+c)^(1/2))
/b^2/d^2+12*cosh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^2-2*c*cosh(a+b*(d*x
+c)^(1/2))*(d*x+c)^(1/2)/b/d^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

= {5472, 5394, 3377, 2717}

$$\int x\sinh\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{12\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{12\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{6(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{2c\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{2(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}$$

```
[In] Int[x*Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (12*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*c*Sqrt[c + d*x]
*Cosh[a + b*Sqrt[c + d*x]])/(b*d^2) + (2*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c
+ d*x]])/(b*d^2) - (12*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (2*c*Sinh[a +
 b*Sqrt[c + d*x]])/(b^2*d^2) - (6*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^2
*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5394

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5472

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (-c+x)\sinh\left(a+b\sqrt{x}\right)\,dx, x, c+dx\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int x(-c+x^2)\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \left(-cx\sinh(a+bx)+x^3\sinh(a+bx)\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int x^3\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{(2c)\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= -\frac{2c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}\\
&\quad - \frac{6\text{Subst}\left(\int x^2\cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd^2}\\
&\quad + \frac{(2c)\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd^2}\\
&= -\frac{2c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}\\
&\quad + \frac{2c\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{6(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}\\
&\quad + \frac{12\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^2d^2}\\
&= \frac{12\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}\\
&\quad + \frac{2(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2c\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}\\
&\quad - \frac{6(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{12\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^3d^2}\\
&= \frac{12\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}\\
&\quad + \frac{2(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{12\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}\\
&\quad + \frac{2c\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{6(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.43

$$
\int x \sinh \left(a+b\sqrt{c+dx}\right) dx = \frac{2b\sqrt{c+dx}(6+b^2dx)\cosh \left(a+b\sqrt{c+dx}\right) - 2(6+b^2(2c+3dx))\sinh \left(a+b\sqrt{c+dx}\right)}{b^4d^2}
$$

```
[In] Integrate[x*Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[c + d*x]*(6 + b^2*d*x)*Cosh[a + b*Sqrt[c + d*x]] - 2*(6 + b^2*(2*
c + 3*d*x))*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(149) = 298.

Time = 2.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.80

method	result
parts	$\frac{2x\sqrt{dx+c}\,\cosh\left(a+b\sqrt{dx+c}\right)}{db} - \frac{2x\sinh\left(a+b\sqrt{dx+c}\right)}{d\,b^2} - \frac{2\left(\frac{2\left(a+b\sqrt{dx+c}\right)^2\sinh\left(a+b\sqrt{dx+c}\right)-4\left(a+b\sqrt{dx+c}\right)\cosh\left(a+b\sqrt{dx\right.}}{}\right.}{}$ …
derivativedivides	$\frac{\frac{6a^2\left(\left(a+b\sqrt{dx+c}\right)\cosh\left(a+b\sqrt{dx+c}\right)-\sinh\left(a+b\sqrt{dx+c}\right)\right)}{b^2}-\frac{2a^3\cosh\left(a+b\sqrt{dx+c}\right)}{b^2}-\frac{6a\left(\left(a+b\sqrt{dx+c}\right)^2\cosh\left(a+b\sqrt{dx+c}\right)-2\left(a+b\sqrt{dx+\right.}\right.}{b^2}}{}$ …
default	$\frac{\frac{6a^2\left(\left(a+b\sqrt{dx+c}\right)\cosh\left(a+b\sqrt{dx+c}\right)-\sinh\left(a+b\sqrt{dx+c}\right)\right)}{b^2}-\frac{2a^3\cosh\left(a+b\sqrt{dx+c}\right)}{b^2}-\frac{6a\left(\left(a+b\sqrt{dx+c}\right)^2\cosh\left(a+b\sqrt{dx+c}\right)-2\left(a+b\sqrt{dx+\right.}\right.}{b^2}}{}$ …

```
[In] int(x*sinh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b*x*(d*x+c)^(1/2)*cosh(a+b*(d*x+c)^(1/2))-2/d/b^2*x*sinh(a+b*(d*x+c)^(1
/2))-2/d/b^2*(2/d/b^2*((a+b*(d*x+c)^(1/2))^2*sinh(a+b*(d*x+c)^(1/2))-2*(a+b
*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))+2*sinh(a+b*(d*x+c)^(1/2))-a*((a+b*(
d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+c)^(1/2))))-2*a/d/b^2*(
(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+c)^(1/2))-sinh(a+
b*(d*x+c)^(1/2))*a)-2/d/b^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-si
nh(a+b*(d*x+c)^(1/2))-a*cosh(a+b*(d*x+c)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

$$\int x\sinh\left(a+b\sqrt{c+dx}\right)\,dx = \frac{2\left((b^3dx+6\,b)\sqrt{dx+c}\cosh\left(\sqrt{dx+c}b+a\right)-(3\,b^2dx+2\,b^2c+6)\sinh\left(\sqrt{dx+c}b+a\right)\right)}{b^4d^2}$$

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^3*d*x + 6*b)*sqrt(d*x + c)*cosh(sqrt(d*x + c)*b + a) - (3*b^2*d*x + 2
*b^2*c + 6)*sinh(sqrt(d*x + c)*b + a))/(b^4*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int x\sinh\left(a+b\sqrt{c+dx}\right)\,dx = \begin{cases} \frac{x^2\sinh(a)}{2} & \text{fo} \\ \frac{x^2\sinh\left(a+b\sqrt{c}\right)}{2} & \text{fo} \\ \frac{2x\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{4c\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{6x\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{12\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{12\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2} & \text{o} \end{cases}$$

```
[In] integrate(x*sinh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**2*sinh(a)/2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**2*sinh(a
+ b*sqrt(c))/2, Eq(d, 0)), (2*x*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b*
d) - 4*c*sinh(a + b*sqrt(c + d*x))/(b**2*d**2) - 6*x*sinh(a + b*sqrt(c + d*
x))/(b**2*d) + 12*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*
sinh(a + b*sqrt(c + d*x))/(b**4*d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.75

$$\int x\sinh\left(a+b\sqrt{c+dx}\right)\,dx = \frac{2\,d^2x^2\sinh\left(\sqrt{dx+c}b+a\right)-\left(\frac{c^2e^{\left(\sqrt{dx+c}b+a\right)}}{b}-\frac{c^2e^{\left(-\sqrt{dx+c}b-a\right)}}{b}-\frac{2\left((dx+c)b^2e^a-2\sqrt{dx+c}be^a+2\,e^a\right)ce^{\left(\sqrt{dx+c}b\right)}}{b^3}+\frac{2\left((dx+c)\right.}{}\right.}{}$$

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d^2*x^2*sinh(sqrt(d*x + c)*b + a) - (c^2*e^(sqrt(d*x + c)*b + a)/b -
 c^2*e^(-sqrt(d*x + c)*b - a)/b - 2*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*
e^a + 2*e^a)*c*e^(sqrt(d*x + c)*b)/b^3 + 2*((d*x + c)*b^2 + 2*sqrt(d*x + c)
*b + 2)*c*e^(-sqrt(d*x + c)*b - a)/b^3 + ((d*x + c)^2*b^4*e^a - 4*(d*x + c)
^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a + 24*e^a)*e^
(sqrt(d*x + c)*b)/b^5 - ((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2)*b^3 + 12*(d*x
+ c)*b^2 + 24*sqrt(d*x + c)*b + 24)*e^(-sqrt(d*x + c)*b - a)/b^5)*b)/d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(149) = 298$.

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.79

$$\int x \sinh \left(a+b\sqrt{c+dx}\right)\,dx = -\frac{\frac{\left(\left(\sqrt{dx+c}b+a\right)b^2c-ab^2c-\left(\sqrt{dx+c}b+a\right)^3+3\left(\sqrt{dx+c}b+a\right)^2a-3\left(\sqrt{dx+c}b+a\right)a^2+a^3-b^2c+3\left(\sqrt{dx+c}b+a\right)^2-6\left(\sqrt{dx+c}b+a\right)a+3a^2-6\sqrt{dx+}\right.}{b^3d}}{}$$

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -((((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqr
t(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 - b^2*c + 3*(sqrt
(d*x + c)*b + a)^2 - 6*(sqrt(d*x + c)*b + a)*a + 3*a^2 - 6*sqrt(d*x + c)*b
+ 6)*e^(sqrt(d*x + c)*b + a)/(b^3*d) + ((sqrt(d*x + c)*b + a)*b^2*c - a*b^2
*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x +
c)*b + a)*a^2 + a^3 + b^2*c - 3*(sqrt(d*x + c)*b + a)^2 + 6*(sqrt(d*x + c)*
b + a)*a - 3*a^2 - 6*sqrt(d*x + c)*b - 6)*e^(-sqrt(d*x + c)*b - a)/(b^3*d))
/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x \sinh \left(a+b\sqrt{c+dx}\right)\,dx = \int x\,\mathrm{sinh}\left(a+b\,\sqrt{c+d\,x}\right) \,d x$$

```
[In] int(x*sinh(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] int(x*sinh(a + b*(c + d*x)^(1/2)), x)
```

3.95 $\int \sinh \left(a+b\sqrt{c+dx}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \sinh \left(a+b\sqrt{c+dx}\right)\,dx=\frac{2\sqrt{c+dx}\cosh \left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\sinh \left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] -2*sinh(a+b*(d*x+c)^(1/2))/b^2/d+2*cosh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/
d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5418, 5412, 3377, 2717}

$$\int \sinh \left(a+b\sqrt{c+dx}\right)\,dx=\frac{2\sqrt{c+dx}\cosh \left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\sinh \left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[In] Int[Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b*d) - (2*Sinh[a + b*Sqrt[c +
d*x]])/(b^2*d)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5412

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Modul
e[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Sinh[c + d*x^(k*
n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] && I
ntegerQ[p]
```

Rule 5418

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sinh\left(a+b\sqrt{x}\right)\,dx,x,c+dx\right)}{d}\\
&= \frac{2\text{Subst}\left(\int x\sinh(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\text{Subst}\left(\int \cosh(a+bx)\,dx,x,\sqrt{c+dx}\right)}{bd}\\
&= \frac{2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \sinh\left(a+b\sqrt{c+dx}\right)\,dx=\frac{2\left(b\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)-\sinh\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

```
[In] Integrate[Sinh[a + b*Sqrt[c + d*x]],x]
[Out] (2*(b*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]] - Sinh[a + b*Sqrt[c + d*x]]))
/(b^2*d)
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{2(a+b\sqrt{dx+c})\cosh(a+b\sqrt{dx+c})-2\sinh(a+b\sqrt{dx+c})-2a\cosh(a+b\sqrt{dx+c})}{b^2d}$	63
default	$\frac{2(a+b\sqrt{dx+c})\cosh(a+b\sqrt{dx+c})-2\sinh(a+b\sqrt{dx+c})-2a\cosh(a+b\sqrt{dx+c})}{b^2d}$	63

```
[In] int(sinh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2)
)-a*cosh(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sinh\left(a+b\sqrt{c+dx}\right)\,dx=\frac{2\left(\sqrt{dx+c}b\cosh\left(\sqrt{dx+c}b+a\right)-\sinh\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(d*x + c)*b*cosh(sqrt(d*x + c)*b + a) - sinh(sqrt(d*x + c)*b + a))/(
b^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \sinh\left(a+b\sqrt{c+dx}\right)\,dx = \begin{cases} x\sinh(a) & \text{for } b=0\wedge(b=0\vee d=0) \\ x\sinh\left(a+b\sqrt{c}\right) & \text{for } d=0 \\ \frac{2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*sinh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sinh(a + b*sqrt
(c)), Eq(d, 0)), (2*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b*d) - 2*sinh(
a + b*sqrt(c + d*x))/(b**2*d), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(48) = 96.

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.06

$$\int \sinh\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{b\left(\frac{\left((dx+c)b^2e^a-2\sqrt{dx+c}be^a+2e^a\right)e^{\left(\sqrt{dx+c}b\right)}}{b^3}-\frac{\left((dx+c)b^2+2\sqrt{dx+c}b+2\right)e^{\left(-\sqrt{dx+c}b-a\right)}}{b^3}\right)-2\,(dx+c)\sinh\left(\sqrt{dx+c}b+a\right)}{2\,d}$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*e^a + 2*e^a)*e^(sqrt(d*x +
c)*b)/b^3 - ((d*x + c)*b^2 + 2*sqrt(d*x + c)*b + 2)*e^(-sqrt(d*x + c)*b - a
)/b^3) - 2*(d*x + c)*sinh(sqrt(d*x + c)*b + a))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \sinh\left(a+b\sqrt{c+dx}\right)\,dx = \frac{\left(\sqrt{dx+c}b-1\right)e^{\left(\sqrt{dx+c}b+a\right)}}{b^2d}+\frac{\left(\sqrt{dx+c}b+1\right)e^{\left(-\sqrt{dx+c}b-a\right)}}{b^2d}$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] (sqrt(d*x + c)*b - 1)*e^(sqrt(d*x + c)*b + a)/(b^2*d) + (sqrt(d*x + c)*b +
1)*e^(-sqrt(d*x + c)*b - a)/(b^2*d)
```

Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \sinh\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{2\left(\sinh\left(a+b\sqrt{c+d\,x}\right)-b\cosh\left(a+b\sqrt{c+d\,x}\right)\sqrt{c+d\,x}\right)}{b^2\,d}$$

```
[In] int(sinh(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] -(2*(sinh(a + b*(c + d*x)^(1/2)) - b*cosh(a + b*(c + d*x)^(1/2))*(c + d*x)^
(1/2)))/(b^2*d)
```

3.96 $\int \frac{\sinh(a+b\sqrt{c+dx})}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 124

$$\begin{aligned}\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}\,dx &= \text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sinh\left(a-b\sqrt{c}\right)\\ &\quad+\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\sinh\left(a+b\sqrt{c}\right)\\ &\quad-\cosh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\\ &\quad+\cosh\left(a-b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\end{aligned}$$

```
[Out] -cosh(a+b*c^(1/2))*Shi(b*(c^(1/2)-(d*x+c)^(1/2)))+cosh(a-b*c^(1/2))*Shi(b*(
c^(1/2)+(d*x+c)^(1/2)))+Chi(b*(c^(1/2)+(d*x+c)^(1/2)))*sinh(a-b*c^(1/2))+Ch
i(b*(c^(1/2)-(d*x+c)^(1/2)))*sinh(a+b*c^(1/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5472, 5400, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}\,dx &= \sinh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\\ &\quad+\sinh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\\ &\quad-\cosh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\\ &\quad+\cosh\left(a-b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\end{aligned}$$

```
[In] Int[Sinh[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])]*Sinh[a - b*Sqrt[c]] + CoshIntegra
l[b*(Sqrt[c] - Sqrt[c + d*x])]*Sinh[a + b*Sqrt[c]] - Cosh[a + b*Sqrt[c]]*Si
nhIntegral[b*(Sqrt[c] - Sqrt[c + d*x])] + Cosh[a - b*Sqrt[c]]*SinhIntegral[
b*(Sqrt[c] + Sqrt[c + d*x])]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5472

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{\sinh\left(a+b\sqrt{x}\right)}{-c+x}\,dx, x, c+dx\right)\\ &= 2\text{Subst}\left(\int \frac{x\sinh(a+bx)}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\end{aligned}$$

$$= 2\text{Subst}\left(\int\left(-\frac{\sinh(a+bx)}{2\left(\sqrt{c}-x\right)}+\frac{\sinh(a+bx)}{2\left(\sqrt{c}+x\right)}\right)dx,x,\sqrt{c+dx}\right)$$

$$= -\text{Subst}\left(\int\frac{\sinh(a+bx)}{\sqrt{c}-x}dx,x,\sqrt{c+dx}\right)+\text{Subst}\left(\int\frac{\sinh(a+bx)}{\sqrt{c}+x}dx,x,\sqrt{c+dx}\right)$$

$$= \cosh\left(a-b\sqrt{c}\right)\text{Subst}\left(\int\frac{\sinh\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}dx,x,\sqrt{c+dx}\right)$$
$$+\cosh\left(a+b\sqrt{c}\right)\text{Subst}\left(\int\frac{\sinh\left(b\sqrt{c}-bx\right)}{\sqrt{c}-x}dx,x,\sqrt{c+dx}\right)$$
$$+\sinh\left(a-b\sqrt{c}\right)\text{Subst}\left(\int\frac{\cosh\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}dx,x,\sqrt{c+dx}\right)$$
$$-\sinh\left(a+b\sqrt{c}\right)\text{Subst}\left(\int\frac{\cosh\left(b\sqrt{c}-bx\right)}{\sqrt{c}-x}dx,x,\sqrt{c+dx}\right)$$

$$= \text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sinh\left(a-b\sqrt{c}\right)+\text{Chi}\left(b\sqrt{c}-b\sqrt{c+dx}\right)\sinh\left(a+b\sqrt{c}\right)$$
$$+\cosh\left(a-b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)-\cosh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\sqrt{c}-b\sqrt{c+dx}\right)$$

Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05

$$\int\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}dx=\frac{1}{2}e^{-a-b\sqrt{c}}\left(-\text{ExpIntegralEi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\right.$$
$$+e^{2\left(a+b\sqrt{c}\right)}\text{ExpIntegralEi}\left(b\left(-\sqrt{c}+\sqrt{c+dx}\right)\right)$$
$$-e^{2b\sqrt{c}}\text{ExpIntegralEi}\left(-b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)$$
$$\left.+e^{2a}\text{ExpIntegralEi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\right)$$

```
[In] Integrate[Sinh[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] (E^(-a - b*Sqrt[c])*(-ExpIntegralEi[b*(Sqrt[c] - Sqrt[c + d*x])] + E^(2*(a
+ b*Sqrt[c]))*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] - E^(2*b*Sqrt[c])
*ExpIntegralEi[-(b*(Sqrt[c] + Sqrt[c + d*x]))] + E^(2*a)*ExpIntegralEi[b*(S
qrt[c] + Sqrt[c + d*x])]))/2
```

Maple [F]

$$\int \frac{\sinh \left(a+b\sqrt{dx+c}\right)}{x}dx$$

[In] int(sinh(a+b*(d*x+c)^(1/2))/x,x)

[Out] int(sinh(a+b*(d*x+c)^(1/2))/x,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(102) = 204$.

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.75

$$\begin{aligned}\int \frac{\sinh \left(a+b\sqrt{c+dx}\right)}{x}\,dx &= \frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}b-\sqrt{b^2c}\right)-\mathrm{Ei}\left(-\sqrt{dx+c}b+\sqrt{b^2c}\right)\right)\cosh\left(a+\sqrt{b^2c}\right)\\ &\quad+\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}b+\sqrt{b^2c}\right)-\mathrm{Ei}\left(-\sqrt{dx+c}b-\sqrt{b^2c}\right)\right)\cosh\left(-a+\sqrt{b^2c}\right)\\ &\quad+\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}b-\sqrt{b^2c}\right)+\mathrm{Ei}\left(-\sqrt{dx+c}b+\sqrt{b^2c}\right)\right)\sinh\left(a+\sqrt{b^2c}\right)\\ &\quad-\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}b+\sqrt{b^2c}\right)+\mathrm{Ei}\left(-\sqrt{dx+c}b-\sqrt{b^2c}\right)\right)\sinh\left(-a+\sqrt{b^2c}\right)\end{aligned}$$

[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x,x, algorithm="fricas")

[Out] 1/2*(Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) - Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*cosh(a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) - Ei(-sqrt(d*x + c)*b - sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) + Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)) - 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) + Ei(-sqrt(d*x + c)*b - sqrt(b^2*c)))*sinh(-a + sqrt(b^2*c))

Sympy [F]

$$\int \frac{\sinh \left(a+b\sqrt{c+dx}\right)}{x}\,dx=\int \frac{\sinh \left(a+b\sqrt{c+dx}\right)}{x}\,dx$$

[In] integrate(sinh(a+b*(d*x+c)**(1/2))/x,x)

[Out] Integral(sinh(a + b*sqrt(c + d*x))/x, x)

Maxima [F]

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \int \frac{\sinh\left(\sqrt{dx+c}b+a\right)}{x}\,dx$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x,x, algorithm="maxima")
[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x, x)
```

Giac [F]

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \int \frac{\sinh\left(\sqrt{dx+c}b+a\right)}{x}\,dx$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x,x, algorithm="giac")
[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \int \frac{\sinh\left(a+b\,\sqrt{c+d\,x}\right)}{x}\,dx$$

```
[In] int(sinh(a + b*(c + d*x)^(1/2))/x,x)
[Out] int(sinh(a + b*(c + d*x)^(1/2))/x, x)
```

3.97 $\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 182

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \frac{bd\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}} - \frac{bd\cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} - \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - \frac{bd\sinh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}} - \frac{bd\sinh\left(a-b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}}$$

```
[Out] -sinh(a+b*(d*x+c)^(1/2))/x-1/2*b*d*Chi(b*(c^(1/2)+(d*x+c)^(1/2)))*cosh(a-b*
c^(1/2))/c^(1/2)+1/2*b*d*Chi(b*(c^(1/2)-(d*x+c)^(1/2)))*cosh(a+b*c^(1/2))/c
^(1/2)-1/2*b*d*Shi(b*(c^(1/2)+(d*x+c)^(1/2)))*sinh(a-b*c^(1/2))/c^(1/2)-1/2
*b*d*Shi(b*(c^(1/2)-(d*x+c)^(1/2)))*sinh(a+b*c^(1/2))/c^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {5472, 5396, 5389, 3384, 3379, 3382}

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \frac{bd\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}} - \frac{bd\cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} - \frac{bd\sinh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}} - \frac{bd\sinh\left(a-b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} - \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}$$

```
[In] Int[Sinh[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] (b*d*Cosh[a + b*Sqrt[c]]*CoshIntegral[b*(Sqrt[c] - Sqrt[c + d*x])])/(2*Sqrt
[c]) - (b*d*Cosh[a - b*Sqrt[c]]*CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])])/
(2*Sqrt[c]) - Sinh[a + b*Sqrt[c + d*x]]/x - (b*d*Sinh[a + b*Sqrt[c]]*SinhIn
tegral[b*(Sqrt[c] - Sqrt[c + d*x])])/(2*Sqrt[c]) - (b*d*Sinh[a - b*Sqrt[c]]
*SinhIntegral[b*(Sqrt[c] + Sqrt[c + d*x])])/(2*Sqrt[c])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5396

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5472

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\text{Subst}\left(\int \frac{\sinh\left(a+b\sqrt{x}\right)}{(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\text{Subst}\left(\int \frac{x\sinh(a+bx)}{(c-x^2)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - (bd)\text{Subst}\left(\int \frac{\cosh(a+bx)}{c-x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - (bd)\text{Subst}\left(\int \left(\frac{\cosh(a+bx)}{2\sqrt{c}\left(\sqrt{c}-x\right)}+\frac{\cosh(a+bx)}{2\sqrt{c}\left(\sqrt{c}+x\right)}\right)\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - \frac{(bd)\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}}\\
&\quad - \frac{(bd)\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}}\\
&= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - \frac{\left(bd\cosh\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}}\\
&\quad - \frac{\left(bd\cosh\left(a+b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(b\sqrt{c}-bx\right)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}}\\
&\quad - \frac{\left(bd\sinh\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}}\\
&\quad + \frac{\left(bd\sinh\left(a+b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(b\sqrt{c}-bx\right)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}}
\end{aligned}$$

$$= -\frac{bd\cosh\left(a-b\sqrt{c}\right)\operatorname{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} + \frac{bd\cosh\left(a+b\sqrt{c}\right)\operatorname{Chi}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}} - \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - \frac{bd\sinh\left(a-b\sqrt{c}\right)\operatorname{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} - \frac{bd\sinh\left(a+b\sqrt{c}\right)\operatorname{Shi}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}}$$

Mathematica [A] (verified)

Time = 3.59 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.12

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$$
$$= \frac{e^{-a}\left(2\sqrt{c}e^{-b\sqrt{c+dx}} - 2\sqrt{c}e^{2a+b\sqrt{c+dx}} + bde^{-b\sqrt{c}}x\operatorname{ExpIntegralEi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right) + bde^{2a+b\sqrt{c}}x\operatorname{ExpIntegr}\right.}{}$$

```
[In] Integrate[Sinh[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] ((2*Sqrt[c])/E^(b*Sqrt[c + d*x]) - 2*Sqrt[c]*E^(2*a + b*Sqrt[c + d*x]) + (b
*d*x*ExpIntegralEi[b*(Sqrt[c] - Sqrt[c + d*x])])/E^(b*Sqrt[c]) + b*d*E^(2*a
 + b*Sqrt[c])*x*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] - b*d*E^(b*Sqrt
[c])*x*ExpIntegralEi[-(b*(Sqrt[c] + Sqrt[c + d*x]))] - b*d*E^(2*a - b*Sqrt[
c])*x*ExpIntegralEi[b*(Sqrt[c] + Sqrt[c + d*x])])/(4*Sqrt[c]*E^a*x)
```

Maple [F]

$$\int \frac{\sinh\left(a+b\sqrt{dx+c}\right)}{x^2}dx$$

```
[In] int(sinh(a+b*(d*x+c)^(1/2))/x^2,x)
```

```
[Out] int(sinh(a+b*(d*x+c)^(1/2))/x^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(142) = 284.

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.73

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$$
$$= \frac{\left(\sqrt{b^2c}dx\mathrm{Ei}\left(\sqrt{dx+c}b-\sqrt{b^2c}\right)+\sqrt{b^2c}dx\mathrm{Ei}\left(-\sqrt{dx+c}b+\sqrt{b^2c}\right)\right)\cosh\left(a+\sqrt{b^2c}\right)-\left(\sqrt{b^2c}dx\mathrm{Ei}\left(\sqrt{dx\right.\right.}{}$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) + sqrt(b^2*c)*d*x*E
i(-sqrt(d*x + c)*b + sqrt(b^2*c)))*cosh(a + sqrt(b^2*c)) - (sqrt(b^2*c)*d*x
*Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) + sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b -
sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) - 4*c*sinh(sqrt(d*x + c)*b + a) + (sqr
t(b^2*c)*d*x*Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) - sqrt(b^2*c)*d*x*Ei(-sqrt(d
*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)) + (sqrt(b^2*c)*d*x*Ei(sqrt(
d*x + c)*b + sqrt(b^2*c)) - sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b - sqrt(b^2*
c)))*sinh(-a + sqrt(b^2*c)))/(c*x)
```

Sympy [F]

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$$

```
[In] integrate(sinh(a+b*(d*x+c)**(1/2))/x**2,x)
```

```
[Out] Integral(sinh(a + b*sqrt(c + d*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \int \frac{\sinh\left(\sqrt{dx+c}b+a\right)}{x^2}\,dx$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \int \frac{\sinh\left(\sqrt{dx+c}b+a\right)}{x^2}\,dx$$

[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="giac")

[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \int \frac{\sinh\left(a+b\,\sqrt{c+d\,x}\right)}{x^2}\,dx$$

[In] int(sinh(a + b*(c + d*x)^(1/2))/x^2,x)

[Out] int(sinh(a + b*(c + d*x)^(1/2))/x^2, x)

3.98 $\int x^2 \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 537

$$\begin{aligned}\int x^2\sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx=&\frac{120960\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3}+\frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}\\&-\frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}\\&+\frac{60480(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3}\\&+\frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}\\&-\frac{120c(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}\\&+\frac{5040(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}\\&-\frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}\\&+\frac{168(c+dx)^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}\\&+\frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}\\&+\frac{720c\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3}\\&-\frac{120960\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3}\\&-\frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}\\&+\frac{360c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}\\&-\frac{20160(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3}\\&+\frac{30c(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}\\&-\frac{1008(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}\\&-\frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}\end{aligned}$$

```
[Out] 120960*cosh(a+b*(d*x+c)^(1/3))/b^9/d^3+6*c^2*cosh(a+b*(d*x+c)^(1/3))/b^3/d^
3-720*c*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)^(1/3))/b^5/d^3+60480*(d*x+c)^(2/3)*c
osh(a+b*(d*x+c)^(1/3))/b^7/d^3+3*c^2*(d*x+c)^(2/3)*cosh(a+b*(d*x+c)^(1/3))/
b/d^3-120*c*(d*x+c)*cosh(a+b*(d*x+c)^(1/3))/b^3/d^3+5040*(d*x+c)^(4/3)*cosh
(a+b*(d*x+c)^(1/3))/b^5/d^3-6*c*(d*x+c)^(5/3)*cosh(a+b*(d*x+c)^(1/3))/b/d^3
+168*(d*x+c)^2*cosh(a+b*(d*x+c)^(1/3))/b^3/d^3+3*(d*x+c)^(8/3)*cosh(a+b*(d*
x+c)^(1/3))/b/d^3+720*c*sinh(a+b*(d*x+c)^(1/3))/b^6/d^3-120960*(d*x+c)^(1/3
)*sinh(a+b*(d*x+c)^(1/3))/b^8/d^3-6*c^2*(d*x+c)^(1/3)*sinh(a+b*(d*x+c)^(1/3
))/b^2/d^3+360*c*(d*x+c)^(2/3)*sinh(a+b*(d*x+c)^(1/3))/b^4/d^3-20160*(d*x+c
)*sinh(a+b*(d*x+c)^(1/3))/b^6/d^3+30*c*(d*x+c)^(4/3)*sinh(a+b*(d*x+c)^(1/3)
)/b^2/d^3-1008*(d*x+c)^(5/3)*sinh(a+b*(d*x+c)^(1/3))/b^4/d^3-24*(d*x+c)^(7/
3)*sinh(a+b*(d*x+c)^(1/3))/b^2/d^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{5472, 1607, 5394, 3377, 2718, 2717\}$

$$\int x^2 \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{120960\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} - \frac{120960\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} + \frac{60480(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} - \frac{20160(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{720c\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{5040(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{1008(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{360c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{168(c+dx)^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{120c(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{30c(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$

```
[In] Int[x^2*Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (120960*Cosh[a + b*(c + d*x)^(1/3)])/(b^9*d^3) + (6*c^2*Cosh[a + b*(c + d*x
)^(1/3)])/(b^3*d^3) - (720*c*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^(1/3)])/(
b^5*d^3) + (60480*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b^7*d^3) +
(3*c^2*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^3) - (120*c*(c + d
*x)*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^3) + (5040*(c + d*x)^(4/3)*Cosh[a +
 b*(c + d*x)^(1/3)])/(b^5*d^3) - (6*c*(c + d*x)^(5/3)*Cosh[a + b*(c + d*x)^
(1/3)])/(b*d^3) + (168*(c + d*x)^2*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^3) +
 (3*(c + d*x)^(8/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^3) + (720*c*Sinh[a +
b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x
)^(1/3)])/(b^8*d^3) - (6*c^2*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)])/(
b^2*d^3) + (360*c*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^4*d^3) -
(20160*(c + d*x)*Sinh[a + b*(c + d*x)^(1/3)])/(b^6*d^3) + (30*c*(c + d*x)^(
4/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2*d^3) - (1008*(c + d*x)^(5/3)*Sinh[a
+ b*(c + d*x)^(1/3)])/(b^4*d^3) - (24*(c + d*x)^(7/3)*Sinh[a + b*(c + d*x)^
(1/3)])/(b^2*d^3)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5394

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5472

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (-c+x)^2 \sinh\left(a+b\sqrt[3]{x}\right)\,dx, x, c+dx\right)}{d^3}\\
&= \frac{3\text{Subst}\left(\int \left(-cx+x^4\right)^2 \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}\\
&= \frac{3\text{Subst}\left(\int x^2\left(-c+x^3\right)^2 \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}\\
&= \frac{3\text{Subst}\left(\int \left(c^2x^2\sinh(a+bx)-2cx^5\sinh(a+bx)+x^8\sinh(a+bx)\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}\\
&= \frac{3\text{Subst}\left(\int x^8 \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}\\
&\quad - \frac{(6c)\text{Subst}\left(\int x^5 \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}\\
&\quad + \frac{\left(3c^2\right)\text{Subst}\left(\int x^2 \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}\\
&= \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}\\
&\quad + \frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{24\text{Subst}\left(\int x^7 \cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^3}\\
&\quad + \frac{(30c)\text{Subst}\left(\int x^4 \cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^3}\\
&\quad - \frac{\left(6c^2\right)\text{Subst}\left(\int x \cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^3}
\end{aligned}$$

$$= \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$+ \frac{30c(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$+ \frac{168\text{Subst}\left(\int x^6\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$- \frac{(120c)\text{Subst}\left(\int x^3\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$+ \frac{(6c^2)\,\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3}$$

$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$- \frac{120c(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{168(c+dx)^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$- \frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{30c(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$- \frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$- \frac{1008\text{Subst}\left(\int x^5\cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^3d^3}$$
$$+ \frac{(360c)\text{Subst}\left(\int x^2\cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^3d^3}$$

$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{120c(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{168(c+dx)^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{30c(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{1008(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{5040\text{Subst}\left(\int x^4\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{(720c)\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^4d^3}$$

$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{120c(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{5040(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{168(c+dx)^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{30c(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{1008(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{20160\text{Subst}\left(\int x^3\cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{(720c)\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^5d^3}$$

$$
\begin{aligned}
&= \frac{6c^2 \cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&+ \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{120c(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&+ \frac{5040(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&- \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{168(c+dx)^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&+ \frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{720c\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} \\
&- \frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} \\
&- \frac{20160(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{30c(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&- \frac{1008(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&+ \frac{60480\text{Subst}\left(\int x^2\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^6d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&\quad + \frac{60480(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&\quad - \frac{120c(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{5040(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&\quad - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{168(c+dx)^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&\quad + \frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{720c\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} \\
&\quad - \frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} \\
&\quad - \frac{20160(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{30c(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&\quad - \frac{1008(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&\quad - \frac{120960\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^7d^3}
\end{aligned}
$$

$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}$$
$$+ \frac{60480(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3}$$
$$+ \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{120c(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}$$
$$+ \frac{5040(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{168(c+dx)^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{720c\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{120960\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3}$$
$$- \frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}$$
$$- \frac{20160(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{30c(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$- \frac{1008(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$+ \frac{120960\text{Subst}\left(\int\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^8d^3}$$

$$= \frac{120960\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} + \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{60480(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{120c(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{5040(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{168(c+dx)^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{720c\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{120960\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} - \frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{20160(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{30c(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{1008(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.66

$$\int x^2\sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$= \frac{3e^{-a-b\sqrt[3]{c+dx}}\left(40320\left(1+e^{2\left(a+b\sqrt[3]{c+dx}\right)}\right) - 40320b\left(-1+e^{2\left(a+b\sqrt[3]{c+dx}\right)}\right)\sqrt[3]{c+dx} + 20160b^2\left(1+e^{2\left(a+b\sqrt[3]{c\cdots}\right.}\right.\right.}{}$$

```
[In] Integrate[x^2*Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*E^(-a - b*(c + d*x)^(1/3))*(40320*(1 + E^(2*(a + b*(c + d*x)^(1/3)))) -
40320*b*(-1 + E^(2*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(1/3) + 20160*b^2*(1
 + E^(2*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(2/3) + b^8*d^2*(1 + E^(2*(a +
b*(c + d*x)^(1/3))))*x^2*(c + d*x)^(2/3) - 2*b^7*d*(-1 + E^(2*(a + b*(c + d
*x)^(1/3))))*x*(c + d*x)^(1/3)*(3*c + 4*d*x) + 240*b^4*(1 + E^(2*(a + b*(c
+ d*x)^(1/3))))*(c + d*x)^(1/3)*(6*c + 7*d*x) - 24*b^5*(-1 + E^(2*(a + b*(c
 + d*x)^(1/3))))*(c + d*x)^(2/3)*(9*c + 14*d*x) - 240*b^3*(-1 + E^(2*(a + b
*(c + d*x)^(1/3))))*(27*c + 28*d*x) + 2*b^6*(1 + E^(2*(a + b*(c + d*x)^(1/3
))))*(9*c^2 + 36*c*d*x + 28*d^2*x^2)))/(2*b^9*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. 2(477) = 954.

Time = 2.39 (sec) , antiderivative size = 1815, normalized size of antiderivative = 3.38

method	result	size
derivativedivides	Expression too large to display	1815
default	Expression too large to display	1815
parts	Expression too large to display	2940

```
[In] int(x^2*sinh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^3/b^3*(-8/b^6*a^7*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b
*(d*x+c)^(1/3)))+28/b^6*a^6*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-
2*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3)))-56
/b^6*a^5*((a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-3*(a+b*(d*x+c)^(1/3
))^2*sinh(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-
6*sinh(a+b*(d*x+c)^(1/3)))+70/b^6*a^4*((a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+
c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(
1/3))^2*cosh(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/
3))+24*cosh(a+b*(d*x+c)^(1/3)))-56/b^6*a^3*((a+b*(d*x+c)^(1/3))^5*cosh(a+b*
(d*x+c)^(1/3))-5*(a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x
+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x
+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-120*sinh(a+b*(d*
x+c)^(1/3)))+2/b^3*a^5*c*cosh(a+b*(d*x+c)^(1/3))-20/b^3*c*a^2*((a+b*(d*x+c)
^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-3*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^
(1/3))+6*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-6*sinh(a+b*(d*x+c)^(1/
3)))+10/b^3*c*a*((a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3))-4*(a+b*(d*x+
c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+
c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+24*cosh(a+b*(d*x+c
)^(1/3)))-10/b^3*a^4*c*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-sinh(a+
b*(d*x+c)^(1/3)))+20/b^3*a^3*c*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3
))-2*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3)))
+28/b^6*a^2*((a+b*(d*x+c)^(1/3))^6*cosh(a+b*(d*x+c)^(1/3))-6*(a+b*(d*x+c)^(
1/3))^5*sinh(a+b*(d*x+c)^(1/3))+30*(a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(
1/3))-120*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+360*(a+b*(d*x+c)^(1
/3))^2*cosh(a+b*(d*x+c)^(1/3))-720*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/
3))+720*cosh(a+b*(d*x+c)^(1/3)))-8/b^6*a*((a+b*(d*x+c)^(1/3))^7*cosh(a+b*(d
*x+c)^(1/3))-7*(a+b*(d*x+c)^(1/3))^6*sinh(a+b*(d*x+c)^(1/3))+42*(a+b*(d*x+c
)^(1/3))^5*cosh(a+b*(d*x+c)^(1/3))-210*(a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*x+
c)^(1/3))+840*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-2520*(a+b*(d*x+
c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))+5040*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+
c)^(1/3))-5040*sinh(a+b*(d*x+c)^(1/3)))-2*c^2*a*((a+b*(d*x+c)^(1/3))*cosh(a
+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3)))-2/b^3*c*((a+b*(d*x+c)^(1/3))^5*c
osh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))+20*(
```

```
a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*sinh(
a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-120*sinh
(a+b*(d*x+c)^(1/3)))+c^2*a^2*cosh(a+b*(d*x+c)^(1/3))+c^2*((a+b*(d*x+c)^(1/3
))^2*cosh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+
2*cosh(a+b*(d*x+c)^(1/3)))+1/b^6*a^8*cosh(a+b*(d*x+c)^(1/3))+1/b^6*((a+b*(d
*x+c)^(1/3))^8*cosh(a+b*(d*x+c)^(1/3))-8*(a+b*(d*x+c)^(1/3))^7*sinh(a+b*(d*
x+c)^(1/3))+56*(a+b*(d*x+c)^(1/3))^6*cosh(a+b*(d*x+c)^(1/3))-336*(a+b*(d*x+
c)^(1/3))^5*sinh(a+b*(d*x+c)^(1/3))+1680*(a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*
x+c)^(1/3))-6720*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+20160*(a+b*(
d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-40320*(a+b*(d*x+c)^(1/3))*sinh(a+b*
(d*x+c)^(1/3))+40320*cosh(a+b*(d*x+c)^(1/3))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.34

$$\int x^2 \sinh\left(a + b\sqrt[3]{c+dx}\right) dx = \frac{3\left(\left(56\,b^6d^2x^2 + 72\,b^6cdx + 18\,b^6c^2 + (b^8d^2x^2 + 20160\,b^2)(dx+c)^{\frac{2}{3}} + 240\,(7\,b^4dx + 6\,b^4c)(dx+c)^{\frac{1}{3}} + 4032\right.\right.}{}$$

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((56*b^6*d^2*x^2 + 72*b^6*c*d*x + 18*b^6*c^2 + (b^8*d^2*x^2 + 20160*b^2)*
(d*x + c)^(2/3) + 240*(7*b^4*d*x + 6*b^4*c)*(d*x + c)^(1/3) + 40320)*cosh((
d*x + c)^(1/3)*b + a) - 2*(3360*b^3*d*x + 3240*b^3*c + 12*(14*b^5*d*x + 9*b
^5*c)*(d*x + c)^(2/3) + (4*b^7*d^2*x^2 + 3*b^7*c*d*x + 20160*b)*(d*x + c)^(
1/3))*sinh((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

Sympy [F]

$$\int x^2 \sinh\left(a + b\sqrt[3]{c+dx}\right) dx = \int x^2 \sinh\left(a + b\sqrt[3]{c+dx}\right) dx$$

```
[In] integrate(x**2*sinh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x**2*sinh(a + b*(c + d*x)**(1/3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.20

$$\int x^2 \sinh\left(a + b\sqrt[3]{c+dx}\right)\,dx$$
$$= \frac{2\,d^3x^3\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)+\left(\frac{c^3e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{b}-\frac{c^3e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b}-\frac{3\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\right.}{b^4}\right.}{}$$

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*d^3*x^3*sinh((d*x + c)^(1/3)*b + a) + (c^3*e^((d*x + c)^(1/3)*b + a)
/b - c^3*e^(-(d*x + c)^(1/3)*b - a)/b - 3*((d*x + c)*b^3*e^a - 3*(d*x + c)^
(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c^2*e^((d*x + c)^(1/3)*b)/
b^4 + 3*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*c
^2*e^(-(d*x + c)^(1/3)*b - a)/b^4 + 3*((d*x + c)^2*b^6*e^a - 6*(d*x + c)^(5
/3)*b^5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x
 + c)^(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*c*e^((d*x + c)^(
1/3)*b)/b^7 - 3*((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x + c)^(4/
3)*b^4 + 120*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x + c)^(1/3)*
b + 720)*c*e^(-(d*x + c)^(1/3)*b - a)/b^7 - ((d*x + c)^3*b^9*e^a - 9*(d*x +
 c)^(8/3)*b^8*e^a + 72*(d*x + c)^(7/3)*b^7*e^a - 504*(d*x + c)^2*b^6*e^a +
3024*(d*x + c)^(5/3)*b^5*e^a - 15120*(d*x + c)^(4/3)*b^4*e^a + 60480*(d*x +
 c)*b^3*e^a - 181440*(d*x + c)^(2/3)*b^2*e^a + 362880*(d*x + c)^(1/3)*b*e^a
 - 362880*e^a)*e^((d*x + c)^(1/3)*b)/b^10 + ((d*x + c)^3*b^9 + 9*(d*x + c)^
(8/3)*b^8 + 72*(d*x + c)^(7/3)*b^7 + 504*(d*x + c)^2*b^6 + 3024*(d*x + c)^(
5/3)*b^5 + 15120*(d*x + c)^(4/3)*b^4 + 60480*(d*x + c)*b^3 + 181440*(d*x +
c)^(2/3)*b^2 + 362880*(d*x + c)^(1/3)*b + 362880)*e^(-(d*x + c)^(1/3)*b - a
)/b^10)*b)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2162 vs. $2(477) = 954$.

Time = 0.33 (sec) , antiderivative size = 2162, normalized size of antiderivative = 4.03

$$\int x^2 \sinh\left(a + b\sqrt[3]{c+dx}\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*((((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*c
^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3)*
```

b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c)^(
1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*b^3*
c - 2*((d*x + c)^(1/3)*b + a)*b^6*c^2 + 2*a*b^6*c^2 + ((d*x + c)^(1/3)*b +
a)^8 - 8*((d*x + c)^(1/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 5
6*((d*x + c)^(1/3)*b + a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d
*x + c)^(1/3)*b + a)^3*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c
)^(1/3)*b + a)*a^7 + a^8 + 10*((d*x + c)^(1/3)*b + a)^4*b^3*c - 40*((d*x +
c)^(1/3)*b + a)^3*a*b^3*c + 60*((d*x + c)^(1/3)*b + a)^2*a^2*b^3*c - 40*((d
*x + c)^(1/3)*b + a)*a^3*b^3*c + 10*a^4*b^3*c + 2*b^6*c^2 - 8*((d*x + c)^(1
/3)*b + a)^7 + 56*((d*x + c)^(1/3)*b + a)^6*a - 168*((d*x + c)^(1/3)*b + a)
^5*a^2 + 280*((d*x + c)^(1/3)*b + a)^4*a^3 - 280*((d*x + c)^(1/3)*b + a)^3*
a^4 + 168*((d*x + c)^(1/3)*b + a)^2*a^5 - 56*((d*x + c)^(1/3)*b + a)*a^6 +
8*a^7 - 40*((d*x + c)^(1/3)*b + a)^3*b^3*c + 120*((d*x + c)^(1/3)*b + a)^2*
a*b^3*c - 120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c + 40*a^3*b^3*c + 56*((d*x +
 c)^(1/3)*b + a)^6 - 336*((d*x + c)^(1/3)*b + a)^5*a + 840*((d*x + c)^(1/3)
*b + a)^4*a^2 - 1120*((d*x + c)^(1/3)*b + a)^3*a^3 + 840*((d*x + c)^(1/3)*b
 + a)^2*a^4 - 336*((d*x + c)^(1/3)*b + a)*a^5 + 56*a^6 + 120*((d*x + c)^(1/
3)*b + a)^2*b^3*c - 240*((d*x + c)^(1/3)*b + a)*a*b^3*c + 120*a^2*b^3*c - 3
36*((d*x + c)^(1/3)*b + a)^5 + 1680*((d*x + c)^(1/3)*b + a)^4*a - 3360*((d*
x + c)^(1/3)*b + a)^3*a^2 + 3360*((d*x + c)^(1/3)*b + a)^2*a^3 - 1680*((d*x
 + c)^(1/3)*b + a)*a^4 + 336*a^5 - 240*((d*x + c)^(1/3)*b + a)*b^3*c + 240*
a*b^3*c + 1680*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3)*b + a)^3*a
 + 10080*((d*x + c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*b + a)*a^3 +
 1680*a^4 + 240*b^3*c - 6720*((d*x + c)^(1/3)*b + a)^3 + 20160*((d*x + c)^(
1/3)*b + a)^2*a - 20160*((d*x + c)^(1/3)*b + a)*a^2 + 6720*a^3 + 20160*((d*
x + c)^(1/3)*b + a)^2 - 40320*((d*x + c)^(1/3)*b + a)*a + 20160*a^2 - 40320
*(d*x + c)^(1/3)*b + 40320)*e^((d*x + c)^(1/3)*b + a)/(b^8*d^2) + (((d*x +
c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*c^2 + a^2*b^6*c
^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3)*b + a)^4*a*b^3
c - 20((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^2*
a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*b^3*c + 2*((d*x +
c)^(1/3)*b + a)*b^6*c^2 - 2*a*b^6*c^2 + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x
 + c)^(1/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(
1/3)*b + a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*
b + a)^3*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)
a^7 + a^8 - 10((d*x + c)^(1/3)*b + a)^4*b^3*c + 40*((d*x + c)^(1/3)*b + a
)^3*a*b^3*c - 60*((d*x + c)^(1/3)*b + a)^2*a^2*b^3*c + 40*((d*x + c)^(1/3)*
b + a)*a^3*b^3*c - 10*a^4*b^3*c + 2*b^6*c^2 + 8*((d*x + c)^(1/3)*b + a)^7 -
 56*((d*x + c)^(1/3)*b + a)^6*a + 168*((d*x + c)^(1/3)*b + a)^5*a^2 - 280*(
(d*x + c)^(1/3)*b + a)^4*a^3 + 280*((d*x + c)^(1/3)*b + a)^3*a^4 - 168*((d*
x + c)^(1/3)*b + a)^2*a^5 + 56*((d*x + c)^(1/3)*b + a)*a^6 - 8*a^7 - 40*((d
*x + c)^(1/3)*b + a)^3*b^3*c + 120*((d*x + c)^(1/3)*b + a)^2*a*b^3*c - 120*
((d*x + c)^(1/3)*b + a)*a^2*b^3*c + 40*a^3*b^3*c + 56*((d*x + c)^(1/3)*b +
a)^6 - 336*((d*x + c)^(1/3)*b + a)^5*a + 840*((d*x + c)^(1/3)*b + a)^4*a^2
- 1120*((d*x + c)^(1/3)*b + a)^3*a^3 + 840*((d*x + c)^(1/3)*b + a)^2*a^4 -

```
336*((d*x + c)^(1/3)*b + a)*a^5 + 56*a^6 - 120*((d*x + c)^(1/3)*b + a)^2*b^
3*c + 240*((d*x + c)^(1/3)*b + a)*a*b^3*c - 120*a^2*b^3*c + 336*((d*x + c)^
(1/3)*b + a)^5 - 1680*((d*x + c)^(1/3)*b + a)^4*a + 3360*((d*x + c)^(1/3)*b
 + a)^3*a^2 - 3360*((d*x + c)^(1/3)*b + a)^2*a^3 + 1680*((d*x + c)^(1/3)*b
+ a)*a^4 - 336*a^5 - 240*((d*x + c)^(1/3)*b + a)*b^3*c + 240*a*b^3*c + 1680
*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3)*b + a)^3*a + 10080*((d*x
 + c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*b + a)*a^3 + 1680*a^4 - 24
0*b^3*c + 6720*((d*x + c)^(1/3)*b + a)^3 - 20160*((d*x + c)^(1/3)*b + a)^2*
a + 20160*((d*x + c)^(1/3)*b + a)*a^2 - 6720*a^3 + 20160*((d*x + c)^(1/3)*b
 + a)^2 - 40320*((d*x + c)^(1/3)*b + a)*a + 20160*a^2 + 40320*(d*x + c)^(1/
3)*b + 40320)*e^(-(d*x + c)^(1/3)*b - a)/(b^8*d^2))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh\left(a + b\sqrt[3]{c + dx}\right) \, dx = \int x^2 \sinh\left(a + b\,(c + d\,x)^{1/3}\right) \, dx$$

```
[In] int(x^2*sinh(a + b*(c + d*x)^(1/3)),x)
[Out] int(x^2*sinh(a + b*(c + d*x)^(1/3)), x)
```

3.99 $\int x \sinh\left(a+b\sqrt[3]{c+dx}\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 261

$$\begin{aligned}\int x \sinh\left(a+b\sqrt[3]{c+dx}\right) dx = &-\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\ &+\frac{360\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} \\ &-\frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\ &+\frac{60(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\ &+\frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\ &-\frac{360\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2} \\ &+\frac{6c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} \\ &-\frac{180(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} \\ &-\frac{15(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}\end{aligned}$$

```
[Out] -6*c*cosh(a+b*(d*x+c)^(1/3))/b^3/d^2+360*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)^(1/
3))/b^5/d^2-3*c*(d*x+c)^(2/3)*cosh(a+b*(d*x+c)^(1/3))/b/d^2+60*(d*x+c)*cosh
(a+b*(d*x+c)^(1/3))/b^3/d^2+3*(d*x+c)^(5/3)*cosh(a+b*(d*x+c)^(1/3))/b/d^2-3
```

```
60*sinh(a+b*(d*x+c)^(1/3))/b^6/d^2+6*c*(d*x+c)^(1/3)*sinh(a+b*(d*x+c)^(1/3)
)/b^2/d^2-180*(d*x+c)^(2/3)*sinh(a+b*(d*x+c)^(1/3))/b^4/d^2-15*(d*x+c)^(4/3
)*sinh(a+b*(d*x+c)^(1/3))/b^2/d^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5472, 5394, 3377, 2718, 2717}

$$\int x\sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx=-\frac{360\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}+\frac{360\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2}-\frac{180(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}+\frac{60(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{15(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{6c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}-\frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}$$

```
[In] Int[x*Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-6*c*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (360*(c + d*x)^(1/3)*Cosh[a
+ b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*c*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)
^(1/3)])/(b*d^2) + (60*(c + d*x)*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (
3*(c + d*x)^(5/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^2) - (360*Sinh[a + b*(c
 + d*x)^(1/3)])/(b^6*d^2) + (6*c*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)
])/(b^2*d^2) - (180*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^4*d^2)
- (15*(c + d*x)^(4/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5394

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5472

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (-c+x)\sinh\left(a+b\sqrt[3]{x}\right)\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{3\text{Subst}\left(\int x^2(-c+x^3)\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^2}$$

$$= \frac{3\text{Subst}\left(\int \left(-cx^2\sinh(a+bx)+x^5\sinh(a+bx)\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d^2}$$

$$= \frac{3\text{Subst}\left(\int x^5\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^2} - \frac{(3c)\text{Subst}\left(\int x^2\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^2}$$

$$=-\frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}-\frac{15\text{Subst}\left(\int x^4\cosh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{bd^2}+\frac{(6c)\text{Subst}\left(\int x\cosh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{bd^2}$$

$$=-\frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{6c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{15(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{60\text{Subst}\left(\int x^3\sinh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{(6c)\text{Subst}\left(\int \sinh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^2d^2}$$

$$=-\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{60(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{6c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{15(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180\text{Subst}\left(\int x^2\cosh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^3d^2}$$

$$=-\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{60(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{6c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}-\frac{15(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{360\text{Subst}\left(\int x\sinh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^4d^2}$$

$$= -\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{360\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{6c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{180(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} - \frac{15(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{360\text{Subst}\left(\int\cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^5d^2}$$

$$= -\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{360\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{360\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2} + \frac{6c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{180(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} - \frac{15(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.45

$$\int x\sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3b\left(120\sqrt[3]{c+dx}+b^4dx(c+dx)^{2/3}+2b^2(9c+10dx)\right)\cosh\left(a+b\sqrt[3]{c+dx}\right)-3\left(120+60b^2(c+dx)^{2/3}+b\right.}{b^6d^2}$$

```
[In] Integrate[x*Sinh[a + b*(c + d*x)^(1/3)],x]
[Out] (3*b*(120*(c + d*x)^(1/3) + b^4*d*x*(c + d*x)^(2/3) + 2*b^2*(9*c + 10*d*x))
*Cosh[a + b*(c + d*x)^(1/3)] - 3*(120 + 60*b^2*(c + d*x)^(2/3) + b^4*(c + d
*x)^(1/3)*(3*c + 5*d*x))*Sinh[a + b*(c + d*x)^(1/3)])/(b^6*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(231) = 462$.

Time = 2.65 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.52

method	result
derivativedivides	$-\frac{3a^5\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)}{b^3}+\frac{15a^4\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)-\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)}{b^3}-\frac{30a^3\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\cosh\left(a\right.\right.}{}$
default	$-\frac{3a^5\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)}{b^3}+\frac{15a^4\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)-\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)}{b^3}-\frac{30a^3\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\cosh\left(a\right.\right.}{}$
parts	Expression too large to display

```
[In] int(x*sinh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^2/b^3*(-1/b^3*a^5*cosh(a+b*(d*x+c)^(1/3))+5/b^3*a^4*((a+b*(d*x+c)^(1/3)
)*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3)))-10/b^3*a^3*((a+b*(d*x+c)
^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1
/3))+2*cosh(a+b*(d*x+c)^(1/3)))+10/b^3*a^2*((a+b*(d*x+c)^(1/3))^3*cosh(a+b*
(d*x+c)^(1/3))-3*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+
c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-6*sinh(a+b*(d*x+c)^(1/3)))-5/b^3*a*((a+b*
(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(
d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x
+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+24*cosh(a+b*(d*x+c)^(1/3)))+1/b^3*((a+b*
(d*x+c)^(1/3))^5*cosh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x+c)^(1/3))^4*sinh(a+b*(
d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x
+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+
c)^(1/3))-120*sinh(a+b*(d*x+c)^(1/3)))-c*a^2*cosh(a+b*(d*x+c)^(1/3))+2*c*a*
((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3)))-c*((a
+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*sinh(a+b*
(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.42

$$\int x\sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{3\left(\left(\left((dx+c)^{\frac{2}{3}}b^5dx+20\,b^3dx+18\,b^3c+120\,(dx+c)^{\frac{1}{3}}b\right)\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(60\,(dx+c)^{\frac{2}{3}}b^2+(5\,b^4\right.\right.\right.}{b^6d^2}$$

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(((d*x + c)^(2/3)*b^5*d*x + 20*b^3*d*x + 18*b^3*c + 120*(d*x + c)^(1/3)*b
)*cosh((d*x + c)^(1/3)*b + a) - (60*(d*x + c)^(2/3)*b^2 + (5*b^4*d*x + 3*b^
4*c)*(d*x + c)^(1/3) + 120)*sinh((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

Sympy [F]

$$\int x\sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int x\sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

```
[In] integrate(x*sinh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x*sinh(a + b*(c + d*x)**(1/3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.42

$$\int x\sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$=\frac{2\,d^2x^2\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(\frac{c^2e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{b}-\frac{c^2e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b}-\frac{2\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\,e^{a}\cdots\right.}{b^4}\cdots\right.}{\cdots}$$

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d^2*x^2*sinh((d*x + c)^(1/3)*b + a) - (c^2*e^((d*x + c)^(1/3)*b + a)
/b - c^2*e^(-(d*x + c)^(1/3)*b - a)/b - 2*((d*x + c)*b^3*e^a - 3*(d*x + c)^
(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c*e^((d*x + c)^(1/3)*b)/b^
4 + 2*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*c*e
^(-(d*x + c)^(1/3)*b - a)/b^4 + ((d*x + c)^2*b^6*e^a - 6*(d*x + c)^(5/3)*b^
5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x + c)^
(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*e^((d*x + c)^(1/3)*b)/
b^7 - ((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x + c)^(4/3)*b^4 + 1
20*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x + c)^(1/3)*b + 720)*e
^(-(d*x + c)^(1/3)*b - a)/b^7)*b)/d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(231) = 462$.

Time = 0.31 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.70

$$\int x \sinh\left(a + b\sqrt[3]{c+dx}\right) dx =$$

$$-\frac{3\left(\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2 b^3c-2\left((dx+c)^{\frac{1}{3}}b+a\right)ab^3c+a^2b^3c-\left((dx+c)^{\frac{1}{3}}b+a\right)^5+5\left((dx+c)^{\frac{1}{3}}b+a\right)^4a-10\left((dx+c)^{\frac{1}{3}}b+a\right)^3a^2+10\left((dx+c)^{\frac{1}{3}}b+\cdots\right.\right.}{}\right.}{}$$

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3/2*(((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^3*c
+ a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a - 1
0*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*((d*
x + c)^(1/3)*b + a)*a^4 + a^5 - 2*((d*x + c)^(1/3)*b + a)*b^3*c + 2*a*b^3*c
 + 5*((d*x + c)^(1/3)*b + a)^4 - 20*((d*x + c)^(1/3)*b + a)^3*a + 30*((d*x
+ c)^(1/3)*b + a)^2*a^2 - 20*((d*x + c)^(1/3)*b + a)*a^3 + 5*a^4 + 2*b^3*c
- 20*((d*x + c)^(1/3)*b + a)^3 + 60*((d*x + c)^(1/3)*b + a)^2*a - 60*((d*x
+ c)^(1/3)*b + a)*a^2 + 20*a^3 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*((d*x +
 c)^(1/3)*b + a)*a + 60*a^2 - 120*(d*x + c)^(1/3)*b + 120)*e^((d*x + c)^(1/
3)*b + a)/(b^5*d) + (((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b
 + a)*a*b^3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*
b + a)^4*a - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^
2*a^3 - 5*((d*x + c)^(1/3)*b + a)*a^4 + a^5 + 2*((d*x + c)^(1/3)*b + a)*b^3
*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 + 20*((d*x + c)^(1/3)*b + a)^3
*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d*x + c)^(1/3)*b + a)*a^3 - 5*
a^4 + 2*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3 + 60*((d*x + c)^(1/3)*b + a)^2
*a - 60*((d*x + c)^(1/3)*b + a)*a^2 + 20*a^3 - 60*((d*x + c)^(1/3)*b + a)^2
 + 120*((d*x + c)^(1/3)*b + a)*a - 60*a^2 - 120*(d*x + c)^(1/3)*b - 120)*e^
(-(d*x + c)^(1/3)*b - a)/(b^5*d))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x \sinh\left(a + b\sqrt[3]{c+dx}\right) dx = \int x \sinh\left(a + b\,(c + d\,x)^{1/3}\right) dx$$

```
[In] int(x*sinh(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] int(x*sinh(a + b*(c + d*x)^(1/3)), x)
```

3.100 $\int \sinh \left(a+b\sqrt[3]{c+dx}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \sinh \left(a+b\sqrt[3]{c+dx}\right) \, dx = \frac{6\cosh \left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{3(c+dx)^{2/3}\cosh \left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\sqrt[3]{c+dx}\sinh \left(a+b\sqrt[3]{c+dx}\right)}{b^2d}$$

```
[Out] 6*cosh(a+b*(d*x+c)^(1/3))/b^3/d+3*(d*x+c)^(2/3)*cosh(a+b*(d*x+c)^(1/3))/b/d
-6*(d*x+c)^(1/3)*sinh(a+b*(d*x+c)^(1/3))/b^2/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5418, 5412, 3377, 2718}

$$\int \sinh \left(a+b\sqrt[3]{c+dx}\right) \, dx = \frac{6\cosh \left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{6\sqrt[3]{c+dx}\sinh \left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{3(c+dx)^{2/3}\cosh \left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[In] Int[Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d) + (3*(c + d*x)^(2/3)*Cosh[a + b*(c
+ d*x)^(1/3)])/(b*d) - (6*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2
*d)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5412

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Modul
e[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Sinh[c + d*x^(k*
n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] && I
ntegerQ[p]
```

Rule 5418

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sinh\left(a+b\sqrt[3]{x}\right)\,dx, x, c+dx\right)}{d}\\
&= \frac{3\text{Subst}\left(\int x^2\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\
&= \frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd}\\
&= \frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}\\
&\quad + \frac{6\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d}\\
&= \frac{6\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} + \frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd}\\
&\quad - \frac{6\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\left(2+b^2(c+dx)^{2/3}\right)\cosh\left(a+b\sqrt[3]{c+dx}\right)-6b\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}$$

```
[In] Integrate[Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*(2 + b^2*(c + d*x)^(2/3))*Cosh[a + b*(c + d*x)^(1/3)] - 6*b*(c + d*x)^(1
/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{3a^2\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)-\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)+3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\cosh\left(a+b\cdots\right.}{b^3d}$
default	$\frac{3a^2\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)-\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)+3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\cosh\left(a+b\cdots\right.}{b^3d}$

```
[In] int(sinh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d/b^3*(a^2*cosh(a+b*(d*x+c)^(1/3))-2*a*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x
+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3)))+(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^
(1/3))-2*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/
3)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{3\left(2\,(dx+c)^{\frac{1}{3}}b\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^2+2\right)\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*(2*(d*x + c)^(1/3)*b*sinh((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^2
+ 2)*cosh((d*x + c)^(1/3)*b + a))/(b^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\begin{cases} x\sinh(a) & \text{for } b=0 \wedge (b=0 \vee d \\ x\sinh\left(a+b\sqrt[3]{c}\right) & \text{for } d=0 \\ \frac{3(c+dx)^{\frac{2}{3}}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd}-\frac{6\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Piecewise((x*sinh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sinh(a + b*c**(
1/3)), Eq(d, 0)), (3*(c + d*x)**(2/3)*cosh(a + b*(c + d*x)**(1/3))/(b*d) -
6*(c + d*x)**(1/3)*sinh(a + b*(c + d*x)**(1/3))/(b**2*d) + 6*cosh(a + b*(c
+ d*x)**(1/3))/(b**3*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.61

$$\int \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx =$$
$$-\frac{b\left(\frac{\left((dx+c)b^3e^a-3(dx+c)^{\frac{2}{3}}b^2e^a+6(dx+c)^{\frac{1}{3}}be^a-6e^a\right)e^{\left((dx+c)^{\frac{1}{3}}b\right)}}{b^4}-\frac{\left((dx+c)b^3+3(dx+c)^{\frac{2}{3}}b^2+6(dx+c)^{\frac{1}{3}}b+6\right)e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b^4}\right) -}{2d}$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*((((d*x + c)*b^3*e^a - 3*(d*x + c)^(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)
*b*e^a - 6*e^a)*e^((d*x + c)^(1/3)*b)/b^4 - ((d*x + c)*b^3 + 3*(d*x + c)^(2
/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*e^(-(d*x + c)^(1/3)*b - a)/b^4) - 2*(d*x
 + c)*sinh((d*x + c)^(1/3)*b + a))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.51

$$\int \sinh \left(a+b\sqrt[3]{c+dx}\right) dx = \frac{3\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2-2\,(dx+c)^{\frac{1}{3}}b+2\right)e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{2\,b^3d} + \frac{3\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2+2\,(dx+c)^{\frac{1}{3}}b+2\right)e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{2\,b^3d}$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*(((d*x + c)^(1/3)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2*(d*x
 + c)^(1/3)*b + 2)*e^((d*x + c)^(1/3)*b + a)/(b^3*d) + 3/2*(((d*x + c)^(1/3
)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 + 2*(d*x + c)^(1/3)*b + 2)*e
^(-(d*x + c)^(1/3)*b - a)/(b^3*d)
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \sinh \left(a+b\sqrt[3]{c+dx}\right) dx = \frac{6\cosh\left(a+b\,(c+d\,x)^{1/3}\right)}{b^3\,d} + \frac{3\cosh\left(a+b\,(c+d\,x)^{1/3}\right)\,(c+d\,x)^{2/3}}{b\,d} - \frac{6\sinh\left(a+b\,(c+d\,x)^{1/3}\right)\,(c+d\,x)^{1/3}}{b^2\,d}$$

```
[In] int(sinh(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] (6*cosh(a + b*(c + d*x)^(1/3)))/(b^3*d) + (3*cosh(a + b*(c + d*x)^(1/3))*(c
 + d*x)^(2/3))/(b*d) - (6*sinh(a + b*(c + d*x)^(1/3))*(c + d*x)^(1/3))/(b^2
*d)
```

3.101 $\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 232

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \mathrm{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)\sinh\left(a+b\sqrt[3]{c}\right) + \mathrm{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right) + \mathrm{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right) - \cosh\left(a+b\sqrt[3]{c}\right)\mathrm{Shi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right) - \cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\mathrm{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right) + \cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\mathrm{Shi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)$$

```
[Out] -cosh(a+b*c^(1/3))*Shi(b*(c^(1/3)-(d*x+c)^(1/3)))-cosh(a+(-1)^(2/3)*b*c^(1/
3))*Shi(b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))+cosh(a-(-1)^(1/3)*b*c^(1/3))*
Shi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))+Chi(b*(c^(1/3)-(d*x+c)^(1/3)))*si
nh(a+b*c^(1/3))+Chi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*sinh(a-(-1)^(1/3)
*b*c^(1/3))+Chi(-b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*sinh(a+(-1)^(2/3)*b*
c^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used

$= \{5472, 5400, 3384, 3379, 3382\}$

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \sinh\left(a+b\sqrt[3]{c}\right)\operatorname{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right) + \sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\operatorname{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right) + \sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\operatorname{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right) - \cosh\left(a+b\sqrt[3]{c}\right)\operatorname{Shi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right) - \cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\operatorname{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right) + \cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\operatorname{Shi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)$$

```
[In] Int[Sinh[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))]*Sinh[a + b*c^(1/3)] + CoshInteg
ral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))]*Sinh[a - (-1)^(1/3)*b*c^(1/3)
] + CoshIntegral[-(b*((-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3)))]*Sinh[a + (-1)
^(2/3)*b*c^(1/3)] - Cosh[a + b*c^(1/3)]*SinhIntegral[b*(c^(1/3) - (c + d*x)
^(1/3))] - Cosh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3
) - (c + d*x)^(1/3))] + Cosh[a - (-1)^(1/3)*b*c^(1/3)]*SinhIntegral[b*((-1)
^(1/3)*c^(1/3) + (c + d*x)^(1/3))]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5472

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\sinh\left(a+b\sqrt[3]{x}\right)}{-c+x}\,dx, x, c+dx\right)$$

$$= 3\text{Subst}\left(\int \frac{x^2\sinh(a+bx)}{-c+x^3}\,dx, x, \sqrt[3]{c+dx}\right)$$

$$= 3\text{Subst}\left(\int\left(-\frac{\sinh(a+bx)}{3\left(\sqrt[3]{c}-x\right)}-\frac{\sinh(a+bx)}{3\left(-\sqrt[3]{-1}\sqrt[3]{c}-x\right)}-\frac{\sinh(a+bx)}{3\left((-1)^{2/3}\sqrt[3]{c}-x\right)}\right)dx, x, \sqrt[3]{c+dx}\right)$$

$$= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right) - \text{Subst}\left(\int \frac{\sinh(a+bx)}{-\sqrt[3]{-1}\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right) - \text{Subst}\left(\int \frac{\sinh(a+bx)}{(-1)^{2/3}\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)$$

$$= \cosh\left(a+b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{\sinh\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right) + \left(i\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\sin\left((-1)^{5/6}b\sqrt[3]{c}+ibx\right)}{-\sqrt[3]{-1}\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right) + \left(i\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\sqrt[6]{-1}b\sqrt[3]{c}+ibx\right)}{(-1)^{2/3}\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right) - \sinh\left(a+b\sqrt[3]{c}\right)\text{Sub}$$

$$= \text{Chi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\sinh\left(a+b\sqrt[3]{c}\right) + \text{Chi}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right) + \text{Chi}\left(-(-1)^{2/3}b\sqrt[3]{c} + b\sqrt[3]{c+dx}\right)\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right) - \cosh\left(a+b\sqrt[3]{c}\right)\text{Shi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right) - \cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Shi}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right) + \cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Shi}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c}\right.$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \frac{1}{2}\Big(-\text{RootSum}\Big[c-\#1^3\&,\cosh(a+b\#1)\text{Chi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big) - \text{Chi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big)\sinh(a+b\#1) - \cosh(a+b\#1)\text{Shi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big) + \sinh(a+b\#1)\text{Shi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big)\&\Big] + \text{RootSum}\Big[c-\#1^3\&,\cosh(a+b\#1)\text{Chi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big) + \text{Chi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big)\sinh(a+b\#1) + \cosh(a+b\#1)\text{Shi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big) + \sinh(a+b\#1)\text{Shi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big)\&\Big]\Big)$$

```
[In] Integrate[Sinh[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] (-RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[b*((c + d*x)^(1/3) - #1)
] - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a + b*#1] - Cosh[a + b*#1]*
SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b*#1]*SinhIntegral[b*((c
+ d*x)^(1/3) - #1)] & ] + RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[
b*((c + d*x)^(1/3) - #1)] + CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a +
 b*#1] + Cosh[a + b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b
*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] & ])/2
```

Maple [F]

$$\int \frac{\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)}{x}dx$$

```
[In] int(sinh(a+b*(d*x+c)^(1/3))/x,x)
```

```
[Out] int(sinh(a+b*(d*x+c)^(1/3))/x,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(182) = 364$.

Time = 0.26 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.17

$$
\begin{aligned}
\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = {} & -\frac{1}{2}\operatorname{Ei}\Bigg(-(dx+c)^{\frac{1}{3}}b \\
& \quad -\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\Bigg)\cosh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)-a\right) \\
& +\frac{1}{2}\operatorname{Ei}\Bigg((dx+c)^{\frac{1}{3}}b \\
& -\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\Bigg)\cosh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+a\right) \\
& -\frac{1}{2}\operatorname{Ei}\Bigg(-(dx+c)^{\frac{1}{3}}b \\
& \quad +\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\Bigg)\cosh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)+a\right) \\
& +\frac{1}{2}\operatorname{Ei}\Bigg((dx+c)^{\frac{1}{3}}b \\
& +\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\Bigg)\cosh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)-a\right) \\
& -\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b+\left(b^3c\right)^{\frac{1}{3}}\right)\cosh\left(a+\left(b^3c\right)^{\frac{1}{3}}\right) \\
& +\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b+\left(-b^3c\right)^{\frac{1}{3}}\right)\cosh\left(-a+\left(-b^3c\right)^{\frac{1}{3}}\right) \\
& -\frac{1}{2}\operatorname{Ei}\Bigg(-(dx+c)^{\frac{1}{3}}b \\
& \quad -\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\Bigg)\sinh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)-a\right) \\
& +\frac{1}{2}\operatorname{Ei}\Bigg((dx+c)^{\frac{1}{3}}b \\
& -\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\Bigg)\sinh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+a\right) \\
& +\frac{1}{2}\operatorname{Ei}\Bigg(-(dx+c)^{\frac{1}{3}}b \\
& \quad +\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\Bigg)\sinh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)+a\right) \\
& -\frac{1}{2}\operatorname{Ei}\Bigg((dx+c)^{\frac{1}{3}}b \\
& +\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\Bigg)\sinh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)-a\right) \\
& +\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b+\left(b^3c\right)^{\frac{1}{3}}\right)\sinh\left(a+\left(b^3c\right)^{\frac{1}{3}}\right) \\
& -\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b+\left(-b^3c\right)^{\frac{1}{3}}\right)\sinh\left(-a+\left(-b^3c\right)^{\frac{1}{3}}\right)
\end{aligned}
$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x,x, algorithm="fricas")
```

```
[Out] -1/2*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(b^
3*c)^(1/3)*(sqrt(-3) + 1) - a) + 1/2*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1
/3)*(sqrt(-3) + 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) - 1/2*Ei(-(
d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)^(1/3)
*(sqrt(-3) - 1) + a) + 1/2*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(
-3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) - 1/2*Ei(-(d*x + c)^(
1/3)*b + (b^3*c)^(1/3))*cosh(a + (b^3*c)^(1/3)) + 1/2*Ei((d*x + c)^(1/3)*b
+ (-b^3*c)^(1/3))*cosh(-a + (-b^3*c)^(1/3)) - 1/2*Ei(-(d*x + c)^(1/3)*b - 1
/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a)
 + 1/2*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(
-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) + 1/2*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)
^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) - 1/2*Ei(
(d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-b^3*c)^(1
/3)*(sqrt(-3) - 1) - a) + 1/2*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*sinh(a
 + (b^3*c)^(1/3)) - 1/2*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3))*sinh(-a + (-
b^3*c)^(1/3))
```

Sympy [F]

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx=\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$$

```
[In] integrate(sinh(a+b*(d*x+c)**(1/3))/x,x)
```

```
[Out] Integral(sinh(a + b*(c + d*x)**(1/3))/x, x)
```

Maxima [F]

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx=\int \frac{\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x, x)
```

Giac [F]

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \int \frac{\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x,x, algorithm="giac")
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \int \frac{\sinh\left(a+b\,(c+d\,x)^{1/3}\right)}{x}\,dx$$

```
[In] int(sinh(a + b*(c + d*x)^(1/3))/x,x)
[Out] int(sinh(a + b*(c + d*x)^(1/3))/x, x)
```

3.102 $\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 329

$$\begin{aligned}\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx =& \frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\operatorname{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\ &+ \frac{(-1)^{2/3}bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\operatorname{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\ &- \frac{\sqrt[3]{-1}bd\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\operatorname{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\ &- \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{bd\sinh\left(a+b\sqrt[3]{c}\right)\operatorname{Shi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\ &- \frac{(-1)^{2/3}bd\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\operatorname{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\ &- \frac{\sqrt[3]{-1}bd\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\operatorname{Shi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}\end{aligned}$$

```
[Out] 1/3*b*d*Chi(b*(c^(1/3)-(d*x+c)^(1/3)))*cosh(a+b*c^(1/3))/c^(2/3)-1/3*(-1)^(
1/3)*b*d*Chi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*cosh(a-(-1)^(1/3)*b*c^(1
/3))/c^(2/3)+1/3*(-1)^(2/3)*b*d*Chi(-b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*
cosh(a+(-1)^(2/3)*b*c^(1/3))/c^(2/3)-1/3*b*d*Shi(b*(c^(1/3)-(d*x+c)^(1/3)))
*sinh(a+b*c^(1/3))/c^(2/3)-1/3*(-1)^(1/3)*b*d*Shi(b*((-1)^(1/3)*c^(1/3)+(d*
x+c)^(1/3)))*sinh(a-(-1)^(1/3)*b*c^(1/3))/c^(2/3)-1/3*(-1)^(2/3)*b*d*Shi(b*
```

```
((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*sinh(a+(-1)^(2/3)*b*c^(1/3))/c^(2/3)-si
nh(a+b*(d*x+c)^(1/3))/x
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5472, 5396, 5389, 3384, 3379, 3382}

$$\begin{aligned}
&\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx \\
&= \frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\
&\quad+\frac{(-1)^{2/3}bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\
&\quad-\frac{\sqrt[3]{-1}bd\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\
&\quad-\frac{bd\sinh\left(a+b\sqrt[3]{c}\right)\text{Shi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\
&\quad-\frac{(-1)^{2/3}bd\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\
&\quad-\frac{\sqrt[3]{-1}bd\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Shi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}-\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}
\end{aligned}$$

```
[In] Int[Sinh[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] (b*d*Cosh[a + b*c^(1/3)]*CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))])/(3*c^
(2/3)) + ((-1)^(2/3)*b*d*Cosh[a + (-1)^(2/3)*b*c^(1/3)]*CoshIntegral[-(b*((
-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3)))])/(3*c^(2/3)) - ((-1)^(1/3)*b*d*Cosh[
a - (-1)^(1/3)*b*c^(1/3)]*CoshIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1
/3))])/(3*c^(2/3)) - Sinh[a + b*(c + d*x)^(1/3)]/x - (b*d*Sinh[a + b*c^(1/3
)]*SinhIntegral[b*(c^(1/3) - (c + d*x)^(1/3))])/(3*c^(2/3)) - ((-1)^(2/3)*b
*d*Sinh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3) - (c +
 d*x)^(1/3))])/(3*c^(2/3)) - ((-1)^(1/3)*b*d*Sinh[a - (-1)^(1/3)*b*c^(1/3)]
*SinhIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))])/(3*c^(2/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5396

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5472

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = d\text{Subst}\left(\int \frac{\sinh\left(a+b\sqrt[3]{x}\right)}{(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (3d)\text{Subst}\left(\int \frac{x^2\sinh(a+bx)}{\left(c-x^3\right)^2}\,dx, x, \sqrt[3]{c+dx}\right)$$

$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - (bd)\text{Subst}\left(\int \frac{\cosh(a+bx)}{c-x^3}\,dx, x, \sqrt[3]{c+dx}\right)$$

$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - (bd)\text{Subst}\left(\int\left(\frac{\cosh(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}-x\right)} + \frac{\cosh(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}+\sqrt[3]{-1}x\right)} + \frac{\cosh(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}-(-1)^{2/3}x\right)}\right)dx, x, \sqrt[3]{c+dx}\right)$$

$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{(bd)\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{(bd)\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt[3]{c}+\sqrt[3]{-1}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{(bd)\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt[3]{c}-(-1)^{2/3}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}}$$

$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{\left(bd\cosh\left(a+b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{\left(bd\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\cos\left((-1)^{5/6}b\sqrt[3]{c}+ibx\right)}{\sqrt[3]{c}-(-1)^{2/3}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{\left(bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\sqrt[6]{-1}b\sqrt[3]{c}+ibx\right)}{\sqrt[3]{c}+\sqrt[3]{-1}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{\left(bd\sinh\left(a+b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{\left(ibd\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\sin\left((-1)^{5/6}b\sqrt[3]{c}+ibx\right)}{\sqrt[3]{c}-(-1)^{2/3}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{\left(ibd\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\sqrt[6]{-1}b\sqrt[3]{c}+ibx\right)}{\sqrt[3]{c}+\sqrt[3]{-1}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}}$$

$$= \frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{\sqrt[3]{-1}bd\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Chi}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{(-1)^{2/3}bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Chi}\left(-(-1)^{2/3}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{bd\sinh\left(a+b\sqrt[3]{c}\right)\text{Shi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{(-1)^{2/3}bd\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Shi}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{\sqrt[3]{-1}bd\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Shi}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)}{3c^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 2.86 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.64

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$$

$$= \frac{e^{-a}\left(3e^{-b\sqrt[3]{c+dx}} - 3e^{2a+b\sqrt[3]{c+dx}} + bdx\text{RootSum}\left[c-\#1^3\&, \frac{e^{2a+b\#1}\,\text{ExpIntegralEi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right] + bdx\right.}{}$$

```
[In] Integrate[Sinh[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] (3/E^(b*(c + d*x)^(1/3)) - 3*E^(2*a + b*(c + d*x)^(1/3)) + b*d*x*RootSum[c
- #1^3 & , (E^(2*a + b*#1)*ExpIntegralEi[b*((c + d*x)^(1/3) - #1)])/#1^2 &
] + b*d*x*RootSum[c - #1^3 & , (Cosh[b*#1]*CoshIntegral[b*((c + d*x)^(1/3)
- #1)] - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[b*#1] - Cosh[b*#1]*Sin
hIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[b*#1]*SinhIntegral[b*((c + d*x)^
(1/3) - #1)])/#1^2 & ])/(6*E^a*x)
```

Maple [F]

$$\int \frac{\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)}{x^2}dx$$

[In] int(sinh(a+b*(d*x+c)^(1/3))/x^2,x)

[Out] int(sinh(a+b*(d*x+c)^(1/3))/x^2,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. $2(245) = 490$.

Time = 0.27 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.14

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx = \text{Too large to display}$$

[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="fricas")

[Out] 1/12*(2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*cosh(a + (b^3*c)^(1/3)) - 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3))*cosh(-a + (-b^3*c)^(1/3)) - 2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*sinh(a + (b^3*c)^(1/3)) + 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3))*sinh(-a + (-b^3*c)^(1/3)) - (b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) + (-b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) + (b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) - (-b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) - (b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) + (-b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) - (b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) + (-b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) - 12*c*sinh((d*x + c)^(1/3)*b + a))/(c*x)

Sympy [F]

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$$

```
[In] integrate(sinh(a+b*(d*x+c)**(1/3))/x**2,x)
[Out] Integral(sinh(a + b*(c + d*x)**(1/3))/x**2, x)
```

Maxima [F]

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)}{x^2}\,dx$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="maxima")
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)}{x^2}\,dx$$

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="giac")
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\sinh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{x^2}\,dx$$

```
[In] int(sinh(a + b*(c + d*x)^(1/3))/x^2,x)
[Out] int(sinh(a + b*(c + d*x)^(1/3))/x^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                          convert(ExpnType_result,string)," vs. order ",
                          convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```